THE INFLUENCE OF SPIN-LATTICE COUPLING ON AN EXCITATION IN TRIANGULAR LATTICE HEISENBERG ANTIFERROMAGNET (TLHA) SPIN WAVE EXCITATIONS

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Using the Green function method the elementary excitations in noncolleniar antiferromagnets of the triangular lattice have been investigated. The paper consists of two sections. In the first section the ground state is derived for the case when the angle between projections of magnetic moments of sublattices on a plane is equal to 120°. Then the elementary excitation spectrum in this systems is calculated.

Recently, many theoretical and experimental studies of triangular lattice Heisenberg antiferromagnets have been carried out. As it is known below the Neel point the neighbouring spins in the triangular lattice are arranged in the same crystal plane at 120° between each other. From Fig.1, it is evident that such arrangement of the spins in the ground state corresponds to three sublattices forming 120°angle, that leads to a $\sqrt{3}\times\sqrt{3}$ periodicity. An excitation of the spin wave in TLHA with an easy plane and an axis was previously discussed by Nishimory and Miyake [1,2] and Suzuki and Natsume [3,4].

The paper consists of two sections. The first section is devoted to the calculation of spin wave spectrum. An analytical expression for spin-wave spectrum with $k_z \neq 0$ has been found. So far, the spin waves for such type of systems has been studied numerically [3]. In this case the spin keeps to be in the (zx)-plane due to the single-ion type anisotropy with easy axis (z-axis). The behaviour of this mode under the influence of skin-lattice coupling is considered in the second section.

Spin Wave

We start with the Hamiltonian

$$H=\sum_{\substack{a=1,2\\ b=2,3}}\sum_{\substack{j'_a,j'_b\\ a\neq b}} J(R_{j'_a j'_b})\vec{S}_{j'_a}\vec{S}_{j'_b}-D\sum_{j'_a}\left(S^{z'}_{j'_a}\right)^2 \quad (1)$$

where $S_{j'_1};S_{j'_2};S_{j'_3}$ are the spin operators for the sublattices 1,2 and 3, respectively. Here the primed coordinate system (x',y',z') is defined by the crystal axes. The easy axis single -ion anisotropy field is the z'-directed.

The equilibrium orientation of the spins is shown in figure 1.

We now define three unprimed coordinate systems (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) by taking the z_1, z_2 and z_3 directions to be the equilibrium directions of the spins on the sublattices 1,2 and 3, respectively. The transformation from the primed coordinate system to the unprimed systems is accomplished by clockwise rotations around the y'-axis in the $(z'x')$ plane of 0°, θ and φ, respectively.

Using the relationships between the new and the old coordinate systems, we define the spin components for the three sublattices in terms of the magnon creation and annihilation operators $a^+_k, a_k; b^+_k, b_k$ and c^+_k, c_k, which obey the usual boson commutation relations [5].

Thus

$$S^x_{j_1}=\sqrt{\frac{SN}{6}}\sum_k e^{i\vec{k}\vec{R}_{j}}(a_k+a^+_k);$$

$$S^y_{j_1}=-i\sqrt{\frac{SN}{6}}\sum_k e^{i\vec{k}\vec{R}_{j1}}(a_k-a^+_k)$$

$$S^x_{j_2}=\sqrt{\frac{SN}{6}}\sum_k e^{i\vec{k}\vec{R}_{j_2}}(b_k+b^+_k);$$

$$S^y_{j_2}=-i\sqrt{\frac{SN}{6}}\sum_k e^{i\vec{k}\vec{R}_{j_2}}(b_k-b^+_k)$$

$$S^x_{j_3}=\sqrt{\frac{SN}{6}}\sum_k e^{i\vec{k}\vec{R}_{j_3}}(c_k+c^+_k); \quad (2)$$

$$S^y_{j_3}=-i\sqrt{\frac{SN}{6}}e^{i\vec{k}\vec{R}_{j3}}(c_k-c^+_k)$$

$$S^z_{j_1}=S-\frac{N}{3}\sum_{kk'}e^{i(\vec{k}-\vec{k}')\vec{R}_{j_1}}a^+_k a_{k'};$$

$$S^z_{j_2}=S-\frac{N}{3}\sum_{kk'}e^{-i(\vec{k}-\vec{k}')\vec{R}_{j_2}}b^+_k b_{k'}$$

$$S^z_{j_3}=S-\frac{N}{3}\sum_{kk'}e^{-i(\vec{k}-\vec{k}')\vec{R}_{j_3}}c^+_k c_{k'},$$

where N is the total number of magnetic atoms, R_j is the position of the spin j and S is the magnitude of the spin vector.

