ON OPTICAL DEPTHS OF FRAUNHOFER LINES FORMATION IN THE SOLAR ATMOSPHERE

R.Kh. SALMAN-ZADEH
Shamakhy Astrophysical Observatory
Azerbaijan Academy of Sciences
Observatory, settl. Y. Mamedaliyev, Shamakhy

The formulas for the definition of $\bar{\tau}_\nu(\mu)$ - of optical depth of Fraunhofer lines formation was received for the different mechanisms of formation in atmosphere. For selected FeI lines computation of $\bar{\tau}_\nu(\mu)$ was made using exact formulas and rough estimate as well as calculation of source function. The results between themselves are comparised.

The mean optical depths, $\bar{\tau}_\nu(\mu)$, of formation of monochromatical radiation going out from solar atmosphere is determined as the mathematical expectation of depth in the atmosphere of the Sun from which depth quants with the fixed frequency go out in the given direction without reradiation. Using this method the analytic expression for the determination of $\bar{\tau}_\nu(\mu)$ was obtained:

$$\bar{\tau}_\nu(\mu) = \mu^2 \frac{d}{d\mu} ln\left[\mu I_\nu(0,\mu)\right] \quad . \qquad (1)$$

where $I_\nu(0,\mu)$ is the intensity of monochromatical radiation going out from the atmosphere under the angle $arccos\mu$ to normal direction:

$$I_\nu(0,\mu) = \int_0^\infty S_\nu(\tau_\nu) e^{-\tau_\nu/\mu} d\tau_\nu/\mu \qquad (2)$$

$S_\nu(\tau_\nu)$ is the total source function. In the case of plane - parallel semi-infinite atmosphere source functions in the coherent scattering of radiation in line are satisfied the following integral equation:

$$S_\nu(\tau_\nu) = \frac{\lambda_\nu(\tau_\nu)}{2} \int_0^\infty S_\nu(\tau'_\nu) E_1|\tau' - \tau'_\nu| d\tau'_\nu + \left[1 - \lambda_\nu(\tau_\nu)\right] B_\nu(\tau_\nu) \qquad (3)$$

λ_ν is the probability of photon survive. The equation (1) is valid for any line formation mechanisms.

1. The source function may be substituted by the Plank function assuming the LTE-approximation and pure absorption mechanism in lines. Taking the first three terms in expansion of the Plank function we obtain the expression for determination of $\bar{\tau}_\nu(\mu)$:

$$\bar{\tau}_\nu(\mu) = \mu\left[1 + \frac{\mu + 4\mu^2 C_2/C_1}{\mu + 2\mu^2 C_2/C_1 + C_0/C_1}\right] = \mu\left[1 + \frac{\mu + 2A_\nu(\mu)}{(1+\eta_\nu)/\beta'_{1\nu} + \mu + A_\nu(\mu)}\right] \qquad (4)$$

where C_0, C_1, C_2 are the coefficients or expansion of Plank function in a series:

$$C_0 = B_\nu(T_0)\ , \quad C_1 = B_\nu(T_0)\beta^*_{1\nu}\big/(1+\eta_\nu), \quad C_2 = B_\nu(T_0)\beta^*_{2\nu}\big/(1+\eta_\nu)^2 ,$$

$$A_\nu(\mu) = 2\mu^2 \frac{\beta'_{2\nu}}{\beta'_{1\nu}} \frac{1}{1+\eta_\nu} \quad , \quad \beta'_{1\nu} = \beta_{1\nu}\bar{\alpha}/\alpha_\nu \quad , \quad \beta'_{2\nu} = \beta_{2\nu}\left(\bar{\alpha}/\alpha_\nu\right)^2$$

$$\beta_{1\nu} = \frac{3}{8}\frac{h\nu}{kT_0}\frac{1}{1-exp\left(-h\nu/kT_0\right)} \quad , \beta_{2\nu} = \beta^2_{1\nu} - \frac{3}{16}\beta_{1\nu}\left(\frac{h}{kT_0} + 5\right) \quad ,$$

where $B_\nu(T_0)$ is the Plank function for the Sun's surface temperature T_0, η_ν=σ_ν/α_ν, $\bar{\alpha}$ is the mean absorption coefficient in continuum.

