THE DEPENDENCE OF THE LIGHT MESON'S ELECTROMAGNETIC FORM FACTORS ON THE FACTORIZATION SCALE

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The pseudoscalar (pion, kaon) and vector (ρ_L-meson) mesons electromagnetic form factors $F_M(Q^2)$ are calculated by means of the running coupling constant and infrared matching methods. The dependence of the mesons' distribution amplitudes (DAs) $\Phi_M(x,Q^2)$ on the factorization scale Q^2 is taken into account. It is demonstrated that both of these methods allow one to evaluate power-suppressed corrections to $F_M(Q^2)$. In calculations the mesons' model DAs obtained in the context of QCD sum rules approach are used.

Exclusive electromagnetic (elm) form factors (ff) are a source of the information about the structure of hadrons. The coupling of an elementary particle to the photon is determined by a few parameters, like its total charge and magnetic moment. For a composite particle these constant coefficients are replaced by momentum dependent functions, which we call form factors, reflecting the internal structure of the particle [1]. It is well known that elm ff, at large momentum transfer, can be calculated using methods of the perturbative QCD (pQCD) [1,2]. In our works [3,4] we have calculated the pion, kaon and ρ_L - meson elm ff using the asymptotic and QCD sum rules DAs (only for ρ_L - meson). In recent paper we extend our investigation of $F_M(Q^2)$ by including into consideration the pion, kaon sum rules DAs and taking into account their dependence on the factorization scale.

In the framework of pQCD a meson M elm form factor, due to QCD factorization theorems, can be written as

$$F_M(Q^2) = \int_0^1\int_0^1 dxdy\Phi_N^*(y,\mu_F^2)T_H(x,y;Q^2,\mu_R^2,\mu_F^2)\Phi_M(x,\mu_F^2) \qquad (1)$$

In Eq. (1) $\Phi_M(x,\mu_F^2)$ is the meson DA, that is the probability amplitude for finding the meson state with constituents carrying the longitudinal momenta xp and $(1-x)p$, $T_H(x,y;Q^2,\mu_R^2,\mu_F^2)$ is the hard-scattering $q\bar{q}'+\gamma^*\to q\bar{q}'$ amplitude, calculable in the context of pQCD, $Q^2=-q^2$ is the momentum transfer in the process, μ_R is the renormalization scale and μ_F is the scale at which soft and hard physics factorize, i.e. the factorization scale.

At the leading order T_H does not depend on μ_F^2 and depends on μ_R^2 through $\alpha_S(\mu_R^2)$. At the next-to-leading order of pQCD [5] T_H depends on μ_F^2, μ_R^2 explicitly due to terms proportional to $\ln(Q^2/\mu_F^2)$, $\ln(Q^2/\mu_R^2)$. One of the important problems in pQCD calculations is the proper choice of the factorization and renormalization scales – the choice which minimizes higher-order corrections to $F_M(Q^2)$.

For the factorization scale μ_F^2, a natural choice is

$$\mu_F^2 = Q^2 \quad , \qquad (2)$$

which eliminates the logarithms of Q^2/μ_F^2.

We choose the renormalization scale μ_R^2 to be equal to the gluon virtuality

$$\mu_R^2 = xyQ^2 \ , \ \mu_R^2 = (1-x)(1-y)Q^2 \qquad (3)$$

depending on the Feynman diagram under consideration. Then, at the leading order of pQCD for T_H we get [2]

$$T_H(x,y;Q^2,\alpha_S(\mu_R^2)) = \frac{16\pi C_F}{Q^2}\left[\frac{2}{3}\frac{\alpha_S[(1-x)(1-y)Q^2]}{(1-x)(1-y)} + \frac{1}{3}\frac{\alpha_S(xyQ^2)}{xy}\right], \ C_F = \frac{4}{3}. \qquad (4)$$

Here $\alpha_S(\mu^2)$ is the one-loop QCD coupling constant

$$\alpha_S(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}, \ \beta_0 = 11 - \frac{2n_f}{3}, \qquad (5)$$

β_0 is the QCD beta-function's first coefficient, n_f is the number of quark flavors and Λ is the QCD dimensional parameter $\Lambda=0.2$ GeV.