Note that the number of magnetic atoms belonging to a single sublattice is $\frac{N}{3}$. Substituting the expressions (2) into (1) we represent the Hamiltonian in the form [6]

$$H=H_0+H_1+H_2+\ldots \quad (3)$$

where H_0 denotes the non-interacting term, while H_1 and H_2 denote the linear and quadratic terms for magnon operators, respectively

$$H_0=\frac{N}{3}S^2\left\{2Jz[\cos\theta+\cos\varphi+\cos(\varphi-\theta)]+D(1+\cos^2\theta+\cos^2\varphi)\right\}$$

$$H_1=\sqrt{\frac{NS}{6}}2Jz\{(\sin\theta+\sin\varphi)(a_0+a_0^+)-[\sin\theta-\sin(\varphi-\theta)](b_0+b_0^+)-$$
$$-[\sin(\varphi-\theta)+\sin\varphi](c_0+c_0^+)\}+D[\sin 2\theta(b_0+b_0^+)+\sin 2\varphi(c_0+c_0^+)] \qquad (4)$$
$$H_2=\sum_k\{\varepsilon_{11}a_k^+a_k+\varepsilon_{22}b_k^+b_k+\varepsilon_{33}c_k^+c_k+M_{12}^-(\lambda_k a_k\underline{b}_k+\underline{\lambda}_k a_k^+\underline{b}_k^+)+M_{23}^-(\lambda_k b_k\underline{c}_k+$$
$$\underline{+\lambda}_k b_k^+\underline{c}_k^+)+M_{13}^-(\lambda_k c_k\underline{a}_k^++\underline{\lambda}_k c_k^+\underline{a}_k^+)+M_{12}^+(\lambda_k a_k b_k^++\underline{\lambda}_k a_k^+b_k)+M_{23}^+(\lambda_k b_k c_k^++$$
$$+\underline{\lambda}_k b_k^+c_k)+M_{13}^+(\lambda_k c_k a_k^++\underline{\lambda}_k c_k^+a_k)+D_{12}(b_k\underline{b}_k+b_k^+\underline{b}_k^+)+D_{13}(c_k\underline{c}_k+c_k^+\underline{c}_k^+)\}$$

where

$$\varepsilon_{11}=2S[-Jz(\cos\theta+\cos\varphi)+D];$$
$$M_{12}^{\pm}=JS\cdot z(\cos\theta\pm 1)$$
$$\varepsilon_{22}=2S\{-Jz[\cos\theta+\cos(\varphi-\theta)]+$$
$$+D(3\cos^2\theta-1)\};M_{13}^{\pm}=JS\cdot z(\cos\varphi\pm 1) \qquad (5)$$
$$\varepsilon_{33}=2S\{-Jz[\cos\varphi+\cos(\varphi-\theta)]+$$
$$+D(3\cos^2\varphi-1)\};M_{23}^{\pm}=JS\cdot z[\cos(\varphi-\theta)\pm 1]$$
$$2D_{12}=-DS\cdot\sin^2\theta\;;\;2D_{13}=-DS\cdot\sin^2\varphi\;;$$
$$\lambda_{\pm k}=z^{-1}\sum_\Delta e^{\pm ik\Delta}$$

where $\vec{\Delta}=R_i-R_j$ (shown in fig.1).

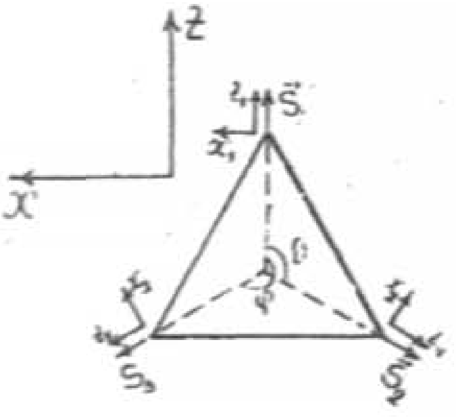

Fig.1. The equilibrium spin configuration for the 120° structure. Upon the inclusion of a *z*-directed anisotropy field *D*.

The ground state of the investigation magnetic system obtainable by the standard method of differentiation H_0 in the polar coordinate

$$\frac{\partial H_0}{\partial\theta}=2Jz[\sin\theta-\sin(\varphi-\theta)]-D\cdot\sin 2\theta=0$$
$$\frac{\partial H_0}{\partial\varphi}=2Jz[\sin(\varphi-\theta)+\sin\varphi]-D\cdot\sin 2\varphi=0 \qquad (6)$$

where *z* is the number of the nearest neighbours.

Hence the exact solutions of these equations can be obtained, but it is practically very complicated due to the cumbrous calculations. To avoid the difficulty we have to perform an approximation with respect to θ and φ.

