PROPAGATION OF ULTRA-SOUND IN InSb-TYPE STRONGLY DEGENERATED SEMICONDUCTORS WITH NON-PARABOLIC BAND

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In the present paper propagation of ultrasound in degenerated semiconductors with non-parabolic band of the InSb-type has been searched. Absorption coefficient and change of ultra sonic wave velocity have been calculated. General expressions for the absorption coefficient and velocity change as functions of acoustic wave frequency and the parameter of non-parabolicity have been obtained. It has been assumed that electrons scatter on acoustic phonons, non-parabolicity of conduction band has been taken into account in the density of states, as well as the matrix element of electronic scattering.

It has been shown that consideration of non-parabolicity in the matrix element of scattering reduces effect of the band non-parabolicity on the absorption coefficient and change of the acoustic wave velocity in comparison with the consideration of non-parabolicity only in the electronic density of states.

Absorption coefficient α and relative change of velocity $\Delta v_s/v_s$ of ultra-sound in semiconductors with non-parabolic band have been investigated in [1-3]. One must note that in [1] a high frequency case $\omega\tau >> 1$ has been considered, i.e. scattering of electrons on phonons and ionized impurities has not been taken into account. And in [3] a general case for electron scattering on ionized impurities has been considered. In these works band non-parabolicity was taken into account only for electronic density of states.

Since in [1] the case $\omega\tau >> 1$ was considered, one can take non-parabolicity into account only in the density of states, because the absorption coefficient does not depend on relaxation time τ. In [3] the general case was investigated, that is why absorption coefficient depends on relaxation time of electrons on impurities. The authors of this work take non-parabolicity into account only for the density of states of electrons. As it has been shown in [4], in the computation of relaxation time of electrons on phonons and impurities, non-parabolicity has been taken into account in the density of states and matrix elements of scattering of electrons. For this reason the results of [3] are incorrect.

In the present work absorption coefficient α and relative change of velocity $\Delta v_s/v_s$ of ultra-sound in strongly degenerated semiconductors of InSb-type are calculated. Low-frequency case $\omega\tau << 1$ is considered under an assumption that electrons are scattered by acoustic phonons. Non-parabolicity of the bands is taken into account in the density of states and matrix element of scattering of electrons by acoustic phonons.

Let us calculate absorption coefficient and change of sound velocity in piezoelectric semiconductors under propagation of acoustic wave. For this purpose we linearize Boltzman equation for electrons under the presence of piezoelectric field induced by acoustic wave with frequency ω and wave vector $\vec{q}$. This equation has the following form [1]:

$$\frac{\partial f}{\partial t} + \frac{\hbar \vec{k}}{m^*}\frac{\partial f}{\partial \vec{r}} - \frac{e}{\hbar}\left(\vec{E}_1 + \frac{\hbar \vec{k}}{m^* C}\vec{B}_1\right)\frac{\partial f}{\partial \vec{k}} = -\frac{f - f_s}{\tau} \quad (1)$$

where $\vec{E}_1$ and $\vec{B}_1$ are vectors of electromagnetic field induced by propagating acoustic wave, $\vec{k}$ is wave vector, m^* - effective mass, τ - relaxation time of electrons which characterizes scattering process, f_s -distribution function which electrons relax after collisions, $\hbar$, e and c are fundamental constants and have regular meanings.

Distribution function f_s can be presented as follows [1]:

$$f_s(\vec{k}) = f_0(\vec{k}, n_0 - n_1) \approx f_0(\vec{k}) + n_1\frac{\partial f_0}{\partial n_0} \quad (2)$$

where f_0 is equilibrium distribution function of electrons, n_0 and n_1 are equilibrium and perturbed electronic concentration, accordingly.

In two-band Kane approximation electronic spectrum has the following form:

$$\hbar^2 k^2 = 2m_n \varepsilon (1 + \varepsilon/\varepsilon_g) \quad (3)$$

Here m_n is effective mass of electrons on the bottom of conduction band, ε_g is the gap width.

It is important to note that electric field $\vec{E}_1$ induced by sound is longitudinal, i.e. $\omega t \vec{E}_1 = 0$ [1]. Therefore, magnetic field $\vec{B}_1 = \frac{c\vec{q}}{\omega}\vec{E}_1$ induced by sound is equal zero.

The solution of equation (1) we seek in the following form:

$$f = f_0(\vec{k}) + f_1 = f_0(\vec{k}) + g(\vec{k})\, e^{i(\vec{q}\vec{r} - \omega t)} \quad (4)$$

where f_1 describe deviation of distribution function from equilibrium function f_0 under propagation of acoustic wave. Substituting (4) in (1), we obtain the following equation for determination of $g(\vec{k})$:

$$\left[\tau^{-1} + i\left(\frac{\hbar \vec{k}\vec{q}}{m^*} - \omega\right)\right] g(\vec{k}) = \frac{e\vec{E}_1}{\hbar}\frac{\partial f_0}{\partial \vec{k}} + \frac{n_1}{\tau}\frac{\partial f_0}{\partial n_0}, \quad (5)$$

where m^* is effective mass of electrons, and in two-band Kane model is defined by the formula

$$m^* = m_n\left(1+2\frac{\varepsilon}{\varepsilon_g}\right) \quad (6)$$

Here we consider strongly degenerated semiconductor of n-InSb type. Therefore $f_0(\vec{k})$ is degenerated Fermi-Dirac distribution:

$$f_0(\vec{k}) = \delta\left[\varepsilon(\vec{k}_F) - \varepsilon(\vec{k})\right] \quad , \quad (7)$$

where

$$\delta(x) = \begin{cases} 1 & \text{если } x>0 \\ 0 & \text{если } x<0 \end{cases} \quad (8)$$