The DAs of the pion [6] and longitudinally polarized ρ_L - meson [7] have the following form

$$\Phi_M(x,\mu_0^2) = \Phi_{asy}^M(x)[a + b(2x-1)^2] \qquad (6)$$

where $\Phi_{asy}^M(x)$ is the meson M asymptotic DA

$$\Phi_{asy}^\pi(x) = \sqrt{3}f_\pi x(1-x) \ , \ \Phi_{asy}^\rho(x) = \sqrt{2}\cdot\sqrt{3}f_\rho^L x(1-x). \qquad (7)$$

In Eqs. (6), (7) the constants take values: f_π=0.093 GeV, a=0, b=5 (Chernyak-Zhitnitsky DA), a = -0.1821, b = 5.91 (DA from Ref. [8]); f_ρ^L=0.141 GeV and a = 0.7, b = 1.5 (Ball-Braun DA). The constants a, b in Eq.(6) were found by means of the QCD sum rules method at the normalization points μ_0=0.5 GeV (pion, kaon) [6] and μ_0=1 GeV (ρ_L - meson) [7].

The meson's DAs evolve in accordance with the expression [2,6]

$$\Phi_M(x, Q^2) = \Phi_{asy}^M(x)\sum_{n=0}^{\infty} r_n C_n^{3/2}(2x - 1)\left[\frac{\alpha_S(Q^2)}{\alpha_S(\mu_0^2)}\right]^{\gamma_n/\beta_0}, \tag{8}$$

where $\{C_n^{3/2}(2x-1)\}$ are the Gegenbauer polynomials, γ_n is the anomalous dimension,

$$\gamma_n = \frac{4}{3}\left[1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1}\frac{1}{j}\right] \tag{9}$$

In our calculations we need expressions only for the first four Gegenbauer polynomials, which are given by the formulae [9]

$$C_0^{3/2}(\xi) = 1,\ C_1^{3/2}(\xi) = 3\xi,\ C_2^{3/2}(\xi) = \frac{3}{2}(5\xi^2 - 1),\ C_3^{3/2}(\xi) = \frac{5\xi}{2}(7\xi^2 - 3), \tag{10}$$

and the corresponding anomalous dimensions take the values

$$\gamma_0 = 0,\ \gamma_1 = \frac{32}{9},\ \gamma_2 = \frac{50}{9},\ \gamma_3 = \frac{314}{45} \tag{11}$$

The DAs (6) can be rewritten in the form (8). But after defining of the coefficients r_n and taking into account the evolution of $\Phi_M(x, Q^2)$ on Q^2, for our purposes, it is useful to give to the DAs their old form, namely

$$\Phi_M(x, Q^2) = \Phi_{asy}^M(x)\left[\alpha(Q^2) + \beta(Q^2)(2x - 1)^2\right]. \tag{12}$$

Now new coefficients $\alpha(Q^2), \beta(Q^2)$ are functions of the factorization scale Q^2,

$$\alpha(Q^2) = a + \frac{b}{5}\left[1 - A_2(Q^2)\right],\ \beta(Q^2) = bA_2(Q^2),$$
$$A_n(Q^2) = \left[\frac{\alpha_S(Q^2)}{\alpha_S(\mu_0^2)}\right]^{\gamma_n/\beta_0}. \tag{13}$$

Using the same method for the kaon we find

$$\Phi_K(x, Q^2) = \Phi_{asy}^K(x)\left[\alpha + \gamma(2x - 1) + \beta(2x - 1)^2 + \delta(2x - 1)^3\right],$$
$$\gamma(Q^2) = \frac{3c}{7}\left[A_1(Q^2) - A_3(Q^2)\right],\qquad \delta(Q^2) = cA_3(Q^2). \tag{14}$$

Here α, β are defined as in Eq.(13) and constants are equal: a=0.4, b=3, c=1.25 (see, Ref.[6]). In Eq.(14) $\Phi_{asy}^K(x)$ has the same form as $\Phi_{asy}^\pi(x)$ in Eq.(7), but with f_π replaced by f_K=0.112 GeV.

For calculation of $F_M(Q^2)$ we have to express the running coupling constant $\alpha_S(\lambda Q^2)$ in terms of $\alpha_S(Q^2)$ [10]

$$\alpha_S(\lambda Q^2) = \frac{\alpha_S(Q^2)}{1 + (\alpha_S(Q^2)\beta_0/4\pi)\ln\lambda}. \tag{15}$$

Integration in Eq.(1) using Eqs.(4), (15) generates infrared divergences and as a result for $F_M(Q^2)$ we get a perturbative series with factorially growing coefficients [11]. This series can be resummed by means of the Borel transformation [12]

$$\left[Q^2F_M(Q^2)\right]^{res} = C\,\frac{(16\pi f_M)^2}{\beta_0}\int_0^\infty du\exp\left[-\frac{4\pi u}{\beta_0\alpha_S(Q^2)}\right]B\left[Q^2F_M\right](u)\ , \qquad (16)$$

where $B\left[Q^2F_M\right](u)$ is the Borel transform of the corresponding perturbative series. The coefficient C is equal to 1 for the pion, kaon and $C=2$ for the ρ_L - meson.

