PION, KAON ELECTROMAGNETIC FORM FACTORS IN THE FRAMEWORK OF THE RUNNING COUPLING CONSTANT METHOD AND INFRARED MATCHING SCHEME

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The pion and kaon electromagnetic form factors $F_M(Q^2)$ are calculated at the leading order of perturbative QCD using the running coupling constant method and infrared matching scheme. In computations the asymptotic distribution amplitudes for mesons are used. The Borel transform and resummed expression for $F_M(Q^2)$ are found. It is shown that the running coupling constant method allows one to estimate power suppressed corrections to $F_M(Q^2)$. Comparison is made with the infrared matching scheme's result and with the experimental data.

Investigation of the infrared (ir) renormalon effects in various exclusive and inclusive processes is one of the important problems in the perturbative QCD [1]. It is well known that all-order resummation of ir renormalons corresponds to the calculation of the one-loop Feynman diagrams with the running coupling constant $\alpha_S(-k^2)$ at the vertices or, alternatively, to calculation of the same diagrams with non-zero gluon mass. Both these approaches are generalization of the Brodsky, Lepage and Mackenzie scale-setting method [2] and are equivalent to absorbing certain vacuum polarization corrections appearing at higher-order calculations into one-loop QCD coupling constant.

Unlike inclusive processes exclusive ones have additional source of ir renormalon contributions [3-5], namely, integration over longitudinal fractional momenta of hadron constituents in the expression of the elm form factor, which generates such corrections.

In this work we consider light mesons (pion, kaon) electromagnetic form factor, which in the context of QCD has the form

$$F_M(Q^2) = \int_0^1\int_0^1 dxdy\phi_M^*(y, Q^2)T_H\left(x, y; Q^2, \alpha\left(\hat{Q}^2\right)\right)\phi_M(x, Q^2) , \tag{1}$$

where $Q^2=-q^2$ is the square of the virtual photon's four-momentum. Here ϕ_M is the meson M distribution amplitude, containing all non-perturbative hadronic binding effects. In (1) T_H is the hard-scattering amplitude of the subprocess $q\bar{q}' + \gamma^* \to q\bar{q}'$ and can be found using QCD.

At the leading order T_H is given by the expression

$$T_H = \frac{16\pi C_F}{Q^2}\left\{\frac{2}{3}\frac{\alpha_S\left[Q^2(1-x)(1-y)\right]}{(1-x)(1-y)} + \frac{1}{3}\frac{\alpha_S\left(Q^2xy\right)}{xy}\right\}, C_F = \frac{4}{3}. \tag{2}$$

One of the important moments in our study is the choice of the meson distribution amplitude $\phi_M(x,Q^2)$. Here for the pion and kaon we use the asymptotic amplitudes

$$\phi_{asy}^{(K)}(x) = \sqrt{3}f_{\pi(K)}x(1-x) \tag{3}$$

and f_π=0.093GeV, f_K=0.122 GeV are the pion and kaon decay constants.

The QCD running coupling constant $\alpha_S\left(\hat{Q}^2\right)$ in Eq.(2) suffers from ir singularities associated with the behaviour of $\alpha_S\left(\hat{Q}^2\right)$ in the soft regions $x\to 0, y\to 0; x\to 1, y\to 1$. To solve this problem in the context of the running coupling constant method let us relate the running coupling $\alpha_S(\lambda Q^2)$ in terms of $\alpha_S(Q^2)$ by means of the renormalization group equation [6]

$$\alpha_S\left(\lambda Q^2\right) \approx \frac{\alpha_S\left(Q^2\right)}{1 + \left[\alpha_S\left(Q^2\right)\beta_0/4\pi\right] \ln(\lambda)}, \tag{4}$$

where $\alpha_S(Q^2)$ is the one-loop QCD coupling constant, $\beta_0=11-2n_f/3$ is the QCD beta-function's one-loop coefficient.