The result is

$$1.\ \sin\theta=0\;;\;\cos\theta=-\frac{J\cdot z}{2(J\cdot z-D)}\quad\text{at}\quad\varphi=-\theta$$
$$(7)$$
$$2.\ \sin\theta=0\;;\;\cos\theta=-\frac{J\cdot z}{2(J\cdot z+D)}\quad\text{at}\quad\varphi=2\theta$$

In the second solution in (7) we neglect terms which contain $\cos^2\theta$ and its higher degrees. It is easy to see that in each case we have two solutions. They correspond to the ground states where in the first case the spins are collinear, while in the second case the spins are to keep 120° -structure. Further we shall consider the second case.

Now we define the law of the spin wave dispersion in the spin system with the 120°-structure. This is conveniently calculated in terms of thermodynamical two-time Green functions, which, in turn, may be obtained from their equations of motion. The required theory of the Green functions is given in monography by S.V. Tyablikov [7].

We introduce the following Green functions are defined by

$$G_{j_1l_1}^{I}(t-t')=\langle\langle S_{j_1}^x(t)/S_{l_1}^y(t')\rangle\rangle;$$
$$G_{j_1l_1}^{II}(t-t')=\langle\langle S_{j_1}^y(t)/S_{l_1}^y(t')\rangle\rangle;$$
$$G_{j_2l_1}^{III}(t-t')=\langle\langle S_{j_2}^x(t)/S_{l_1}^y(t')\rangle\rangle;$$
$$G_{j_2l_1}^{IV}(t-t')=\langle\langle S_{j_2}^y(t)/S_{l_1}^y(t')\rangle\rangle; \qquad (8)$$
$$G_{j_3l_1}^{V}(t-t')=\langle\langle S_{j_3}^x(t)/S_{l_1}^y(t')\rangle\rangle;$$
$$G_{j_3l_1}^{VI}(t-t')=\langle\langle S_{j_3}^y(t)/S_{l_1}^y(t')\rangle\rangle;$$

For convenience, we introduce S_{ik}^{α} $(\alpha=x,y;i=1,2,3)$ as a linear combination of the magnon operators $a_k\pm\underline{a}_k^+$; $b_k\pm\underline{b}_k^+$ and $c_k\pm\underline{c}_k^+$. Therefore, their equation of motion, based on the Hamiltonian H_2, in momentum space is following

$$
\begin{aligned}
-EG_k^{I} &= S + \varepsilon_{11} G_k^{II} + (M_{12}^{+} - M_{12}^{-})\underline{\lambda}_k G_k^{IV} + (M_{13}^{+} - M_{13}^{-})\lambda_k G_k^{VI} \\
-EG_k^{II} &= \varepsilon_{11} G_k^{I} + (M_{12}^{+} + M_{12}^{-})\underline{\lambda}_k G_k^{III} + (M_{13}^{+} + M_{13}^{-})\lambda_k G_k^{V} \\
-EG_k^{III} &= (\varepsilon_{22} - 2D_{12})\, G_k^{IV} + (M_{12}^{+} - M_{12}^{-})\lambda_k G_k^{II} + \\
&\quad + (M_{23}^{+} - M_{23}^{-})\underline{\lambda}_k G_k^{VI} \\
-EG_k^{IV} &= (\varepsilon_{22} + 2D_{12})\, G_k^{III} + (M_{12}^{+} + M_{12}^{-})\lambda_k G_k^{I} + \\
&\quad + (M_{23}^{+} + M_{23}^{-})\underline{\lambda}_k G_k^{V} \\
-EG_k^{V} &= (\varepsilon_{33} - 2D_{13})\, G_k^{VI} + (M_{23}^{+} - M_{23}^{-})\lambda_k G_k^{IV} + \\
&\quad + (M_{13}^{+} - M_{13}^{-})\underline{\lambda}_k G_k^{II} \\
-EG_k^{VI} &= (\varepsilon_{33} + 2D_{13})\, G_k^{V} + (M_{23}^{+} + M_{23}^{-})\lambda_k G_k^{III} + \\
&\quad + (M_{13}^{+} + M_{13}^{-})\underline{\lambda}_k G_k^{I}
\end{aligned}
\qquad (9)
$$

$$M_{12}^{\pm} = zJS\,(\cos\theta \pm 1)\,;$$

$$M_{13}^{\pm} = zJS\,(\cos\varphi \pm 1)\,;$$

$$M_{23}^{\pm} = zJS\,[\cos(\varphi - \theta) \pm 1]\,;$$

Note, that these sets of equations can not be solved exactly for any value of k. Therefore, we shall perform an approximation with respect to k.After this approximation λ_{k}, takes the form

$$\lambda_{k_x} = \underline{\lambda}_{k_x} \frac{1}{3}\left(1 + 2\cos\frac{1}{2}ak_x\right) \qquad \text{at} \qquad k = k_x$$

Note also, the sum is carried out Δ_1, Δ_2 or Δ_3 in figure 1 at obtained λ_{k_x}.

