THE SCREENED POTENTIAL OF THE IMPURITY IN THE THIN SEMICONDUCTOR FILM WITH TWO-DIMENSIONAL CORRELATION OF THE IMPURITY IONS

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The screening of the potential of the point charge in the thin semiconductor film with completely compensated impurities is calculated. The film is considered to be prepared by the vacuum evaporation method with the thermal treatment.

The expression of the screened potential is found and it is shown that for the large distances the screening essentially differs from that of the three-dimensional Debye-Huccel case.

The screening of the potential of the point charge in the semiconductor film with the impurities essentially depends on the method of preparation of the film. In the case of the film cut out from the massive layered crystal, the screening is conditioned by the three-dimensional Debye-Huccel correlation between the impurity ions. Such a correlation existing in the melt, from which the crystal is stretching out, is frozen during the process of crystallization and it leads to the changing of the potential of the point charge in the film [1].

But if the semiconductor film is prepared by the vacuum evaporation method with the thermal treatment, the correlation of the impurity ions feels the difference between the potential of the point charge in the film [2] and the Coulomb potential. In fact, if the rate of diffusion of the impurity ions during the thermal treatment process is sufficient for the establishment of the equilibrium, then their space distribution becomes correlated and remains the same one after the film has been cooled. In this situation the difference between the dielectric properties of the film and the environment leads to the changing of the character of correlation connected with the charged impurities interaction. The screening of the point charge's potential in the case have to be different from the three-dimensional Debye-Huccel screening and the aim of our paper is to demonstrate that.

We consider the completely compensated semiconductor film with impurities, in which the average concentrations of positive and negative ions are equal $\bar{n}_+ = \bar{n}_- = n$.

Let the thickness of the film is d, dielectric constant is ε and dielectric constants of the environment are ε_1 and ε_2 out of two boundaries of the film respectively, z-axis is perpendicular to the film boundaries and the origin of coordinates is chosen to be in the centre of the film.

The potential of the point charge e located on the z-axis in the point $z'\left(|z'| \le \dfrac{d}{2}\right)$ can be evaluated from the Poisson equation taking into account the screening inside the film and using the conditions of the continuation of the potential and z-components of the induction vector on the two boundaries of the film.

The Poisson equation inside the film for the Fourier-component of the potential $\varphi_{\vec{k}}(z,z') = \int \varphi(\vec{\rho},z,z')\, e^{-i\vec{k}\vec{\rho}} d^2\vec{\rho}$, where $\vec{k}$ and $\vec{\rho}$ are two-dimensional vectors in the film plane, in the linear screening approximation has the following from:

$$\frac{d^2\varphi_{\vec{k}}(z,z')}{dz^2} - (k^2+æ^2)\varphi_{\vec{k}}(z,z') = -\frac{4\pi e}{\varepsilon}\delta(z-z') \quad (1)$$

Here $æ = \sqrt{\dfrac{8\pi n e^2}{\varepsilon T}}$ is usual Debye-Huccel expression of the verse screening radius and T is the temperature of thermal treatment.

The solution of equation (1) satisfying the boundary conditions is as follows

$$\varphi_{\vec{k}}(z,z') = \frac{4\pi e}{\varepsilon\tilde{k}\, sh(\tilde{\eta}_1+\tilde{\eta}_2+\tilde{k}d)} \cdot \begin{cases} ch\left[\tilde{k}\left(\frac{d}{2}+z\right)+\tilde{\eta}_1\right]\cdot ch\left[\tilde{k}\left(\frac{d}{2}-z'\right)+\tilde{\eta}_2\right], & z \le z' \\ ch\left[\tilde{k}\left(\frac{d}{2}+z'\right)+\tilde{\eta}_1\right]\cdot ch\left[\tilde{k}\left(\frac{d}{2}-z\right)+\tilde{\eta}_2\right], & z \ge z' \end{cases} \quad (2)$$

where
$$\tilde{k} = \sqrt{k^2+æ^2}, \quad (3)$$

and
$$\tilde{\eta}_{1,2} = \frac{1}{2}\ln\frac{\varepsilon\tilde{k}+\varepsilon_{1,2}k}{\varepsilon\tilde{k}-\varepsilon_{1,2}k}. \quad (4)$$