The technique for calculation of $B\left[Q^2F_M\right](u)$ and $\left[Q^2F_M(Q^2)\right]^{res}$ has been described in details in Refs. [4,13], therefore, here we write down the final expressions,

$$B\left[Q^2F_M\right](u) = \sum_{k=1}^{N}\left[\frac{m_k(Q^2)}{(k-u)^2}+\frac{n_k(Q^2)}{k-u}\right] \qquad (17)$$

and

$$\left[Q^2F_M(Q^2)\right]^{res} = C\,\frac{(16\pi f_M)^2}{\beta_0}\sum_{k=1}^{N}\left\{-\frac{m_k(Q^2)}{k}+\left[n_k(Q^2)+m_k(Q^2)\ln\lambda\right]\frac{li(\lambda^k)}{\lambda^k}\right\} \qquad (18)$$

Here $li(\lambda^k)$ and λ are defined as

$$li(\lambda^k) = P.V.\int_0^{\lambda^k}\frac{dx}{\ln x}\ , \qquad \lambda = Q^2/\Lambda^2 . \qquad (19)$$

The values of $m_k(Q^2)$ and $n_k(Q^2)$ for the pion and ρ_L - meson can be obtained from corresponding formulae of Ref.[4] by replacing $a\to\alpha,\ b\to\beta$. For the kaon the functions $m_k(Q^2)$ and $n_k(Q^2)$ have rather lengthy expressions therefore we omit them here.

The obtained formula (18) is the generalization of the previous results for the pion, kaon and ρ_L - meson elm ff. Indeed, if we switch off the dependence of $\Phi_M(x,Q^2)$ on Q^2 (that is, put $A_n\equiv1$), then Eq. (17) coincides for a=0, b=5 and N=4 with result of Ref.[13] for the pion, for a=0.4, b=3 c=1.25 and N=5 with kaon elm ff from Ref.[14] and for a=0.7, b=1.5, N=4 with ρ_L -meson ff from Ref.[4].

We can choose the renormalization scale also by freezing one of variables x,y in Eq.(3) (for example, $\langle y\rangle$=1/2), which corresponds to the choice $\mu_R^2 = Q^2(1-x)/2$, $\mu_R^2 = Q^2x/2$. Then after some calculations for the kaon we get

$$\left[Q^2F_K(Q^2)\right]^{res} = \frac{(16\pi f_K)^2}{\beta_0}\sum_{k=1}^{5}l_k(Q^2)\frac{li(\tilde\lambda^k)}{\tilde\lambda^k}\ .\ \tilde\lambda = \frac{Q^2}{2\Lambda^2} \qquad (20)$$

The expressions for $l_k(Q^2)$ are given in the Appendix. In the case of the pion and ρ_L -meson the sum in Eq.(20) runs up to N=4 and the functions $l_k(Q^2)$ can be obtained from the corresponding expressions in the Appendix by taking $\gamma\equiv0$, $\delta\equiv0$ (for the ρ_L -meson the factor C=2 in Eq.(20) has to be taken into account).

As we have proved in our previous works [3,4] the running coupling constant method allows us to estimate power-suppressed corrections to $F_M(Q^2)$. The infrared matching scheme [15] also allows one to make such estimation. In the framework of this method one explicitly divides power corrections from the full expression by introducing moments of α_S at low scales as new non-perturbative parameters. By freezing one of variables ($\langle y\rangle$=1/2) in Eq.(4) we can express $Q^2F_M(Q^2)$ in terms of moment integrals $f_p(Q)$ defined as

$$f_p(Q) = \frac{p}{Q^p}\int_0^Q dk\,k^{p-1}\alpha_S(k^2)\ . \qquad (21)$$

After some calculations we get

$$Q^2F_M(Q^2) = C\cdot64\pi f_M^2\sum_{k=1}^{N}\frac{l_k(Q^2)}{k}f_{2k}\left(\frac{Q}{\sqrt2}\right), \qquad (22)$$

where the functions $f_{2k}\left(Q/\sqrt2\right)$ have the form [15]

$$f_{2k}\left(\frac{Q}{\sqrt2}\right) = 2^k\left(\frac{\mu}{Q}\right)^{2k}f_{2k}(\mu)+\alpha_S\left\{1-\Gamma(1,2\kappa z)+\frac{\beta_0\alpha_S}{4\kappa\pi}\left[1-\Gamma(2,2\kappa z)\right]+\right.$$
$$\left.+\left(\frac{\beta_0\alpha_S}{4\kappa\pi}\right)^2\left[2-\Gamma(3,2\kappa z)\right]+\cdots\right\}. \qquad (23)$$

In Eq. (23) μ=2GeV is the infrared matching scale, $z=\ln\left(Q/\sqrt2\mu\right)$, $\alpha_S\equiv\alpha_S\left(Q^2/2\right)$ and $\Gamma(n,x)$ is the incomplete gamma function.

