DETERMINATION OF THE NUCLEAR RADIUS AND SOME PARAMETERS OF PLANETARY NEBULAES

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The method of determination of the central star radius of planetary nebulae has been proposed, when its optical thickness beyond the Layman series considerably exceeds unity.

At known parameters and data, for many investigated nebulae appropriate determined radius of a central star it is possible to estimate other parameters of these objects, which in many cases cannot be determined because of the uncertainty.

It is known, that if the optical thickness of nebulae behind a limit of Layman series is considerably more than unity, then the number of L_c-quantums, radiated by a central star, will be equal to the number of Balmer quantums radiated by the nebula. In this case we have [1]:

$$\int_{x_0}^{\infty} \frac{x^2 dx}{e^x - 1} = \sum_{Ba} A_i \frac{x_i^3}{e^{x_i} - 1}, \quad (1)$$

where

$$x = \frac{h\nu}{kT_*}, \quad x_i = \frac{h\nu_i}{kT_*}, \quad x_o = \frac{h\nu_o}{kT_*}.$$

T_* - the temperature of the central star, ν_i - the frequency of i-th Balmer line, ν_o - the frequency of boundary of Layman series), A_i - dimensionless quantity is expressed by the relation

$$A_i = \frac{E_i}{\nu_i E_i^*}, \quad (2)$$

where E_i - the total energy, radiated by the nebula in i-th Balmer line, E_i^* - the energy radiated by a star in a unit interval of frequencies near i-th Balmer line, which is expressed by a relation

$$E_i^* = 4\pi R_*^2 \pi I_{\nu_i}^* \quad (3)$$

In (3) R_* - the radius of a star, $I_{\nu_i}^*$ - the mean intensity of star radiation corresponding to the frequency ν_i near i-th Balmer line. In (2) E_i - is determined as:

$$E_i = E_{i2} = n_i A_{i2} h\nu_{2i} V, \quad (4)$$

where n_i - the concentration of hydrogen atoms in an i-th state, A_{i2} - Einstein coefficient of spontaneous transitions $i \to 2$, V - volume of the nebula, flashing in Balmer line. The nebula is accepted to be a sphere of radius R and filled homogeneously by ionized substance. As it was mentioned in [2], though it is only simplified model, it apparently, well represents a true state, especially it is difficult to construct more common model, which could easily be applied to all nebulae.

At first we shall estimate an integral in the left part (1), taking into account that

$$\frac{1}{e^x - 1} = \sum_{n=0}^{\infty} e^{-(n+1)x}.$$

Then we shall have

$$\int_{x_0}^{\infty} \frac{x^2 dx}{e^x - 1} = \sum_{n=0}^{\infty} \int_{x_0}^{\infty} e^{-(n+1)x} x^2 dx$$

Restricting by the three addends in the previous expression and substituting it in (1) we have:

$$\sum_{n=0}^{\infty} e^{-(n+1)x_0} \left[\frac{x_0^2}{n+1} + \frac{2x_0}{(n+1)^2} + \frac{2}{(n+1)^3} \right] = \sum_{Ba} A_i \frac{x_i^3}{e^{x_i} - 1} \quad (5)$$

The summation in the right part of (5) is extended to all Balmer series lines and on Balmer continuum. At known temperature of a central star T_*, it is possible to estimate the left part of expression (5); in this case the quantum $\sum_{Ba} A_i \frac{x_i^3}{e^{x_i} - 1}$ can be considered to be known for the given nebula. Taking into consideration (3) and (4) it is possible to present the formula (2) as

$$A_i = \frac{n_i A_{i2} hV}{4\pi^2 I_{\nu_i}^* R_*^2} \quad (6)$$

Here

$$n_i = b_i n_e n^+ \frac{i^2 h^3}{(2\pi m k T_e)^{3/2}} e^{x_i / kT_e}, \quad (7)$$

$$I^*_{\nu_i} = \frac{2h\nu_i^3}{c^2}\frac{1}{e^{h\nu_i/kT_*} - 1}, \quad (8)$$

$$V = \frac{4\pi}{3}R^3. \quad (9)$$

In (9) R - is radius of nebula.

Let us accept $R=d\sin\theta$, where d - distance up to nebula, θ - its angular radius. Thus, at known appropriate parameters it is possible to estimate the quantity

$$B_i = \frac{n_i A_{i2} h V}{4\pi^2 I^*_{\nu_i}}. \quad (10)$$

In other words parameter A_i is expressed by a relation

$$A_i = \frac{B_i}{R_*^2}$$

Then instead of (5), we shall have

$$R_*^2\sum_{n=0}^{\infty} e^{-(n+1)x_0}\left[\frac{x_0^2}{n+1} + \frac{2x_0}{(n+1)^2} + \frac{2}{(n+1)^3}\right] = \sum_{Ba} B_i\frac{x_i^3}{e^{x_i} - 1} \quad (11)$$

With the formula (11) we estimate the radius of the central star.

