SPIN WAVES IN A MAGNETIC SUPERLATTICE

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A superlattice consisting of alternating layers of two simple-cubic Heisenberg antiferromagnets is considered. The dispersion equation for spin waves propagating in the superlattice is derived by the transfer-matrix method. The result is illustrated numerically.

Magnetic multilayer structures have been the subject of growing interest in recent years. Excitation spectra, some fundamental properties of periodic magnetic superstructures or magnetic superlattices have been analysed theoretically in many special cases. For example, detailed theoretical calculations for magnetic multilayers are discussed in Refs. [1-3].

In this paper we consider a simple model of a superlattice, namely alternating simple-cubic antiferromagnets in which the interfaces are (001) planes. Only the nearest-neighbour exchange interaction is taken into account. Our purpose is to derive a general dispersion relation for the spin-wave propagating in the direction perpendicular to the layers by the transfer-matrix method. This theoretical approach is analogous to one from the Ref. [1], where simple-cubic ferromagnets are considered.

Let us describe the geometry of the system shown in Fig.1. The elementary unit of a superlattice consists of atomic layers n_1 of material 1 and atomic layers n_2 of material 2. Both materials are taken to be simple-cubic Heisenberg antiferromagnets having the following bulk parameters: the exchange constants J_i, Lande factors g_i, spins S_i $(i=1,2)$, anisotropy fields $H_i^{(A)}$ and lattice constant a. The exchange constant between constituents is J. We define $m=n_1+n_2$ such that the periodic distance is $D=ma$. Cell l is defined to run from $(l-1)ma+a$ to lma.

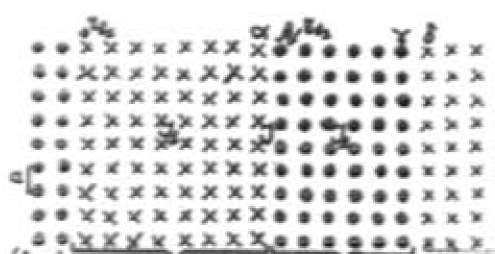

Fig.1. A superlattice model consisting of alternating layers of two simple-cubic Heisenberg antiferromagnetics.

The Hamiltonian of the system can be written in the form

$$H=\sum_{n,k}\sum_{\delta_{11}}J_{n,n}(S_{n,k}S_{n,k+\delta_{11}})+\sum_{n,k}J_{n,n\pm 1}(S_{n,k}S_{n\pm 1,k})-$$
$$-\sum_{n,k}g_n\mu H_n^{(A)}\cdot\left(S_{n,k}^{z(a)}-S_{n,k}^{z(b)}\right)-\mu H_0\sum_{n,k}g_n\left\{S_{n,k}^{z(a)}+S_{n,k}^{z(b)}\right\}, \qquad (1)$$

where the first term describes the intra-atomic layer exchange interactions, the second term describes the inter-atomic layer exchange interactions and the last one includes the Zeeman and magnetic anisotropy energies. n is the index of atomic layer and k describes the position of a lattice site in this layer, the summation over δ_{11} denotes the summation over in-plane nearest neighbours. External static magnetic field H_0 is assumed to be parallel to the axis z.

The dispersion equation for a bulk spin-wave is found within random-phase-approximation (RPA) from the equation of motion for the operator $S_i^+=S_i^x+iS_i^y$. Using the equation of motion, one finds the following system of equations corresponding to sublattices A and B:

$$\begin{cases}\lambda_n^a S_{n,k\uparrow}^+ - J_{n,n}\sum_{\delta_{11}}S_{n,k\downarrow+\delta_{11}}^+ - J_{n,n\pm 1}S_{n\pm 1,k\downarrow}^+ = 0 \ , \\ \lambda_n^b S_{n,k\downarrow}^+ - J_{n,n}\sum_{\delta_{11}}S_{n,k\uparrow+\delta_{11}}^+ - J_{n,n\pm 1}S_{n\pm 1,k\uparrow}^+ = 0 \ , \end{cases} \qquad (2)$$

where

$$\begin{cases}\lambda_n^a=\left(\hbar\omega-\mu g_n\left(H_0+H_n^{(A)}\right)\right)/S_n^a+J_{n,n}z_{11}S_n^b/S_n^a+J_{n,n\pm 1}S_{n\pm 1}^b/S_n^a \ , \\ \lambda_n^b=\left(\hbar\omega-\mu g_n\left(H_0-H_n^{(A)}\right)\right)/S_n^b+J_{n,n}z_{11}S_n^a/S_n^b+J_{n,n\pm 1}S_{n\pm 1}^a/S_n^b \ . \end{cases} \qquad (3)$$