One can see from the expression (2) that the Fourier-component of the potential depends on value of the vector $\vec{k}$. Therefore, we can write for the potential itself:

$$\varphi(\rho,z,z') = \frac{1}{(2\pi)^2}\int \varphi_{\vec{k}}(z,z')\, e^{i\vec{k}\vec{\rho}} d^2\vec{k} = \frac{1}{2\pi}\int_0^\infty \varphi_k(z,z')\cdot J_0(k\rho)\cdot k\cdot dk, \quad (5)$$

where we have integrated over the polar angle and obtained the Bessel function $J_0(k\rho)$. Due to strong oscillations of $J_0(k\rho)$ - function at large values of it's argument, the small values of k $(kd \ll 1)$ are essential ones in the integral over k at large distances $\rho \gg d$. If the Debye-Huccel radius is much larger than the film thickness, that is $æd \ll 1$, then $kd \ll 1$ and one can write for $\varphi_k(z,z')$ the following approximate expression:

$$\varphi_k(z,z') \approx \frac{4\pi e}{\varepsilon \tilde{k}} \cdot \frac{ch\tilde{\eta}_1 \cdot ch\tilde{\eta}_2}{sh(\tilde{\eta}_1+\tilde{\eta}_2)+\tilde{k}d \cdot ch(\tilde{\eta}_1+\tilde{\eta}_2)} \quad (6)$$

In the most interesting case, when the dielectric constant of the film ε is much larger than the dielectric constant of the environment $\varepsilon_{1,2} \ll \varepsilon$

$$\tilde{\eta}_{1,2} \approx \frac{k}{\tilde{k}} \cdot \frac{\varepsilon_{1,2}}{\varepsilon} << 1 \quad (7)$$

and (6) can be simplified as follows

$$\varphi_k(z,z') \approx \frac{4\pi e}{\varepsilon} \cdot \frac{1}{k^2 d + k\dfrac{\varepsilon_1+\varepsilon_2}{\varepsilon} + æ^2 d} \quad (8)$$

Thus, we have in our approximation:

$$\varphi(\rho) \approx \frac{2e}{\varepsilon d}\int_0^\infty \frac{J_0(\alpha x)\cdot x \cdot dx}{x^2+\beta x+1} \equiv \frac{2e}{\varepsilon d} A(\alpha,\beta) \quad (9)$$

where $\alpha = æ\rho$, $\beta = \dfrac{\varepsilon_1+\varepsilon_2}{\varepsilon \cdot æ \cdot d}$

Let us find the asymptotic behaviour of the integral $A(\alpha,\beta)$, when $\alpha \gg 1$ or at the distances $\rho \gg 1/æ$. We consider two cases ($\beta \ll 1$ and $\beta \gg 1$).

1. $\beta \ll 1$.

$$A(\alpha,\beta) \approx A(\alpha,0) - \alpha\beta\left[B(\alpha) + \frac{\alpha}{2}\frac{d}{d\alpha}B(\alpha)\right], \quad (10)$$

where

$$A(\alpha,0) = \int_0^\infty \frac{J_0(\alpha x)\cdot x\cdot dx}{x^2+1},\ B(\alpha) \equiv \int_0^\infty \frac{J_0(\alpha x)\cdot dx}{x^2+1}. \quad (11)$$

$A(\alpha,0)$ is the McDonald function $K_0(\alpha)$ [3], which asymptotic expression at large α has the following form:

$$K_0(\alpha) \approx \sqrt{\frac{\pi}{2\alpha}} \cdot e^{-\alpha} \quad (12)$$

and

$$B(\alpha) = \frac{\pi}{2\alpha}\left[I_0(\alpha) - L_0(\alpha)\right] \quad (13)$$

Using the asymptotic expressions of the Bessel function of imaginary argument $I_0(\alpha)$ and the Struve function $L_0(\alpha)$ [3] we obtain:

$$B(\alpha) \sim \sum_{m=0}^{\infty}\left[(2m-1)!!\right]^2 \cdot \alpha^{-2(m+1)} \quad (14)$$

Thus

$$A(\alpha,\beta) \sim K_0(\alpha) + \frac{\beta}{\alpha}\sum_{m=0}^{\infty} m\cdot\left[(2m-1)!!\right]^2 \cdot \alpha^{-2m} \quad (15)$$

or using (12) and taking into consideration in (15) only the first member of the series we obtain for the potential (9):

$$\varphi(\rho) \approx \frac{2e}{\varepsilon d}\left[\sqrt{\frac{\pi}{2æ\rho}} \cdot e^{-æ\rho} + \frac{\varepsilon_1+\varepsilon_2}{\varepsilon æ d} \cdot \frac{1}{(æ\rho)^3}\right] \quad (16)$$

When ρ increases for the first time the exponential member plays the main role because of the small value of the parameters β. In this case the screening radius coincides with the three-dimensional screening radius $1/æ$. Then, at the largest values of ρ the cubic member becomes essential one and there is no screening radius in this case.

