DIFFERENCE HARMONIC OSCILLATORS. III.

SH.M. NAGIEYV
Institute of Physics of Academy of Sciences of Azerbaijan
Baku, 370143, pr. G. Javid,33

The Meixner difference oscillator model is introduced, using well-known properties of the Meixner polynomials and its difference equation.

6. Meixner difference oscillator

In the previous papers [1,2] we are generalazed the factorization method to the case of the difference Schrodinger equation and constructed the models of q-oscillator. Here we introduce a Meixner difference oscillator model [3]. The Meixner polynomials are solutions of the difference equation ($h=m=\omega=1$) [4]

$$\left[\gamma(x+\beta)e^{\partial_x}+xe^{-\partial_x}-(1+\gamma)\left(x+\beta/\gamma\right)+(1+\gamma)\left(n+\beta/2\right)\right]M_n(x;\beta,\gamma)=0 \quad (6.1)$$

They form a two-parameter family of polynomials for $\beta>0$ and $0<\gamma<1$, and satisfy to the recurrence relation

$$[n+(n+\beta)\gamma-(1-\gamma)x]M_n(x;\beta,\gamma)=(n+\beta)\gamma M_{n+1}(x;\beta,\gamma)+nM_{n-1}(x;\beta,\gamma) \quad (6.2)$$

We consider functions

$$\psi_n(x)=(-1)^n\sqrt{\rho(x)/d_n}\,M_n(x;\beta,\gamma), \quad (6.3)$$

where $\rho(x)$ and d_n are the weight function and square norm for the Meixner polynomials

$$\rho(x)=\frac{(\beta)_x\gamma^x}{x!},\quad d_n=\frac{n!}{\gamma^n(\beta)_x(1-\gamma)^\beta} \quad (6.4)$$

As follows from (6.1) and (6.4), these functions satisfy to the equation $H\psi_n=E_n\psi_n$, where the difference Hamiltonian operator for the Meixner difference oscillator has the form:

$$H(x)=\frac{1+\gamma}{1-\gamma}\left(x+\frac{1}{2}\beta\right)-\frac{\sqrt{\gamma}}{1-\gamma}\left[\mu(x)e^{\partial_x}-e^{-\partial_x}\mu(x)\right],\quad \mu(x)=\sqrt{(x+1)(x+\beta)} \quad (6.5)$$

The energy spectrum of the Meixner difference oscillator is equal to

$$E_n=n+\frac{\beta}{2},\quad n=0,1,2,\dots \quad (6.6)$$

The wave functions (6.3) satisfy to the discrete orthogonality relation

$$\sum_{x=0}^{\infty}\psi_n(x;\beta,\gamma)\psi_m(x;\beta,\gamma)=\delta_{nm} \quad (6.7)$$

From the recurrence relation (6.2), it can be shown that the following limit to the Hermitian polynomials holds:

$$\lim_{\nu\to\infty}(2\nu)^{n/2}M_n\left(\frac{\nu+\sqrt{2\nu}\cdot x}{1-\gamma};\frac{\nu}{\gamma},\gamma\right)=(-1)^nH_n(x) \quad (6.8)$$

Furthermore, measures and normalization coefficients relate as

$$\lim_{\nu\to\infty}\sqrt{(2\nu)}\left(1-\frac{\nu}{\gamma}\right)^{\frac{\nu}{\gamma}-1}\rho\left(\frac{\nu+\sqrt{2\nu}x}{1-\gamma}\right)=\frac{1}{\pi}e^{-x^2},\quad \lim_{\nu\to\infty}(2\nu)^{n/2}\left(1-\frac{\nu}{\gamma}\right)^{\frac{\nu}{2\gamma}}d_n=\sqrt{2^n n!} \quad (6.9)$$

Hence, the wave functions (6.3) with argument $\left(\nu+\sqrt{2\nu x}\right)/(1-\gamma)$ and $\beta=\nu/\gamma$, coincide in the limit $\nu\to\infty$ with the wave functions of the linear harmonic oscillator (3.8) [1].

[1] *Sh.M. Nagiyev.* Fizika,1998, v.4, № 4, pp.39.

[2] *Sh.M. Nagiyev.* Fizika, 2000, v.6, № 1, pp. 57.

[3] *N.M.Atakishiyev, E.I. Jafarov, Sh.M. Nagiyev, K.B.Wolf.* Rev. Mexic.Fisica, 1998, v. 44, pp. 235.

[4] *R.Koekoek, R.F. Swarttouw.* Report 94-05, Delft University of Technology, 1994.

Ş.M. Nağıyev

SONLU-FƏRQ HARMONİK OSSİLYATORLAR.III.

Meyksner çoxhədlilərinin mə'lum xassələri və onların sonlu-fərq tənliyi əsasında Meyksner sonlu-fərq ossilyator modeli qurulmuşdur.

Ш.М. Нагиев

РАЗНОСТНЫЕ ГАРМОНИЧЕСКИЕ ОСЦИЛЛЯТОРЫ.III.

На основе известных свойств полиномов Мейкснера и их разностного уравнения построена разностная модель осциллятора Мейкснера.

Дата поступления: 05.01.00 *Редактор: И.Г. Джафаров*