$\vec{k}_F$ is wave vector on Fermi surface.

From (4)-(7) we can determine current density induced by propagating acoustic wave. Current density is defined by expression

$$\vec{I} = -\frac{e}{4\pi^3}\int\frac{\hbar\vec{k}}{m^*}f(\vec{k})\,d^3k \quad (9)$$

From the other side, $\vec{I}$ satisfies the following equation:

$$\vec{I} = \bar{\sigma}\vec{E}_1 - \vec{R}n_1 e v_s \quad (10)$$

where v_s is ultrasonic wave velocity. In particular, choosing the direction of $\vec{q}$ along z axis, comparing (9) and (10), and using (5), we find for σ_{zz}, the component of $\bar{\sigma}$ conductivity tensor and R_z, the component of $\vec{R}$ diffusion vector, the following expressions:

$$\sigma_{zz}(\vec{q},\omega) = i\left(\frac{e}{\pi\hbar q}\right)^2 k_F m^*(\varepsilon_F)G \quad (11)$$

$$R_z = \left(i/\omega\,\tau(\varepsilon_F)\right)G \quad (12)$$

Complex function G is determined by the expression :

$$G = -1 + \frac{\omega_s}{2\hbar k_F q/m^*(\varepsilon_F)}\ln\frac{\omega_s + \hbar k_F q/m^*(\varepsilon_F)}{\omega_s - \hbar k_F q/m^*(\varepsilon_F)} \ , \quad (13)$$

where $\omega_s = \omega + i\tau^{-1}(\varepsilon_F)$; $\hbar k_F = [2m_n\varepsilon_F(1+\varepsilon_F/\varepsilon_g)]^{1/2}$; $m^*(\varepsilon_F) = m_n(1+2\varepsilon_F/\varepsilon_g)$.

Longitudinal absorption coefficient α_s is determined by the expression [1]:

$$\alpha_s = -\frac{4\pi\,q\beta_1}{\rho\varepsilon_0 v_s^2}\,\mathrm{Im}\left(1-\frac{4\pi\sigma'}{i\omega\varepsilon_0}\right)^{-1} \quad (14)$$

where ρ is density of the crystal, ε_0 is static dielectric constant, β_1 is piezoelectric constant, σ' is effective electric conductivity defined by expression

$$\sigma' = \frac{\sigma_{zz}}{1-R_z} \quad (15)$$

Relative change of velocity of ultrasonic wave caused by interaction of conduction electrons, can be written as

$$\frac{\Delta v_s}{v_s} = \frac{2\pi\beta_1^2}{\rho\varepsilon_0 v_s^2}\,\mathrm{Re}\left(1-\frac{4\pi\sigma'}{i\omega\varepsilon_0}\right)^{-1} \quad (16)$$

We consider low-frequency case $\omega\tau(\varepsilon_F) << 1$ and $\frac{\hbar k_F}{m^*(\varepsilon_F)}\omega\tau(\varepsilon_F) < 1$. In this limit for G from (13) we find:

$$G(\varepsilon_F) \approx -2 + i\omega\tau(\varepsilon_F) \quad (17)$$

Substituting (17) into (11) and (12), for σ' we find

$$\sigma' = -\frac{e^2}{2(\pi\hbar q)^2}k_F m^*(\varepsilon_F)\omega\left[\omega\tau(\varepsilon_F) + 2i\right] \quad (18)$$

Relaxation time $\tau(\varepsilon_F)$ of electrons scattered by acoustic phonons has been calculated in [4] taking into account non-parabolicity of conduction band in the density of states and scattering matrix element. For degenerated semiconductors of n-InSb type relaxation time of electrons scattered by acoustic phonons is determined by the formula

$$\tau(\varepsilon_F) = \tau_{00}(T)\left(\frac{\varepsilon_F}{k_0 T}\right)^{-1/2}\left(1+\frac{\varepsilon_F}{\varepsilon_g}\right)^{-1/2}\left(1+2\frac{\varepsilon_F}{\varepsilon_g}\right)^{-1}\frac{\left(1+2\frac{\varepsilon_F}{\varepsilon_g}\right)^2}{1+\frac{2}{3}\frac{\varepsilon_F}{\varepsilon_g}+\frac{2,3}{3}\left(\frac{\varepsilon_F}{\varepsilon_g}\right)^2} \quad (19)$$

where

$$\tau_{00}(T) = \frac{9\pi}{2}\frac{\rho u_0^2\hbar^4}{C^2(m_n k_0 T)^{3/2}} \quad (20)$$

C - is electron-acoustic phonon interaction constant, u_0 is acoustic phonon velocity.

