ON THE DISTRIBUTION OF ELECTROMAGNETIC FIELDS' IN QUASI-CONDUCTORS

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Possibility of uniform intensity distribution (absence of skin-effect), when electromagnetic wave is falling on quasi-conductor, has been shown and correspondent to this case waveform and it's alternating ratio has been determined. Variational interpretation of considered phenomenon has been provided.

1. INTRODUCTION

Firstly phenomenon of non-uniform electromagnetic field and current distribution has been discovered in conductors, i.e. in substances, in which this effect, called skin-effect, is displaying most definitly. Some authors determine skin-effect as current concentration near surface of conductor [1,2], or as field localization in the thin surface layer [3], that corresponds to its name as well as to its essence in the case of electromagnetic field interaction with typical conductors.

However, undoubtedly non-uniform distribution of current and electromagnetic fields with decreasing into conductor deep is, generally, taking place for any substances with unequal to zero electric conduction in alternating electromagnetic fields. That is why main questions and notions of skin-effect theory have later been generalized on, formally, noncomprising class of substances with relatively low electric conduction s and, first of all, ground [4-10], including polarized ground [6,8,11]. Definition of skin-effect, given in [12], as non-uniform distribution of current on surface is most general, because it allows to comprise cases of electromagnetic field influence on quasi-conductors with relatively low electric conduction.

All the cases considered in sources can be divided into two groups:

1. electromagnetic wave falling from vacuum (or air) on quasi-conductor with finite and unequal to zero electric conduction . This group is characterized by exponential damping of electric and magnetic fields, decreasing of current density along field direction;
2. electromagnetic wave is falling on ideal dielectric (with zero electric conduction) under which there is not any field damping.

Thus, in all cases of electromagnetic wave falling on bodies with finite electric conduction , field damping (under other equal conditions) must be proportional to electric conduction of the body.

At the same time complicated wave resistance of quasi-conductor dependence of frequency of acting field and properties of substance in comparison with typical conductors, extraordinary cases are also possible such as uniform field and current distribution (i.e. absence of skin-effect) when wave falls on quasi-conductor with finite and unequal to zero electric conduction .

As distinct from certain cases of uniform space distribution of alternating fields, taking place in typical dielectrics [1-3,12], substance (for example, quasi-conducting ground), in further considering problem, can possess by the significant (in comparison with dielectrics) specific conductance γ (γ = 10^{-4}-10^{-1} Siemens/m for ground) and inductive resistance ωL under quasi-electric conduction boundary condition $\gamma >> \omega\varepsilon$ (meanings are specified below) which has opposite sign for quasi-conductors. It means mathematically that imaginary parts of complex wave number and complex wave resistance can be essential while imaginary parts of the same values for dielectrics are considerably less. That is why uniform alternating electromagnetic field distribution in quasi-conductors is unlikely to take place and, as we know, has not been considered in sources.

2. THEORY

The following problem is being considered. The plane wave falls on the surface of infinite half-space occupied by homogeneous isotropic quasi-conductor with specific conductance γ, magnetic permeability μ and dielectric constant ε.

Placing coordinate system on the surface of quasi-conductor and directing z - axis into conductor deep (see [12], for instance) from equations:

$$rot\,\overline{E} = -\mu\partial\,\overline{H}/\partial t\quad, \tag{1}$$

$$rot\,\overline{H} = \gamma\overline{E} + \varepsilon\partial\,\overline{E}/\partial t \tag{2}$$

it is possible to get well known (telegraph) equations:

$$\partial^2\,\overline{E}/\partial z^2 - \mu\gamma\partial\,\overline{E}/\partial t - \mu\varepsilon\partial^2\,\overline{E}/\partial t^2 = 0\quad, \tag{3}$$

$$\partial^2\,\overline{H}/\partial z^2 - \mu\gamma\partial\,\overline{H}/\partial t - \mu\varepsilon\partial^2\,\overline{H}/\partial t^2 = 0\quad. \tag{4}$$