Here the RPA $\langle S_{n,k\uparrow}^z\rangle=S_n^a$, $\langle S_{n,k\downarrow}^z\rangle=S_n^b$ and the general solution $S_{n,k\downarrow}^+ \, a\exp(ikr-iwt)$ have been done, also z_{11} is the number of nearest neighbours in the atomic layer ($z_{11}=4$ in the considered case).

In the particular case, after passing to the momentum representation, one can find the bulk spin-wave dispersion curve in medium 1 [4] by using the equations (2) and (3):

$$\hbar\omega=g_1\mu H_0+g_1\mu H_E^{(1)}\left(\left(1+\frac{H_1^{(A)}}{H_E^{(1)}}\right)^2-\gamma_k^2\right)^{1/2} ; J_1Sz=g_1\mu H_E^{(1)}, \qquad (4)$$

$$S_n^a=-S_n^b=S \, ,$$

γ_k is defined us

$$\gamma_k=\gamma_{-k}=\frac{1}{z}\sum_{\delta}e^{ik\delta} \ ,$$

$\delta=\{\delta,\delta_1\}$ is the position vector, which joins the atom under consideration with its nearest neighbour.

We now consider the superlattice and find the spin-wave dispersion equations solving the equations of motion for the spin operators S_i^+ in the RPA. A spin, which isn't in an interface layer, labelled by α, β, γ, δ in

Fig.1, has the same nearest-neighbour environment. Therefore, the spin-wave amplitudes must be given within each component by a linear combination of the positive- and negative- going solutions for the bulk medium:

$$S^{+}_{n,a1} = \left\{A_1^a \exp\left[ik_1\left(r-r_{1_1}\right)\right] + B_1^a \exp\left[-ik_1\left(r-r_{1_1}\right)\right]\right\}\exp(-i\omega t) \quad , \qquad (5)$$
$$S^{+}_{n,b1} = \left\{A_1^b \exp\left[ik_1\left(r-r_{1_1}\right)\right] + B_1^b \exp\left[-ik_1\left(r-r_{1_1}\right)\right]\right\}\exp(-i\omega t) \quad ,$$

in component 1, cell *1*;

$$S^{+}_{n,a2} = \left\{C_1^a \exp\left[ik_2\left(r-r_{1_2}\right)\right] + D_1^a \exp\left[-ik_2\left(r-r_{1_2}\right)\right]\right\}\exp(-i\omega t) \quad , \qquad (6)$$
$$S^{+}_{n,b2} = \left\{C_1^b \exp\left[ik_2\left(r-r_{1_2}\right)\right] + D_1^b \exp\left[-ik_2\left(r-r_{1_2}\right)\right]\right\}\exp(-i\omega t) \quad ,$$

in component 2, cell *1*.

The phase factors $exp(\pm ik_1 r_{1_1})$ and etc. are inserted for the algebraic convenience. Here r_{1_1}, r_{1_2} are the positions of the left-hand layers of the corresponding components in cell *1*. To derive the dispersion equation we use the transfer-matrix method. Solving Eq.(2) for layers α and β, and also γ and δ, we obtain the matrix relations between amplitudes in *A* sublattices

$$H\begin{pmatrix} A_n^a \\ B_n^a \end{pmatrix} = K'\begin{pmatrix} C_n^a \\ D_n^a \end{pmatrix} \quad , \qquad (7)$$

and

$$K\begin{pmatrix} A_{n+1}^a \\ B_{n+1}^a \end{pmatrix} = H'\begin{pmatrix} C_n^a \\ D_n^a \end{pmatrix} \quad , \qquad (8)$$

where the explicit forms of the matrices are given in Appendix. These equations combine to yield a transfer-matrix T^a

$$\begin{pmatrix} A_{n+1}^a \\ B_{n+1}^a \end{pmatrix} = K^{-1}H'K'^{-1}H\begin{pmatrix} A_n^a \\ B_n^a \end{pmatrix} = T^a\begin{pmatrix} A_n^a \\ B_n^a \end{pmatrix} \quad . \qquad (9)$$