As an example we shall consider well -known investigated nebula NGC 7027. Most probable distance up to this nebula 1 кps, angular radius 4",0, i.e. linear radius is very small ($6\cdot10^{16}$ sm); besides a central star is rather weak (apparent visual star value 19^m,4). It is known, that when nebula is very small or the central star is very weak, the continuous radiation in visible region is badly measured color and Zanztra temperature are determined uncertainly. In this case temperature of a central star is determined by a Stoa method; the temperature of a nucleus of nebula NGC 7027 is also determined by this method T_*=295$\cdot10^3$ K. Knowing that electronic temperature T_e in this nebula is $1{,}4\cdot10^4$ K and the electron concentration is equal to $8\cdot10^4$ sm^{-3}, for Balmer lines H_α, H_β, H_γ, H_δ, H_ε, H_8, H_9, H_{10} (and also for Balmer continuum) we estimated E_{k2}, $I^*_{\nu_i}$, $\frac{E_i^*}{R_*^2}$, B_i, $\frac{x_i^3}{e^{x_i}-1}$, $\left(B_i\frac{x_i^3}{e^{x_i}-1}\right)$; taking into account, that in quantums radiated in one H_β line approximately is equal to those of all Balmer continuum [1].

The results of calculations are given in the table:

Table

Balmer lines and continuum	$E_{k2}\times10^{35}$ [erg/s]	$I^*_{\nu_i}\times10^2$ [erg/sm$^2\cdot$s$\cdot$Hz$\cdot$s$_r$]	E_i^*/R_*^2	$B_i\times10^{-21}$	$\frac{x_i^3}{e^{x_i}-1}\times10^2$	$B_i\frac{x_i^3}{e^{x_i}-1}\times10^{-19}$
H_α	11.80	1.83	0.722	2.8700	0.539	1.5500
H_β	3.960	2.34	0.923	0.6960	0.952	0.6630
H_γ	1.980	4.03	1.590	0.1810	1.200	0.2160
H_δ	1.150	4.50	1.780	0.0886	1.320	0.1170
H_ε	0.741	5.04	1.972	0.0443	1.420	0.0629
H_8	0.451	5.64	2.070	0.0221	1.490	0.0329
H_9	0.3090	6.21	2.230	0.0085	1.530	0.0130
H_{10}	0.0704	6.63	2.350	0.0051	1.560	0.0080
Ba_c						0.6630

$$\sum_{Ba} B_i\frac{x_i^3}{e^{x_i} - 1} = 3{,}33\cdot10^{19}$$

If the left part of the equation (11) is equal to $2{,}26\,R_*^2$ then we shall have the follows

$$R_* = 3{,}84\cdot10^9 \text{ sm}$$

From the literature the radius of the nebula NGC 7027 is not known for us, but its luminous is known. According to our estimation we have R for a luminous:

$$\frac{L_*}{L_0} = \left(\frac{R_*}{R_0}\right)^2\left(\frac{T_*}{T_0}\right)^4 \cong 2\cdot10^4$$

According to [2] for this nebula $\frac{L_*}{L_0}\le10^4$, which is obtained from a relation between a luminosity L and total flux F, i.e. from $L=4\pi d^2F=4\pi d^2 10^2F(H_\beta)$ [erg/s]. This relation takes place only for nebulae, which optical thickness is more than unity in Balmer continuum, and the luminosity in this case is determined by total flux of energy radiated by the nebula in lines and in a continuum wave lengths region λ>912 Å, the total flux is approximately estimated as $100F(H_\beta)$, where $F(H_\beta)$ - the radiation flow in a line is H_β. In [2] it is supposed, that this value can differ from true no more, than two times, except for cold stars, for which this estimation is too much lowered.

At d=1 kps=$3{,}086\cdot10^{21}$ sm, $F(H_\beta)=10^{-8{,}81}$[erg/sm$^2\cdot$s] (taking into account the interstar absorption) for a relative luminosity of a nucleus of the nebula NGC 7027 is really obtained

$\sim 5 \cdot 10^3$. At least, according to the above mentioned remarks and with respect to total flux, it is possible to expect, that the value of a relative luminosity of the central star of the considered nebula, obtained by us, should be more competent.

Thus, by determining nuclear radius of the optically thick nebula in Layman continuum, it is possible to estimate its luminosity with a greater accuracy as well.

Now we shall consider questions, connected with determination of some parameters, which in many cases are unknown or cannot be determined at all because of uncertainties.

At known distances and angular sizes of the nebula, by estimating their linear dimensions R and knowing values of the radiuses of the central stars, from the known formula $W = \frac{1}{4}\left(\frac{R_*}{R}\right)^2$ it is also possible to determine the dillusion coefficient W.