2. In the case of coherent scattering of radiation in line and for a quadratic dependence of Plank function on the optical depth the formula for determination of $\tau_\nu(\mu)$ was obtained:

$$\bar{\tau}_\nu(\mu) = \mu\left[1 + \frac{\mu + 2\mu^2 \alpha'_2/\alpha'_1}{\alpha'_0/\alpha'_1 + \mu^2\alpha'_2/\alpha'_1 + \mu} + \mu\frac{\varphi'_\nu(\mu,\lambda_\nu)}{\varphi'(\mu,\lambda_\nu)}\right] \qquad (5)$$

Here $\varphi_\nu(\mu,\lambda_\nu)$ is the Ambartsumyan function. For the definition of coefficients α'_k the recurrent formula was obtained:

$$\frac{\alpha'_k}{1-\lambda} = k!C_k + \sum_{i=k}^{n}\frac{i!}{(i-k)!}C_i I_{i-k}, (k=0,1,2,\ldots,n) \quad (6)$$

For the definition I_k the recurrent formula was derived as well:

$$I_k = \int_0^\infty \Phi(\tau)\tau^k d\tau = \frac{\lambda}{2}\frac{1}{\sqrt{1-\lambda}}\left[\sum_{i=1}^{k}\frac{k!}{(k-i)!}\alpha_i I_{k-i} + k!\alpha_k\right] \quad (7)$$

α_k are the moments of the Ambartsumyan function, λ is the probability of photon survive.

3. The formulas for the intensity of monochromatical radiation going out from the solar atmosphere in the case of non coherent scattering of radiation in line have been received in [1]. Substituting of the found $I_\nu(0,\mu)$ in Eq. (1), we obtain $\bar{\tau}_\nu(\mu)$ for the case of non-coherent scattering mechanisms.

4. The formula for rough estimate of $\bar{\tau}_\nu(\mu)$ is obtained for the coherent scattering mechanisms assuming η_ν=Const and in the representation of Plank function by "n"-th degree polynom from τ_ν:

$$\bar{\tau}_\nu(\mu) = \frac{\mu}{\varphi_\nu(\mu,\lambda_\nu)}\frac{\alpha'_0\psi(\tau_\nu,\lambda_\nu) + \sum_{k=1}^{n}\alpha'_k\cdot\int_0^{\tau_\nu}\cdots\cdots\int_0^{\tau_\nu^{(k-1)}}\psi[\tau_\nu^{(k)},\lambda_\nu]d\tau_\nu^{(k)}}{\sum_{k=0}^{n}\alpha'_k\mu^k} \quad . \quad (8)$$

where is assumed that τ_ν=2μ. Here:

$$\psi(\tau_\nu,\lambda_\nu) = 1 + \int_0^{\tau_\nu}\Phi(\tau'_\nu,\lambda_\nu)d\tau'_\nu \quad . \quad (9)$$

which have been tabulated in [2].

5. In order to determine the exact value of $\bar{\tau}_\nu(\mu)$ in the given model of atmosphere it is required to calculate source functions $S_\nu(\tau_\nu)$ for the considered line. The more simple method is the method of approximating function for the calculation of $S_\nu(\tau_\nu)$. The method of root mean squares and the method of moments give a satisfactory results.Using the formulas (2) and (1), $\bar{\tau}_\nu(\mu)$ was determined after the definition of source function.

The optical depth of formulation of selected FeI lines is calculated according every above mentioned points (Table). In this table τ_ν is the optical depth at the given ν',τ_0 is the optical depth in scale of continuum at λ=5000 Å, h is the geometrical depth. The results was comparised between themselves as well as with the results of [3,4]. One can see that LTE-approximation is no acceptable in high layer of photosphere where the radiation can go away to cosmic space. The systematical difference of our results ($h(km)$) from the ones of [3,4] may be due to the difference of parent parameter which is used in calculations and due to the difference of the definition methods of $\bar{\tau}_\nu(\mu)$. The comparison of results indicates that in most case it is reasonably to determine $\bar{\tau}_\nu(\mu)$ by the rough estimate. We would like to mention that the formula (1) allows to