The condition of solvability of Eq.(2) gives the following expression

$$\frac{\lambda_1^a}{\lambda_1^b} = \frac{\lambda_2^a}{\lambda_2^b} \quad \text{or} \quad \omega_1^2 = \omega_2^2, \quad \text{where:} \qquad (10)$$
$$\omega_1 = \frac{A_n^b}{A_n^a} = \frac{B_n^b}{B_n^a} \quad , \quad \omega_2 = \frac{C_n^b}{C_n^a} = \frac{D_n^b}{D_n^a} \quad .$$

One can show that the transfer-matrix T^b is the same, i.e. $T^a = T^b$, then we can write

$$\begin{pmatrix} A_{n+1}^a \\ B_{n+1}^a \\ A_{n+1}^b \\ B_{n+1}^b \end{pmatrix} = \begin{pmatrix} T^a & 0 \\ 0 & T^b \end{pmatrix}\begin{pmatrix} A_n^a \\ B_n^a \\ A_n^b \\ B_n^b \end{pmatrix} = T\begin{pmatrix} A_n^a \\ B_n^a \\ A_n^b \\ B_n^b \end{pmatrix} \quad . \qquad (11)$$

The matrix T has the property that its determinant is equal to unity. According to Bloch's theorem for a periodic structure, eigenvalues of T should be $exp(\pm iQD)$, where Q is the normal component of the wave-vector describing the propagation along the axis of the *SL*. The dispersion equation for the spin waves in the *SL* can be written in the following form:

$$\cos QD = \frac{1}{2}(T + T^{-1}) = \frac{1}{2}\mathrm{Tr}T^a \quad . \qquad (12)$$

The expression for the elements of the transfer matrices may be defined. We only give in the Appendix the explicit real form of the dispersion equation for the case when k_1 and k_2 are real.

It can be verified from equation A(8) that when both media are identical, $J_1 = J_2 = J$, $d_1 = d_2$ and $k_1 = k_2$, the dispersion equation reduces to

$$\cos QD = \cos 2kd = \cos^2 kd - \sin^2 kd \qquad (13)$$

Equation (12) being the main result of this paper describes the propagation of the spin-waves in the considered magnetic superlattice. For simple numerical illustration, we choose the case of normal incidence, $k_x = k_y = 0$, $k = k_z \neq 0$. The bulk spin-wave dispersion curves of the component media 1 and 2 for a particular choice of parameters are shown in Fig. 2(a), while Fig. 2(b) shows the spin-wave dispersion curve of the superlattice. The dispersion curves are drawn in the frequency range $1 < \hbar\omega / g\mu H_0 < 3$. In the frequency range where the both wave-vectors $k_i\ (i=1,2)$ are real, the dispersion curve of the superlattice exhibits broad pass bands and narrow-stop bands. When at least one of the wave-vectors is complex, the pass bands are narrow and the stop bands are broad. The dispersion curves for both the bulk and the superlattice spin-waves move up with increasing anisotropy field.

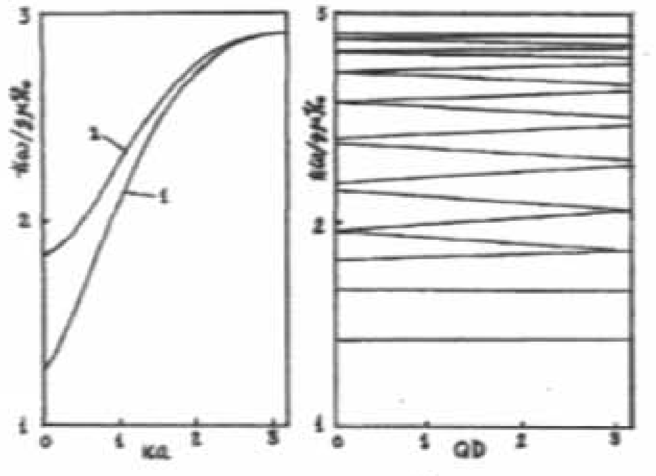

Fig.2. (a) Bulk and (b) superlattice spin-waves dispersion graphs for (001) propagation with parameters $J/J_1 = 3/2;\ J/J_2 = 3/4;\ H_1^{(A)}/H_E^{(1)} = 0.01;$ $H_2^{(A)}/H_E^{(2)} = 0.1;\ H_0/H_E^{(1)} = 0.5;\ H_0/H_E^{(2)} = 0.55.$