We find solutions of (3) and (4) equations, satisfying to following relations

$$\partial\,\overline{E}/\partial z = 0,\ \partial\,\overline{H}/\partial z = 0\quad. \tag{5}$$

If is formally impied that

$$\overline{E}\,(z,t)=E_0(z)\exp(i\omega t) \tag{6}$$

$$\overline{H}(z,t)=H_0(z)\exp(i\omega t) \quad . \tag{7}$$

where $E_0(z), H_0(z)$ - complex amplitudes of intensity of electric and magnetic fields correspondingly; ω - cyclic frequency of monochromatic wave; i - imaginary unit.

Substituting (6) and (7) in to (3) and (4) taking into account relations(5), one can find for complex dielectric constant

$$\varepsilon(\omega)=\varepsilon-i\gamma/\omega = 0 \; . \tag{8}$$

From (6), (7) and (8) with (1) it is possible to write the following equations

$$\overline{E}(z,t)=E_0\exp[(-\gamma/\varepsilon)t] \; , \tag{9}$$

$$\overline{H}(z,t)=0\cdot\exp[(-\gamma/\varepsilon)t] = 0 \; . \tag{10}$$

Thus, with exponential time dependence (9) the uniform $E_x(z,t)$ distribution along z -axis takes place. Under this condition wave resistance of quasi-conductor has the following form:

$$z=[\mu/\varepsilon(\omega)]^{1/2} = \infty \; . \tag{11}$$

It should be mentioned that $\omega=i\gamma/\varepsilon$ is pole of spectral density of exponential function (9), which can be obtained from [13]. Physical mean of relation (10) is total reflection of $\overline{H}$ -component of field. At a glance received result formally resembles with the process of capacitor discharge at self-leakance in case of uniform field distribution with exponential voltage drop.

However, as mentioned above, quasi-conductors have considerably more active electric conduction than dielectrics. This quasi-conductors can have magnetic properties and their time constant $\tau=\varepsilon/\gamma$ are considerably less than those of dielectrics. Therefore the best analogy is comparing with long line regime with leakage conductivity G and capacity C in case of voltage drop $U=U_0\exp[(-G/C)t]$ when wave resistance becomes infinite and current wave is being reflected totally.

In distinction of the case of the total reflection of $\overline{E}$ - and $\overline{H}$ -components when superconductivity (superconductors) takes place and case of total refraction quasi-elastic dipole) the relations (5) have the exponential time dependence for expectional case with damping ratio δ, totally determined by γ and ε parameters of quasi-conducting substance. One should note that in anisotropic substances it is possible to obtain more compound quasi-conductor and field interaction in consequence of dependence $\varepsilon(\omega)$ on direction.

3. VARIATIONAL INTERPRETATION

Now let consider the problem with point of view of fundamental variation principle of the least action. Lagrange function L of electromagnetic field and action S, according to [14], transformed according to System International (SL) have the following forms:

$$L = 1/2\int_{(v)}(\varepsilon E^2 - \mu H^2)dV \quad , \tag{12}$$

$$S = 1/2\int_{t_1}^{t_2}\int_{(v)}(\varepsilon E^2 - \mu H^2)dVdt \, , \tag{13}$$

where dV - volume element of quasi-conductor. To take into account dissipation factor ε in (12), (13) we must use (8) in complex form. Taking into consideration that we are considered monochrome waves propagation changing in time and z-coordinate it is possible to write

$$L = 1/2\int_{(l)}(\varepsilon(\omega)E^2 - \mu H^2)dl \quad , \tag{14}$$

$$S = 1/2\int_{t_1}^{t_2}\int_{(l)}(\varepsilon(\omega)E^2 - \mu H^2)dVdt \, , \tag{15}$$