Substituting (18) and (19) into (13) and (16), we obtain the following expressions for longitudinal absorption coefficient α_s and relative change of the velocity of ultrasonic wave:

$$\alpha_l = -\frac{6\pi\beta_1^2\omega_p^2\tau_{00}}{\rho\varepsilon_g^2 v_s^2}\left(\frac{v_s}{v_T}\right)^2 \cdot \frac{1}{\left[1+\frac{3}{\varepsilon_0}\left(\frac{v_s}{v_T}\right)^2\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\varepsilon_F}{k_0T}\right)^{1/2}\left(1+\frac{\varepsilon_F}{\varepsilon_g}\right)^{1/2}\left(1+2\frac{\varepsilon_F}{\varepsilon_g}\right)\right]^2} \cdot \frac{\left(1+2\frac{\varepsilon_F}{\varepsilon_g}\right)^2}{1+\frac{2}{3}\frac{\varepsilon_F}{\varepsilon_g}+\frac{2,3}{3}\left(\frac{\varepsilon_F}{\varepsilon_g}\right)^2}$$

(21)

$$\frac{\Delta v_s}{v_s} = \frac{2\pi\beta_1^2}{\rho\varepsilon_0 v_s^2} \cdot \frac{1}{\left[1+\frac{3}{\varepsilon_0}\left(\frac{v_s}{v_T}\right)^2\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\varepsilon_F}{k_0T}\right)^{1/2}\left(1+\frac{\varepsilon_F}{\varepsilon_g}\right)^{1/2}\left(1+2\frac{\varepsilon_F}{\varepsilon_g}\right)\right]^2}$$

(22)

where $\omega_p^2 = \frac{4e^2(2m_n k_0 T)^{3/2}}{3\pi m_n \hbar^3}$; $v_T^2 = \frac{2k_0T}{m_n}$.

The last factor in (21) appears due to consideration of non-parabolicity in the matrix element of electronic scattering on acoustic phonons. As it follows from this formula, consideration of non-parabolicity in the scattering matrix element increased the value of longitudinal absorption coefficient α_l of ultrasonic wave.

From (22) it follows that consideration of non-parabolicity in the matrix element of electronic scattering on acoustic phonons does not affect the relative change of velocity of ultrasonic wave.

Let us consider the case when $\varepsilon_F \geq \varepsilon_g$, in which non-parabolicity of conduction band is more essential. In this case consideration of non-parabolicity in the matrix element of scattering increases the value of α_l by almost 4 times.

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QEYRİ PARABOLİK ZONALI ANTİMONİD-İNDİUM TİP GÜCLÜ CIRLAŞMIŞ YARIMKEÇİRİCİLƏRLƏ ULTRASƏSİN YAYILMASI

İşdə qeyri-parabolik zonalı antimonid-indium tip cırlaşmış yarımkeçiricilərlə ultrasəs dalğalarının yayılması tədqiq olunur. Ultrasəsin udulma əmsalı və yayılma sür'ətinin dəyişməsi hesablanmışdır. Udulma əmsalının və yayılma sür'ətinin dəyişməsinin akustik dalğanın tezliyindən və qeyri-paraboliklik parametrindən asılılığının ümumi ifadəsi alınmışdır. Hesablamalarla elektronların akustik fononlardan səpilməsinə baxılır. Keçiricilik zonasının qeyri-parabolik olması həm elektronların hal sıxlığından, həm də səpilmə matrisi elementində nəzərə alınmışdır.

Göstərilmişdir ki, qeyri-parabolikliyin matris elementində nəzərə alınması ultrasəs dalğalarının udulma əmsalına və sür'ətinin dəyişməsinə zonanın qeyri-parabolik olmasının tə'sirini azaldır.

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РАСПРОСТРАНЕНИЕ УЛЬТРАЗВУКА В СИЛЬНОВЫРОЖДЕННЫХ ПОЛУПРОВОДНИКАХ С НЕПАРАБОЛИЧЕСКОЙ ЗОНОЙ ТИПА АНТИМОНИДА ИНДИЯ

В статье исследуется распространение ультразвука в вырожденных полупроводниках с непараболической зоной типа антимонида индия. Вычислены коэффициент поглощения и изменение скорости ультразвуковой волны. Получены общие выражения для зависимости коэффициента поглощения и изменения скорости от частоты акустической волны и параметра непараболичности. Предполагается, что рассеяние электронов происходит на акустических фононах , непараболичность зоны проводимости учитывается в плотности состояний, а также в матричном элементе рассеяния электронов.

Показано, что учет непараболичности в матричном элементе рассеяния уменьшает влияние непараболичности зоны на коэффициент поглощения и изменения скорости акустической волны по сравнению с учетом непараболичности только в плотности состояний электронов.

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