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APPENDIX

The matrices appearing in equations (7) and (8) are

$$H=\begin{pmatrix}\bar{f}_1 t_1 v_1 & f_1\bar{t}_1\bar{v}_1\\ \bar{f}_1 t_1 J & f_1\bar{t}_1 J\end{pmatrix}\quad, \qquad A(1)$$

$$H'=\begin{pmatrix}\bar{f}_2 t_2 v_2 & f_2\bar{t}_2\bar{v}_2\\ \bar{f}_2 t_2 J & f_2\bar{t}_2 J\end{pmatrix}\quad, \qquad A(2)$$

$$K=\begin{pmatrix}J & J\\ \bar{v}_1 & v_1\end{pmatrix}\quad, \qquad A(3)$$

$$K'=\begin{pmatrix}J & J\\ \bar{v}_2 & v_2\end{pmatrix}\quad, \qquad A(4)$$

where

$$f_i=e^{ik_i a}\ ,\ \bar{f}_i=1/f_i\ ,\ i=1,2$$
$$t_i=e^{in_i k_i a}\ ,\ \bar{t}_i=1/t_i\ ,\ i=1,2 \qquad A(5)$$

$$v_i=J_i(P_i+f_i)\ ,\ P_i=\sqrt{x_i^a x_i^b}-z\gamma_{k_i}\ , \qquad A(6)$$

We can find the simpler forms for P_i using Eq.(3) and (4)

$$P_i=\frac{1}{J_i}\sqrt{\left(\frac{J}{J_i}-1\right)^2+2Z\left(\frac{J}{J_i}-1\right)\left(1+\frac{H_i^{(A)}}{H_E^{(i)}}\right)+Z^2\gamma_{k_i}^2}-Z\gamma_{k_i}\cdot \qquad A(7)$$

For the case when k_1 and k_2 are both real, equation (12) can be written in the explicit form

$$\cos QD=\frac{1}{2J_1J_2J^2\sin k_1 a\,\sin k_2 a}\cdot Re\,(T_{11}^{a})\ , \qquad A(8)$$

with

$$\begin{aligned}Re\,(T_{11}^{a})&=J_1^2J_2^2\left[P_1^2\sin a_1k_1+2p_1\sin d_1k_1+\sin\bar{a}_1k_1\right]\cdot\\ &\cdot\left[P_2^2\sin a_2k_2+2p_2\sin d_2k_2+\sin\bar{a}_2k_2\right]-2J^2J_1J_2\cdot\\ &\cdot\left[P_1\sin a_1k_1+\sin d_1k_1\right]\left[P_2\sin a_2k_2+\sin d_2k_2\right]+\\ &+J^4\sin a_1k_1\sin a_2k_2\ .\end{aligned} \qquad A(9)$$

$$a_i=d_i-a\ ,\ i=1,2\ ;$$
$$\bar{a}_i=d_i+a\ ,\ i=1,2\ . \qquad A(10)$$

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İFRAT MAQNİT QƏFƏSLƏRİNDƏ SPİN DALĞALARI

İki sadə kubik Heyzenberq antiferromaqnetikdən təşkil olunmuş ifrat qəfəs halına baxılmışdır. Çevirmə-matrisa metodun-dan istifadə edərək belə ifrat qəfəsdə yayılan spin dalğaları üçün dispersiya tənliyi alınmışdır. Nəticə kəmiyyətcə qiymətləndirilmişdir.

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СПИНОВЫЕ ВОЛНЫ В МАГНИТНЫХ СВЕРХРЕШЕТКАХ

Рассматривается сверхрешетка, состоящая из чередующихся слоев двух простых кубических Гейзенберговских антиферромагнетиков. С помощью метода трансфер-матрицы выводится дисперсионное уравнение, описывающее распространение спиновых волн в сверхрешетке. Результаты иллюстрируются количественно.

Редактор: Р.Р. Гусейнов