Particularly, for the nebula NGC 7027 $W = 10^{-15}$. Certainly, it is necessary to have in mind, that here constancy of W for every nebula, which can take place only at geometrically small thickness of the nebula in comparison with its radius.

According to formula (7) by estimating the concentration of the hydrogen atoms in i-th state n_i, with formula of distribution of hydrogen atoms in excited states, i.e. on

$$\frac{n_i}{n_1} = PW\frac{T_*}{T_e} b_i \frac{g_i}{g_1} e^{\frac{\chi_i}{kT_*} - \frac{\chi_1}{kT_*}}, \qquad (12)$$

it is possible to estimate concentration of atoms of hydrogen in the ground state n_1. In (7) and (12) quantum b_i indicating in how many times the ratio of the concentration of hydrogen atoms in a i-th state, n_i to product of concentrations of free electrons and protons n^+ in the nebulae differs from value such relation $\frac{n_i}{n_e n^+}$ in a state of thermodynamic equilibrium with the electron temperature T_e; in (12) factors P- the portion of electron captures on the first level. It is also accepted $P = 1/2$; besides in conditions of nebulae it is possible to consider, that $n^+ = 0{,}8 n_e$; at last, the magnitude of the value $b_i e^{\chi_i / kT_e}$ has been calculated by Citon [1].

For the nebula NGC 7027 it is obtained $n_1 = 1{,}94 \cdot 10^2 \text{sm}^{-3}$.

Knowing n_1, according to the formula of ionization in nebula with the high optical thickness τ, i.e. on

$$n_e \frac{n^+}{n_1} = \frac{g^+}{g_1} W \sqrt{\frac{T_e}{T_*}} \frac{2(2\pi mkT_*)^{3/2}}{h^3} e^{-\frac{h\nu_1}{kT_*}} e^{-\tau} \qquad (13)$$

it is possible to estimate the optical distance τ from a star beyond the border of basic series, in particular Layman series, corresponding to some average absorption coefficient.

Knowing $\frac{n_1}{n^+ + n_1}$ and also R and n_e [2], according to the

$$\tau_0 = \chi \frac{n_1}{n^+ + n_1} n_e R \qquad (14)$$

it is possible to estimate an optical thickness of the nebula in Layman continuum; in (14) $\chi = 6{,}3 \cdot 10^{-18} \text{sm}^2$ – the photoelectric absorption coefficient of hydrogen at once behind Layman series limit.

For the nebula NGC 7027 by known values from (14) $\tau_0 = 91$ has been obtained. Let's note, that the value $\frac{n_1}{n^+ + n_1}$ up to now is not the measurable value, as for as the estimation of this value needs the knowledge of a radiation flow of a central star in Layman continuum. For this reason at an estimation of corresponding values for all nebulae $\frac{n_1}{n^+ + n_1} = 10^{-3}$, while for the nebula NGC 7027 we have $\frac{n_1}{n^+ + n_1} = 3 \cdot 10^{-3}$, i.e. is eliminated uncertainty in numerical value of the expression $\frac{n_1}{n^+ + n_1}$.

Applying shown in this work method of determination of the radius of the central star of planetary nebulae and receiving ways of an estimation of some parameters on the example of NGC 7027 nebula to the planetary nebulae with major optical thickness in L_c-e, in particular we can construct more exact Herthshprung-Ressel diagram for central stars in the units of Solar, to determine the numerical values for many selected nebulae of such parameters, as the coefficient dillusion, concentration of hydrogen atoms in the ground state, optical thickness and optical distance directly behind the Layman series and may be some other parameters.

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PLANETAR DUMANLIQLARIN NÜVƏSİNİN RADİUSU VƏ BƏ'Zİ PARAMETRLƏRİNİN TƏ'YİNİ

Layman seriyasının sərhəddindən e'tibarən planetar dumanlığın optik qalınlığı vahiddən xeyli böyük olan halda mərkəzi ulduzun radiusunun tə`yini metodu təklif edilmişdir.

Tədqiq olunan dumanlıqların çoxunun uyğun parametrlərinə istinadən mərkəzi ulduzun tapılan radiuslarına görə bu obyektlərin indiyədək qeyri-müəyyənlik nəticəsində tə`yini mümkün olmayan digər parametrlərini də qiymətləndirmək olur.

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ОПРЕДЕЛЕНИЕ РАДИУСА ЯДРА И НЕКОТОРЫХ ПАРАМЕТРОВ ПЛАНЕТАРНЫХ ТУМАННОСТЕЙ

Предложен метод определения радиуса центральной звезды планетарной туманности, когда её оптическая толщина за границей серии Лаймана значительно превосходит единицу.

При известных для многих изученных туманностей соответствующих параметров и данных по найденному радиусу центральной звезды можно оценить и другие параметры этих объектов, которые во многих случаях невозможно определить из-за неопределенностей.

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