As it was shown in the works [3-5], integration in (1) using (2) and (4) generates ir divergences and as a result for $F_M(Q^2)$ we get a perturbative series with factorially growing coefficients. This series can be resummed using the Borel transformation [7]

$$\left[Q^2F_M\left(Q^2\right)\right]^{res} = \frac{\left(16\pi f_M\right)^2}{\beta_0}\int_0^\infty du\, exp\left(-\frac{4\pi u}{\beta_0\alpha_S}\right)B\left[Q^2F_M\right](u), \tag{5}$$

where $B[Q^2F_M](u)$ is the Borel transform of the corresponding perturbative series.

When we take both of the variables x, y in Eq.(2) as the running ones, for $B[Q^2F_M](u)$ we find

$$B\left[Q^2 F_M\right](u) = \frac{1}{(1-u)^2} + \frac{1}{(2-u)^2} - \frac{2}{1-u} + \frac{2}{2-u} \qquad (6)$$

The Borel transform (6) has double and single poles at u=1,2 which are the ir renormalon poles. The resummed expression (5) can be calculated with the help of the principal value prescription [6]

$$\left[Q^2 F_M\left(Q^2\right)\right]^{res} = \frac{\left(16\pi f_M\right)^2}{\beta_0}\left[-\frac{3}{2} + (\ln\lambda - 2)\frac{li(\lambda)}{\lambda} + (2+\ln\lambda)\frac{li(\lambda^2)}{\lambda^2}.\right] \qquad (7)$$

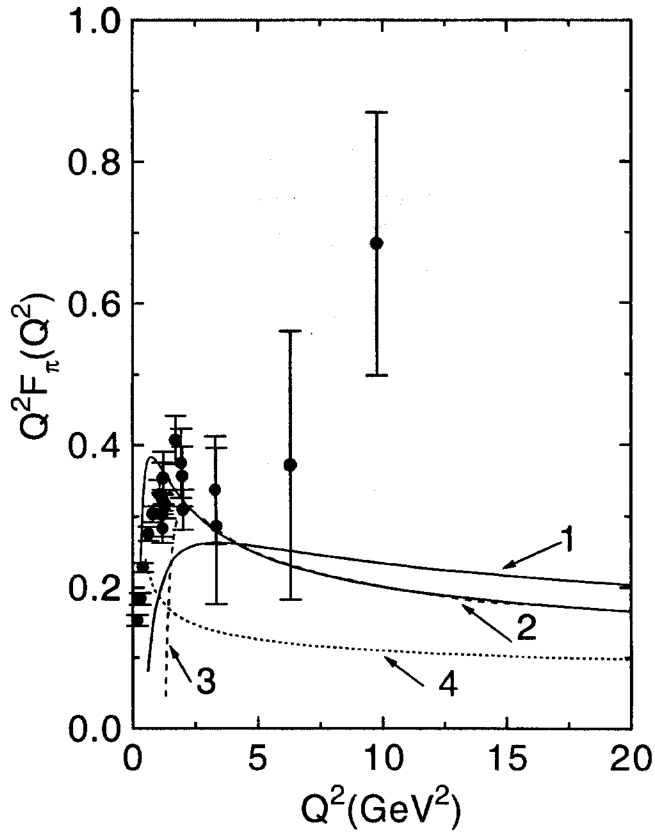

Fig. The pion elm form factor calculated using $\Phi^{\pi}_{asy}(x)$ (3). The curves correspond to the following computational schemes: 1 - resummed expression (7), 2 - resummed expression (9),dashed curve 3 - ir matching scheme, short-dashed curve 4 - frozen coupling ap proximation.

Here $li(\lambda)$ is the logarithmic integral

$$li(\lambda) = P.V.\int_0^{\lambda}\frac{dx}{\ln x}, \lambda = \frac{Q^2}{\Lambda^2} \qquad (8)$$

In the case with one frozen (for example,y) and one running (x) variables we get

$$\left[Q^2 F_M\left(Q^2\right)\right]^{res} = \frac{\left(16\pi f_M\right)^2}{2\beta_0}\left[\frac{li(\tilde{\lambda})}{\tilde{\lambda}} - \frac{li(\tilde{\lambda}^2)}{\tilde{\lambda}^2}\right], \tilde{\lambda} = \frac{Q^2}{2\Lambda^2} \qquad (9)$$