Using this, the solutions of the equ.(9) can be rewritten as follows

$$\left\{-E^2 + \omega_{1S}^2(k_x)\right\}\left\{E^4 - \gamma_1^2(k_x)\,E^2 + \gamma_2^2(k_x)\right\} = 0 \qquad (10)$$

where

$$
\begin{aligned}
\omega_{1S}^2(k_x) &= \left[2JS\cdot z\left(1 - \lambda_{k_x}\right) + DS/2\right]\times \\
&\quad \times\left[2JS\cdot z\left(1 + \lambda_{k_x}/2\right) - DS\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_1^2(k_x) &= 2\left\{(JSz)^2\left(1 - \lambda_{k_x}\right)\left(4 + 5\lambda_{k_x}\right) + \frac{DS}{4}\times\right. \\
&\quad \left.\times\left\{JS\cdot z\left(17 - 2\lambda_{k_x}\right) + 2DS\right\}\right\} \\
\gamma_2^2(k_x) &= 2\left\{(JSz)^2\left(1 - \lambda_{k_x}\right)\left(2 + \lambda_{k_x}\right) + \frac{DS}{4}\times\right. \\
&\quad \left.\times\left\{JS\cdot z\left(7 - 4\lambda_{k_x}\right) - DS\right\}\right\}\times \\
&\quad \times\left\{(2JSz)^2\left(1 - \lambda_{k_x}\right)\left(1 + 2\lambda_{k_x}\right) + DS\times\right. \\
&\quad \left.\times\left\{JS\cdot z\left(5 + 4\lambda_{k_x}\right) + DS\right\}\right\}
\end{aligned}
\qquad (11)
$$

The results of the solutions (7) at the terms of expression (11) have been also used. At $k_x = 0$ the roots of the equation (10) lead to the following expression

$$E_1^2 = \omega_{1S}^2\left(k_x = 0\right) = \frac{DS}{2}(3JS\cdot z - DS) \qquad (12)$$

$$E_{2,3}^2 \approx \frac{DS}{4}(15JS\cdot z + 2DS) \pm \frac{DS}{4}(3JS\cdot z + 18DS)$$

We note that the obtained second and third terms, terms type $-39\,(DS)^3\,(3JS\cdot z + 18DS)^{-1}$ and its higher degrees are neglected.

The spin wave excitation energy for this triangular structure has been calculated in the paper [3] at $k=0$. However, it is not very accurate calculations (one of the solutions as to E^2 is zero, but in our calculation all three solutions differ from zero).

We note that the solutions of the equ. (9) may be obtained in another way. If to apply the group theory to our spin system, we may construct a unitary matrix. With the help of this matrix it may be factorized the determinant constructing from coefficient of the Green functions in (9). As a result, we obtain the factorized determinant as in (10). We also note that construction of the unitary matrix in (9) based on irreducible representations of the D_3 group [8,9].

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QEYRİ-KOLLİNEAR ÜÇ ALTQƏFƏSLİ HEYZENBERQ ANTİFERROMAQNİTLƏRİNDƏ ELEMENTAR OYANMALARA SPİN-FONON QARŞILIQLI TƏ'SİRİ

Baxılan maqnit sistemlərinnin əsas halı tədqiq edilmişdir. Mə'lum olmuşdur ki, belə sistemlər özünə məxsus maqnit nizamlanmasına malik olurlar. Belə ki, yarımqəfəslərin maqnit momentlərinin hər hansı kristallik müstəvi üzərində alınmış proyeksiya-

ları arasındakı bucaqlar 120° olur. Belə sistemlərdə oyanan spin dalğalarının dispersiya qanunu Qrin funksiya üsulu ilə tapılmışdır.

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ВЛИЯНИЕ СПИН-ФОНОННОГО ВЗАИМОДЕЙСТВИЯ НА ЭЛЕМЕНТАРНЫЕ ВОЗБУЖДЕНИЯ В НЕКОЛЛИНЕАРНЫХ ТРЕХПОДРЕШЕТОЧНЫХ АНТИФЕРРОМАГНЕТИКАХ

Методом функции Грина исследованы элементарные возбуждения в неколлинеарных трехподрешеточных антиферромагнетиках. Работа состоит из двух частей. В первой части определено основное состояние, для которого угол между проекциями магнитных моментов подрешеток на одной из кристаллических плоскостей составляет 120°. Во второй части вычислен спектр элементарного возбуждения в таких системах.

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