# BULK SPIN-WAVE REGIONS IN A FERROMAGNETIC SUPERLATTICE

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A superlattice consisting of alternating layers of two simple-cubic Heiserberg ferromagnetics is considered. The bulk-spin wave regions for spin waves propagating in a general direction in the superlattice are derived by the Green function method. The numerical results are illustrated.

There has been a growing interest in magnetic superlattices in the last few years. Theoretically, many investigations have been done with different models and methods [1-3]. The qualitative features of spin waves are most easily illustrated for the simple cubic structure. Some features of the layered structures can be explained in terms of modified single-film properties. The aim of this paper is to study properties of a ferromagnetic superlattice with quantum Heisenberg spins at finite temperature by the Green function method.

We consider a simple cubic ferromagnetic superlattice model in which the atomic planes of material 1 alternate with atomic planes of material 2. Each atomic plane is assumed to be the [001] planes. The materials are taken to be single-cubic Heisenberg ferromagnetic, having exchange constant $I_1$ and $I_2$ lattice constant $a$. The exchange constant between constituents is $I$.

The Heisenberg Hamiltonian for the system is

$$H = -\frac{1}{2}\sum_{i,j} I_{ij}(\vec{S}_i \vec{S}_j) - \sum_i g m_B (H_0 + H_i^{(A)}) S_i^z \quad , \qquad (1)$$

where $H_0$ is the internal field, which is assumed to be parallel to the spins along the $z$ axis and $H_i^{(A)}$ ($i$=1,2) anisotropy field for a ferromagnetic with simple uniaxial anisotropy along the $z$ axis. The definition of retarded Green function in real space is $G_{ij}(t,t') = \langle\langle S_i^+(t); S_j^-(t')\rangle\rangle$. Writing the equation of motion for $G_{ij}(t,t')$ and employing the random-phase approximation one obtains equations for Green function. Furthermore, to exphasize the layered structure we shall use the following the frequency and two-dimensional Fourier transformations

$$G_{ij}(t,t') = \frac{1}{\pi^2}\int dk_{11}\, exp\left[ik_{11}(r_i - r_j)\int dw G_{nn}(w,k_{11})\, exp[-iw(t-t')]\right] \quad , \qquad (2)$$

where $k_{11}$ is two-dimensional wave vector, $w$ is spin-wave frequency, $n$ and $n'$ are indices of the layers to which $r_i$ and $r_j$ belong, respectively. Assuming that $n$-th layer is of the material 1 and $(n+1)$-th layer is of the material 2, one obtains the following set of equations

$$\begin{cases}(E - A_1)g_{n,n'} + r(g_{n+1,n'} + g_{n-1,n'}) = 2S_n\delta_{n,n'} \\ (E - A_2)g_{n+1,n'} + r(g_{n+2,n'} + g_{n,n'}) = 2S_{n+1}\delta_{n+1,n'}\end{cases} \quad , \qquad (3)$$

where $E=(w-m_B H_o)/6I_1S$ and $g=G(w,k)\cdot 6I_1S$.

The system is also periodic in the $z$ direction, which lattice constant is $d=2a$. According to Bloch's theorem we introduce the following plane waves [4]

$$g_{n+2,n'} = g_{n,n'}\, exp(ik_z d) \quad . \qquad (4)$$

Using (4) the set of equation (3) may be written under the following matrix form

$$\begin{pmatrix} E - A_1 & T^* \\ T & E - A_2 \end{pmatrix}\begin{pmatrix} g_{n,n'} \\ g_{n+1,n'} \end{pmatrix} = \begin{pmatrix} 2S_n\delta_{n,n'} \\ 2S_{n+1}\delta_{n+1,n'} \end{pmatrix} \quad , \qquad (5)$$

where $T=r(1+exp(ik_z d)$ and $T^*$ is the complex conjugate of $T$. The Green functions are obtained by solving the equation (5).

$$g_{nn} = \frac{C_1}{E - E_{k1}} + \frac{C_2}{E - E_{k2}}$$
$$g_{n+1,n+1} = \frac{D_1}{E - E_{k1}} + \frac{D_2}{E - E_{k2}} \qquad (6)$$

The spin -wave spectrum was obtained as like the poles of he Green function.

$$E_k = 0.5 \cdot [A_1 + A_2 \pm \sqrt{(A_1 - A_2)^2 + 8 \cdot r^2(1 + \cos k_z d)}] \quad . \qquad (7)$$

Equations (6-7) are the main result of this paper. It can be verified from equations (7) that when both media are identical, $I_1=I_2=I$ the well-known expression of bulk-spin waves dispersion equation for constituents is obtained. In fig.1-3 the result numerically illustrated for a particular choice of parameters. Fig.2 shows the spin-wave regions for the superlattice as a function of the quantity $q$, while fig. 1 shows those for the components 1 and 2. Fig.3 describe the spin-wave regions for the superlattice as a function of $\varepsilon$ for $q$=0.2 and $q$=1. All these figures correspond $-1 \le \cos k_z d \le 1$. The analysis of the results was shows that the width of the bulk-spin waves regions in the superlattice was depended on transverse components of the wave vectors and exchange interaction between constituents. As shown in the Fig.1 the width of the bulk-spin waves regions of constituents does not depend on transverse components of the wave vectors.

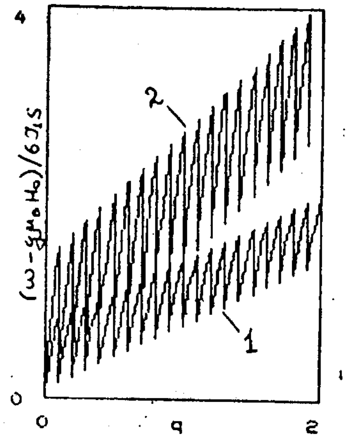


*Fig.1.* The bulk spin-wave regions for the components 1 and 2 as a function of transverse components of the wave vectors. $d_1$=0.007, $d_2$=0.008, $\alpha$=2.