Table

Computation method	depth	λ(Å)						
		4808.152	5250.216	5253.500	5269.547	5302.307	5324.188	6302.496
Pure absorp.	τ_ν	1.985	1.726	1.335	1.003	1.068	1.017	1.184
	τ_0	0.557	$0.870\cdot10^{-2}$	$0.147\cdot10^{-1}$	$0.140\cdot10^{-3}$	$0.140\cdot10^{-2}$	$0.310\cdot10^{-3}$	$0.933\cdot10^{-2}$
	h	31	301	274	552	427	501	298
Coh. scatter.	τ_ν	1.963	1.778	1.663	1.652	1.653	1.648	1.588
	τ_0	0.547	$0.911\cdot10^{-2}$	$0.150\cdot10^{-1}$	$0.140\cdot10^{-3}$	$0.161\cdot10^{-2}$	$0.420\cdot10^{-3}$	$0.101\cdot10^{-1}$
	h	32	299	271	552	403	484	292
Non-coh. scat.	τ_ν	1.958	1.773	1.665	1.676	1.673	1.662	1.596
	τ_0	0.544	$0.907\cdot10^{-2}$	$0.150\cdot10^{-1}$	$0.140\cdot10^{-3}$	$0.163\cdot10^{-2}$	$0.420\cdot10^{-3}$	$0.103\cdot10^{-1}$
	h	33	300	271	552	402	484	291
Rough estim.	τ_ν	1.752	1.686	1.583	1.696	1.557	1.676	1.550
	τ_0	0.445	$0.840\cdot10^{-2}$	$0.141\cdot10^{-1}$	$0.140\cdot10^{-3}$	$0.152\cdot10^{-2}$	$0.420\cdot10^{-3}$	$0.977\cdot10^{-2}$
	h	44	303	275	552	409	484	295
Calcul. $S(\tau)$	τ_ν	1.989	1.762	1.650	1.658	1.662	1.630	1.592
	τ_0	0.558	$0.898\cdot10^{-2}$	$0.148\cdot10^{-1}$	$0.140\cdot10^{-3}$	$0.162\cdot10^{-2}$	$0.410\cdot10^{-3}$	$0.102\cdot10^{-1}$
	h	31	300	274	552	403	486	292
Gurtovenko [4]	τ_ν							
	τ_0							
	h	179	409	372				381
Luibimkov [3]	τ_ν							
	τ_0							
	h	105	353			480		320

using the results of exact observations of Fraunhofer line profiles with a high spatial resolution for different angular distances from the solar disc centre. The values of $\bar{\tau}_\nu(\mu)$ which are defined using the results of exact observations will be maximum closely correspond to their own real values.

Using (1) a formula for the determination of optical depths of flux radiation formation from stars was obtained as well:

$$\bar{\tau}_\nu^{H} = \frac{\varphi_\nu(1, \lambda_\nu)\sum_{k=0}^{n} \alpha'_k - 2\sum_{k=0}^{n} \alpha'_k \alpha_{k+2}}{\sum_{k=0}^{n} \alpha'_k \alpha_{k+1}} \qquad (10).$$

---

[1] *V.V. Sobolev.* The trans. of rad. in stell. and plan atm., M., 1956.

[2] *M.A. Heaslet, R.F. Warwing.* Astroph. Space Sci., 1968, 1, 460.

[3] *L.C. Luibimkov.* Izvestiya Kr. AO, 1976, 54, 248.

[4] *E.A. Gurtovenko, R.I. Kostik.* The Fraunhofer Spectrum and the System of the Solar Oscillator Strengths, Kiev, Naukova Dumka, 1989.

R.X. Salman-zadə

GÜNƏŞ ATMOSFERİNDƏ FRAUNHOFER XƏTLƏRİNİN YARANMA DƏRİNLİKLƏRİ HAQQINDA

Günəş atmosferində,Fraunhofer xətlərinin, müxtəlif yaranma mexanizmləri çərçivəsində optik dərinliklərinin, $\bar{\tau}_\nu(\mu)$ -nün hesablanması üçün düsturlar alınmışdır. Seçilmiş FeI xətləri üçün, dəqiq və təqribi düsturların köməyi ilə və həmçinin mənbə funksiyasının hesablanması yolu ilə $\bar{\tau}_\nu(\mu)$ tə`yin edilmişdir. Alınmış nəticələr bir-birilə müqayisə edilir.

Р.Х. Салман-заде

ОБ ОПТИЧЕСКИХ ГЛУБИНАХ ОБРАЗОВАНИЯ ФРАУНГОФЕРОВЫХ ЛИНИЙ В АТМОСФЕРЕ СОЛНЦА

Получены формулы для определения оптических глубин образования фраунгоферовых линий в атмосфере Солнца - $\bar{\tau}_\nu(\mu)$ - для различных механизмов их образования. Произведены расчеты $\bar{\tau}_\nu(\mu)$ для избранных линий FeI, по точным и приближенным формулам, а также по расчетам функции источников. Полученные результаты сравнивались между собой.

Дата поступления: 10.11.97

Редактор: Ф.М. Гашимзаде