where dl – length element along z-axis

In order to use Euler's variational equation in (14) and (15) one must change Lagrange function. Therefore we introduce the charge density function which is equal to z - coordinate charge partial derivative $q(z,t)$, i.e.:

$$\lambda(z,t)=\partial q(z,t)/\partial z \; , \tag{16}$$

and having therefore the dimension of linear charge density, i.e. Cl/m. Intensities of electric and magnetic fields are proportional to the line charge density and its time derivative correspondingly:

$$E \sim (\partial\lambda/\partial z)/\varepsilon(\omega) \quad , \tag{17}$$

$$H \sim (\partial\lambda/\partial t) \quad . \tag{18}$$

Then Lagrange function (14) has the following form:

$$L = 1/2\int_{(l)}\left[A_1(\partial\lambda/\partial z)^2/\varepsilon(\omega) - A_2\mu(\partial\lambda/\partial t)^2\right]dl \quad , \tag{19}$$

where A_1 and A_2 dimensionless proportional coefficients

Designating partial derivatives in (19) on λ_z' and λ_t' correspondingly we have:

$$\partial/\partial z \cdot \partial l/\partial\lambda_{z'} = [2A_1/\varepsilon(\omega)]\int_{(l)}(\partial^2\lambda/\partial z^2)dl \quad , \tag{20}$$

$$\partial/\partial t \cdot \partial 1/\partial\lambda_{t'} = -2A_2\mu\int_{(1)}(\partial^2\lambda\ /\partial t^2)dl \qquad (21)$$

Taking into account (20) and (21) we can obtained from Euler's equation the following expression:

$$\int_{(1)}\left[-\ A_1(\partial^2\lambda\ /\partial z^2)/\varepsilon(\ \omega) + A_2\mu((\partial^2\lambda\ /\partial t^2)\right] = 0 \qquad (22)$$

As long as (7) equation shall be truth at any segment 1 along z - axis, we have:

$$\partial^2\lambda/\partial z^2-\{\ (A_2/A_1)^{1/2}[\varepsilon(\omega)\mu]^{1/2}\}\partial^2\lambda/\partial t^2=0 \qquad (23)$$

This equation is single measure wave equation (not using complex dielectric constant meaning we would have obtained the telegraph type equation, such as [1] and [3]).

According to solving problem we are not interested the solution of the boundary-value problem corresponding to (23). We just, need prove existence of uniform field distribution in guasi-conductor under the electromagnetic wave influence.

Indeed, from relation $\partial\lambda/\partial z=0$, taking into consideration

$$\partial^2\lambda/\partial t^2\neq 0\ ,\ \mu\neq 0 \qquad (24)$$

we have the following result: $\varepsilon(\omega)=0$, i.e relation (8) obtained above is based on principle of least action. Moreover, from (23) it follows that in case of alternating electromagnetic field $\partial\lambda/\partial t=var$ skin-effect takes place, i.e. $\partial\lambda/\partial z=var$ (except of the case of exponential wave damping ratio considered above). Last statement is based on equations we used, which can be obtained from Schvartchild's variational principle. Thus distributions of fields in substances (for example in quasi-conductors) satisfy to relation of minimum action.

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DƏYİŞƏN SAHƏLƏRİN KVAZİKEÇİRİCİLƏRDƏ PAYLANMASI HAQQINDA

Dəyişən sahələrin kvazikeçiricilərdə müntəzəm paylanmasının (səth effektinin mövcud olmamasının) mümkünlüyü göstərılm göstərilmişdir. Səth effekti ən kiçik tə'sir variasiya prinsipi nöqteyi nəzərindən yozulmuşdur.

Т.М. Лазимов

О РАСПРЕДЕЛЕНИИ ПЕРЕМЕННЫХ ПОЛЕЙ В КВАЗИПРОВОДНИКАХ

Показана возможность равномерного распределения переменных полей (отсутствие скин-эффекта) в квазипроводниках. Дана вариационная интерпретация скин-эффекта.

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