The ir renormalon carried out analysis allows one to estimate power corrections to the light mesons' elm form factor. Another way of such estimation is the infrared matching scheme, in the context of which one explicitly divides power corrections from the full expression by introducing moments of α_S at low scales as new non-perturbative parameters [8]. By freezing one of variables (y) we can express $Q^2F_M(Q^2)$ in terms of moment integrals f_p defined by the formula

$$f_p(Q) = \frac{p}{Q^p}\int_0^Q dk k^{p-1}\alpha_s\left(k^2\right). \qquad (10)$$

After simple calculations we get

$$Q^2 F_M(Q^2) = 64\pi f_M^2 \left\{ \left(\frac{\mu}{Q}\right)^2 f_2(\mu) - \left(\frac{\mu}{Q}\right)^4 f_4(\mu) + \frac{\alpha_S}{4}\left[1 - 2\Gamma(1,2z) + \Gamma(1,4z)\right] + \right.$$

$$\left. + \frac{\alpha_S^2 \beta_0}{32\pi}\left[3 - 4\Gamma(2,2z) + \Gamma(2,4z)\right] + \ldots \right\}$$

where μ=2 GeV is the ir matching scale, $z=\ln(Q/\sqrt{2}\mu)$, $\alpha_S(Q^2/2)$ and $\Gamma(n,x)$ is the incomplete gamma function.

Results of numerical calculations for the pion are shown in Fig. The similar results can be obtained also for the kaon. In our calculations we take for non-perturbative parameters $f_2(\mu)$ and $f_4(\mu)$ the following values $f_2(\mu\text{=2GeV})=0.5$, $f_4(\mu\text{=2 GeV})=0.347$. As is seen, Eq.(9) and ir matching scheme give approximately the same results, excluding a region of small Q^2. In general, the ir renormalon effects enhance the ordinary perturbative predictions for the pion, kaon elm form factors approximately two times, partly, explain experimental data [9].

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PİON, KAONUN ELEKTROMAQNİT FORM FAKTORLARI DƏYİŞƏN TƏ'SİR SABİTİ ÜSULU VƏ İNFRAQIRMIZI İN'İKAS SXEMİ ÇƏRÇİVƏSİNDƏ

Pion və kaon $F_M(Q^2)$ elektromaqnit form faktorladı perturbativ KXD-nin əsas yaxınlaşmasında dəyişən tə'sir sabiti üsulu və infraqırmızı in'ikas sxeminin köməyi ilə hesablanmışdır. Hesablamalarda mezonların asimptotik paylanma funksiyalarından istifadə olunmuşdur. Üçün Borel çevrilməsi və yenidəncəmlənmə ifadələri tapılmışdır. Göstərilmişdir ki, dəyişən tə'sir sabiti üsulu üstlü azalan düzəlişlərin $F_M(Q^2)$-na verdiyi əlavələri qiymətləndirməyə imkanverir. İnfraqırmızı in'ikas sxeminin nəticələri və təcrübi nəticələrlə müqayisə apalırlmışdır.

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ПИОННЫЙ, КАОННЫЙ ЭЛЕКТРОМАГНИТНЫЕ ФОРМ-ФАКТОРЫ В РАМКАХ МЕТОДА БЕГУЩЕЙ КОНСТАНТЫ СВЯЗИ И СХЕМЫ ИНФРАКРАСНОГО ОТОБРАЖЕНИЯ

Электромагнитные форм-факторы пиона, каона $F_M(Q^2)$ вычислены в главном приближении пертурбативной КХД с помощью метода бегущей константы связи и схемы инфракрасного отображения. При вычислениях использованы асимптотические функции распределения. Для $F_M(Q^2)$ найдены Борелевское преобразование и просуммированное выражения. Показано, что метод бегущей константы связи позволяет оценить вклад в $F_M(Q^2)$ степенно-подавленных поправок. Проведено сравнение с результатом схемы инфракрасного отображения и с экспериментальными данными.

Дата поступления: 15.01.99 *Редактор: И.Г. Джафаров*