The value of the non-perturbative parameter f_2 (2GeV)≈0.5 is obtainable from the experimental data [16]. For $p\ge4$ we have to use the running coupling constant method for their calculation. It is not difficult to find that

$$f_4(2\text{GeV}) \approx 0.347,\ f_6(2\text{GeV}) \approx 0.329,\ \ f_8(2\text{GeV}) \approx 0.322,\ \ f_{10}(2\text{GeV}) \approx 0.318\ . \tag{24}$$

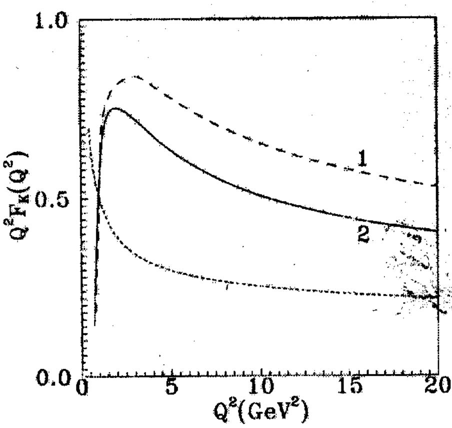

Fig.1. The K^+ meson elm form factor as a function of Q^2. The curve 2 is $Q^2F_K(Q^2)$ calculated using Eq. (20), the curve 1 is the same ff, but obtained by neglecting the dependence of $\Phi_K(x,Q^2)$ on Q^2. The short-dashed curve in frozen coupling pQCD result (Q^2 dependence of $\Phi_K(x,Q^2)$ is taken into account).

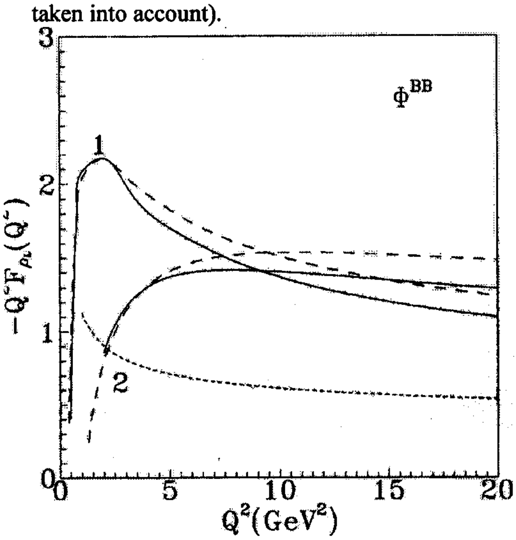

Fig.2. The ρ_L^+-meson ff as a function of Q^2. The curves 1 (solid and dashed) are found using Eq.(20),the curves 2–Eq.(18). The solid curves are obtained by taking into account the dependence of the ρ_L^+-meson DA on the factorization scale Q^2, the dashed ones –neglecting this dependence.The short-dashed curve is ordinary pQCD result.

The some results of our numerical calculations are shown in Figs.1, 2. As is seen, the dependence of the mesons' DAs on the factorization scale Q^2 changes the elm form factors $Q^2F_M(Q^2)$ considerably; in particular, this is true for the kaon. Indeed, the form factors $Q^2F_M(Q^2)$ calculated in the framework of the running coupling constant method by including the dependence of $\Phi_M(x,Q^2)$ on Q^2, in the region of small Q^2, are larger than the ones obtained neglecting this dependence and become smaller in the region of large values of Q^2. The size of these regions depends on the choosen renormalization scale (1 or 2 running variables x, y) and on the meson under consideration. Thus, for the kaon in the region $0.7\text{GeV}^2 \le Q^2 \le 1.2\text{GeV}^2$ the ff with Q^2 dependence is larger than old one, whereas for the ρ_L-meson (1 running variable case) this region is $0.5\text{GeV}^2 \le Q^2 \le 0.7\text{GeV}^2$. In the both cases for Q^2 larger than upper bounds of these domains, the elm ff with Q^2 dependence are smaller than the old ones. In the case with two running variables (or $\mu_R^2 = Q^2(1-x)(1-y)$) the situation is the same, but the regions under discussion are shifted towards large values of Q^2; for example, for the ρ_L-meson now we have: $2\text{GeV}^2 \le Q^2 \le 4\text{GeV}^2$. The qualitatively same picture is valid also for the pion (does not depicted).