It have to be pointed out, that only the exponential member for the potential screened by the degenerate electron gas has been obtained in the paper [4]. And because of the formal analogy between the problem considered in [4] and our problem, one can consider the formula (16) as more precise one than the result of [4].

2. $\beta \gg 1$.

This case corresponds to a very small values of the film thickness in comparison with the Debye-Huccel radius ($æd \ll \frac{\varepsilon_1+\varepsilon_2}{\varepsilon} \ll 1$). At $\alpha \gg 1$ such values $x \le 1$ give the main contribution to the integral (9). Therefore, for $\beta \gg 1$ we can neglect x^2 in the denominator of the integral and then we have [5]:

$$A(\alpha,\beta) \approx \int_0^\infty \frac{J_0(\alpha x)\cdot x\cdot dx}{\beta x+1} = \frac{1}{\beta}\left\{\frac{1}{\alpha} - \frac{\pi}{2\beta}\left[H_0\left(\frac{\alpha}{\beta}\right) - N_0\left(\frac{\alpha}{\beta}\right)\right]\right\} \quad (17)$$

where H_0 is the Struve and N_0 - Neumann functions. Using the asymptotic expressions of these functions at the large values of their arguments and their expressions at small values of argument [4], we have

$$A(\alpha,\beta) \approx \frac{1}{\alpha\beta} \cdot \begin{cases} \dfrac{\beta^2}{\alpha^2}, & \alpha >> \beta \\ 1, & \alpha << \beta \end{cases} \quad (18)$$

For the potential we obtain in this case

$$\varphi(\rho) \approx \frac{2e}{\varepsilon_1+\varepsilon_2}\cdot\frac{1}{\rho}\cdot\begin{cases} \left(\frac{\varepsilon_1+\varepsilon_2}{\varepsilon æ d}\right)^2 \cdot \frac{1}{æ^2\rho^2} & , æ\rho >> \frac{\varepsilon_1+\varepsilon_2}{\varepsilon æ d} >> 1 \\ 1 & , \frac{\varepsilon_1+\varepsilon_2}{\varepsilon æ d} >> æ\rho >> 1 \end{cases} \tag{19}$$

Formulas (16) and (19) correspond to the general principle according to which the screened potential in two-dimensional system at large distances must decrease as $1/\rho$.

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İKİÖLÇÜLÜ KORRELYASİYALI AŞQAR İONLARI OLAN NAZİK YARIMKEÇİRİCİ TƏBƏQƏDƏ AŞQARLARIN EKRANLAŞMIŞ POTENSİALI

Tam kompensasiya olunmuş aşqarlı nazik yarımkeçirici təbəqədə nöqtəvi yükün ekranlaşması hesablanmışdır. Hesab olunur ki, təbəqə "buxarlandırma-közərmə" metodu ilə hazırlanmışdır. Ekranlaşdırılmış potensialın ifadəsi tapılmış və onun böyük məsafələrdə üçölçülü Debay-Hyukkel ekranlaşdırmasından əsaslı şəkildə fərqlənməsi göstərilmişdir.

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ЭКРАНИРОВАННЫЙ ПОТЕНЦИАЛ ПРИМЕСИ В ТОНКОЙ ПОЛУПРОВОДНИКОВОЙ ПЛЕНКЕ С ДВУМЕРНОЙ КОРРЕЛЯЦИЕЙ ПРИМЕСНЫХ ИОНОВ.

Рассчитана экранировка точечного заряда в тонкой полупроводниковой пленке с полностью компенсированными примесями. Считается, что пленка приготовлена методом напыления с последующим отжигом. Найдено выражение экранированного потенциала и показано, что на больших расстояниях экранировка существенно отличается от трехмерной Дебай-Хюккелевской экранировки.

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