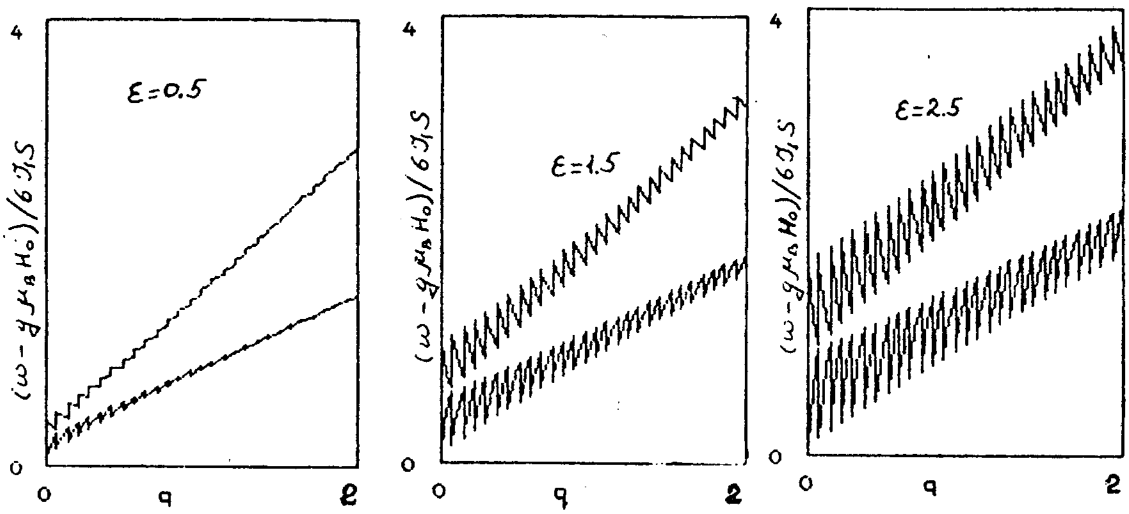


*Fig.2.* The bulk spin-wave regions in the superlattice as a function of transverse components of the wave vectors for different values of $c$ $d_1$ =0007, $d_2$=0.008, $\alpha$=2.

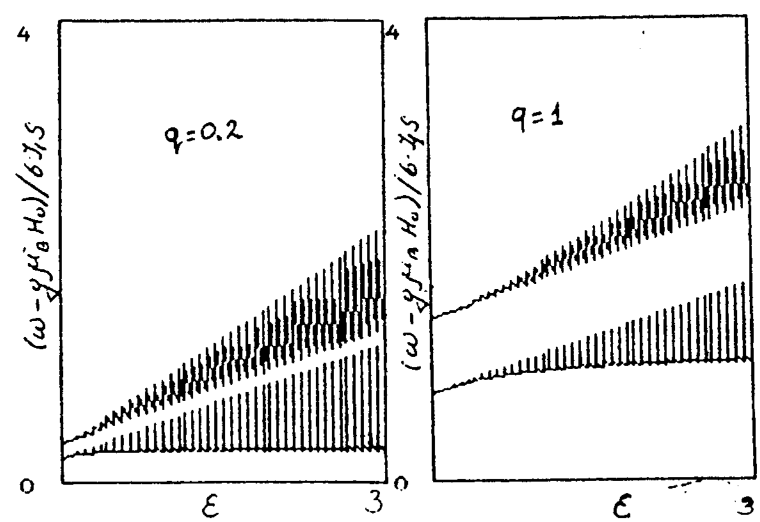


*Fig.3* The bulk spin-wave regions in the superlattice as a function of exchange interaction between constituents, $d_1$=0.007, $d_2$ =0.008, $\alpha$=2.

## APPENDIX

The terms appearing in the equations (3-7) are

$$A_1 = d_1 + \frac{1}{3}\varepsilon + \frac{2}{3}q$$

$$A_2 = d_2 + \frac{1}{3}\varepsilon + \frac{2}{3}q$$

where

$$d_i = \frac{g\mu_B H_i^{(A)}}{6J_1 S} \qquad (i = 1,2);$$

$$q = 1 - \frac{1}{2}(\cos k_x a + \cos k_y a)$$

$$\delta = \frac{J}{J_1}, \quad \alpha = \frac{J_2}{J_1}, \quad r = \frac{J}{6J_1},$$

$$C_1 = \frac{-2S(E_{k1} - A_2)}{E_{k1} - E_{k1'}}, D_1 = \frac{-2S(E_{k1} - A_1)}{E_{k1} - E_{k1'}}$$

$$(1, 1' = 1,2; 1 \neq 1')$$


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## FERROMAQNİT İFRAT QƏFƏSDƏ HƏCM SPİN DALĞA ZONASI

İki sadə kubik Heyzenberq ferromaqnitdən təşkil olunmuş ifrat qəfəsə baxılır. Qrin funksiyası metodundan istifadə edərək ifrat qəfəsin oxu boyu yayılan spin dalğaları üçün həcm spin-dalğa zonaları müəyyən edilmişdir. Nəticə kəmiyyətcə təsvir olunmuşdur.

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## ОБЛАСТЬ ОБЪЕМНЫХ СПИНОВЫХ ВОЛН В ФЕРРОМАГНИТНЫХ СВЕРХРЕШЕТКАХ

Рассмотрена сверхрешетка, состоящая из чередующихся слоев простых кубических Гейзенберговских ферромагнетиков. Методом функции Грина определена область спиновых волн, распространяющихся вдоль оси сверхрешетки. Результаты представлены численно.

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