As we have demonstrated in our works [3,4] (see, also [13,14]) both of the methods used in this paper allow us to estimate power-suppressed corrections to $F_M(Q^2)$. These corrections enhance the ordinary pQCD (i.e., frozen coupling constant approximation) predictions approximately two times. This conclusion is also reliable, when we take into account Q^2 dependence of the mesons' DAs.

The more detailed analysis of $F_M(Q^2)$ requires considering the next-to-leading order correction to T_H [5], as well as the next term in expansion of $\alpha_S(\lambda Q^2)$ in terms of $\alpha_S(Q^2)$. These problems will be studied in our forthcoming publications.

APPENDIX

The functions $I_k(Q^2)$ for kaon have the following expressions:

$$I_1(Q^2) = \left(\frac{\alpha}{2} + \frac{\beta}{6}\right)(\alpha + \beta) + \left(\frac{\gamma}{6} + \frac{\delta}{10}\right)(\gamma + \delta) + \frac{\delta}{5}\left(\alpha + \frac{4}{9}\beta\right) + \frac{\gamma}{9}(2\alpha + \beta),$$

$$I_2(Q^2) = -\left(\frac{\alpha}{2} + \frac{\beta}{6}\right)(\alpha + 5\beta) - \left(\frac{\gamma}{6} + \frac{\delta}{10}\right)(3\gamma + 7\delta) - \frac{\gamma}{9}(5\alpha + 4\beta) - \delta\left(\frac{6\alpha}{5} + \frac{5\beta}{9}\right),$$

$$I_3(Q^2) = 4\beta\left(\alpha + \frac{\beta}{3}\right) + \left(\frac{\gamma}{3} + \frac{\delta}{5}\right)(\gamma + 9\delta) + \delta\left(3\alpha + \frac{19}{15}\beta\right) + \frac{\gamma}{3}\left(\alpha + \frac{5}{3}\beta\right),$$

$$I_4(Q^2) = -2\beta\left(\alpha + \frac{\beta}{3}\right) - 10\delta\left(\frac{\gamma}{3} + \frac{\delta}{5}\right) - \frac{2}{9}\beta\gamma - \frac{2\delta}{3}\left(5\alpha + \frac{28}{15}\beta\right),$$

$$I_5(Q^2) = \frac{4\delta}{3}\left(\alpha + \gamma + \frac{1}{3}\beta + \frac{1}{15}\delta\right),$$

where $\alpha, \gamma, \beta, \delta$ are given in Eqs.(13), (14).

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YÜNGÜL MEZONLARIN ELEKTROMAQNİT FORMFAKTORLARININ FAKTORİZASİYA MİQYASINDAN ASILILIĞI

Məqalədə dəyişən qarşılıqlı tə`sir sabiti və infraqırmızı in`ikas metodları vasitəsilə psevdoskalyar (pion, kaon) və vektor (ρ_L – mezon) mezonların elektromaqnit formfaktorları $F_M(Q^2)$ hesablanmışdır. Mezonların paylanma funksiyalarının $\Phi_M(x,Q^2)$ faktorizasiya miqyası Q^2–dan asılılığı nəzərə alınmışdır. Hər iki metod üstlü azalan düzəlişlərin $F_M(Q^2)$–ə verdiyi düzəlişləri qiymətləndirməyə imkan verdiyi göstərilmişdir. Hesablamalarda mezonların KXD cəmləmə qaydaları çərçıvəsində alınmış paylanma funksiyalarından istifadə olunmuşdur.

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ЗАВИСИМОСТЬ ЭЛЕКТРОМАГНИТНЫХ ФОРМ-ФАКТОРОВ ЛЕГКИХ МЕЗОНОВ ОТ МАСШТАБА ФАКТОРИЗАЦИИ

Электромагнитные форм-факторы $F_M(Q^2)$ псевдоскалярных (пион, каон) и векторных (ρ_L –мезон) мезонов вычислены с помощью методов бегущей константы связи и инфракрасного отображения. Учтена зависимость мезонных функций распределений $\Phi_M(x,Q^2)$ от масштаба факторизации Q^2. Продемонстрировано, что оба метода позволяют оценить вклад в $F_M(Q^2)$ степенно-подавленных поправок. При вычислениях использованы мезонные функции распределения, полученные в рамках правил сумм КХД.

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