THERMOELECTROMOTIVE FORCE OF NONDEGENERATE KANE SEMICONDUCTORS UNDER THE CONDITIONS OF MUTUAL ELECTRON-PHONON DRAG IN A STRONG ELECTRIC FIELD

M.M. BABAEV, T.M. GASSYM
Institute of Physics of Academy of Sciences of Azerbaijan
370143, Baku, H. Cavid av., 33

The thermoelectromotive force of nondegenerate Kane semiconductors with due regard for the electron and phonon heating and their mutual drag is investigated. The electron spectrum is taken in the Kane two-band form. It is shown that the nonparabolicity of electron spectrum significantly influences on quantity of thermoelectromotive force and leads to the change of its dependence on the electron temperature T_e as well as on the heated electric field. Under the conditions of strong mutual electron-phonon drag, thermoelectromotive force mainly consists of the phonon part. In these conditions for the parabolic case the thermoelectromotive force increases as $\sim T_e^{3/2}$, and for the strong nonparabolic case as $\sim T_e^{3}$.

The interest to the studies of thermoelectromotive force in different systems under the conditions of carrier heating at the high external electric field has recently been intensified [1-3]. Lei [1], Xing, Liu, Dong and Wang [2] were discussed the thermoelectromotive force under the conditions of carrier heating at the external strong electric field neglecting the contribution of the phonon drag, which is very important at low temperatures of lattice [4]. The role of the phonon drag in thermoelectromotive force of hot carriers was studied by Wu, Horing and Cui [3], with taking into account only the drag of electrons by phonons (thermal drag), but the mutual drag of electrons and phonons was neglected. The thermoelectromotive force with regard for both the electron drag by phonons and their mutual drag was studied in [5]. In all the papers [1-3,5] the electron dispersion law was assumed to be parabolic. In publication [6] the thermoelectromotive force of hot electrons in strongly degenerated semiconductors for two-band Kane spectrum of electrons was discussed.

In this paper a theoretical study of thermoelectromotive force of nondegenerate Kane semiconductors placed at strong electric field with taking into account both the drag of heated electrons by phonons and their mutual drag is carried out. The phonons were assumed to be heated or nonheated. Two-band Kane spectrum of electrons is assumed [4]:

$$p(\varepsilon) = (2m_n\varepsilon)^{\frac{1}{2}}\left(1 + \frac{\varepsilon}{\varepsilon_g}\right)^{\frac{1}{2}}, \qquad (1)$$

where m_n is the electron effective mass at the bottom of the conduction band, ε_g is the band gap, p and ε are the electron momentum and energy, respectively.

The considered physical process is the thermoelectric Seebeck effect in the presence of a heated electric field $\vec{E}$ and an electron temperature gradient ∇T_e, which can be produced by the gradient of $\vec{E}$ or by a lattice temperature gradient ∇T.

The basic equations of the problem are the coupled Boltzmann transport equations for electrons and phonons. The case of quasi-elastic electron scattering by acoustic phonons is considered. For the considered case the distribution functions of electrons $f(\vec{p}, \vec{r})$ and phonons $N(\vec{q}, \vec{r})$ may be presented in the form:

$$f(\vec{p}, \vec{r}) = f_0(\varepsilon, \vec{r}) + \vec{f}_1(\varepsilon, \vec{r})\frac{\vec{p}}{p}, \quad |\vec{f}_1| \ll f_0, \qquad (2)$$

$$N(\vec{q}, \vec{r}) = N_0(q, \vec{r}) + \vec{N}_1(q, \vec{r})\frac{\vec{q}}{q}, \quad |\vec{N}_1| \ll N_0. \qquad (3)$$

Here f_o and $\vec{f}_1$, N_o and $\vec{N}_1$ are the isotropic and anisotropic parts of the electron and phonon distribution functions, respectively.

If the inter-electronic collision frequency ν_{ee} is much more than the collision frequency of the electrons for the energy transfer to lattice ν_ε, then $f_o(\varepsilon, \vec{r})$ is the Fermi distribution function with an electron temperature T_e. We consider the case, when for long-wavelength (LW) phonons there is a "thermal reservoir" of short-wavelength (SW) phonons: $q_{max} \approx 2\bar{p} \ll \frac{T}{s_0}$, where s_o is the sound velocity in the crystal, q_{max} is the maximum quasi-momentum of LW phonons. In this case $N_o(q, \vec{r})$ has the form [7]

$$N_0(q, \vec{r}) \approx \frac{T_p(\vec{r})}{s_0 q}, \qquad (4)$$

where T_p is the effective temperature of the LW phonons.

Starting from the Boltzmann transport equations we obtain the following equations for $\vec{f}_1$ and $\vec{N}_1$ in the steady state:

$$\frac{p}{m(\varepsilon)}\nabla f_0 - e\overline{E_c}\,\frac{p}{m(\varepsilon)}\frac{\partial f_0}{\partial \varepsilon} + \nu(\varepsilon)\vec{f}_1 + \frac{2\pi m(\varepsilon)}{(2\pi\hbar)^3 p^2}\frac{\partial f_0}{\partial \varepsilon}\int_0^{2p}\vec{N}_1(q)W(q)\hbar\omega_q q^2 dq = 0\ , \quad (5)$$

$$S_0\nabla N_0 + \beta(q)\vec{N}_1 - \frac{4\pi m(\varepsilon)}{(2\pi\hbar)^3}W(q)N_0(q)\int_{q/2}^{\infty}\vec{f}_1 dp = 0 \quad (6)$$

Here e is the absolute value of the electronic charge, $\vec{E}_c = \vec{E} + \vec{E}_T$, $\vec{E}_T$ is the thermoelectric field, $m(\varepsilon)$ is the electronic effective mass, $\hbar\omega_q = s_0 q$ is the phonon energy, $W(q) = W_0 q^t$ is the square matrix element of the electron-phonon interaction (t=1 for deformation and t=-1 for piezoelectric interaction), $\beta(q)$ and $\nu(\varepsilon)$ are the total phonon and electron momentum scattering rates, respectively.

For the Kane semiconductors with the electron spectrum (1) the expressions of $m(\varepsilon)$ and $\nu(\varepsilon)$ have the form [4]:

$$m(\varepsilon) = m_n\left(1 + \frac{2\varepsilon}{\varepsilon_g}\right)\ , \quad (7)$$

$$\nu(\varepsilon) = \nu_0(T)\left(\frac{T_p}{T}\right)^l\left(1 + \frac{2\varepsilon}{\varepsilon_g}\right)\left(1 + \frac{\varepsilon}{\varepsilon_g}\right)^{-r}\left(\frac{\varepsilon}{T}\right)^{-r}, \quad (8)$$

where $r=3/2, l=0$ for the scattering of electrons by impurity ions and $r=-t/2, l=1$ for the scattering of electrons by acoustic phonons. When LW phonons are scattered by SW phonons or by crystal boundaries, $\beta(q)$ doesn't depend on the spectrum of electrons and has the form [7]

$$\beta_p(q) = \frac{T^4}{4\pi\rho\hbar^4 s_0^4}q\ , \quad \beta_b(q) = \frac{s_0}{L}\ , \quad (9)$$

where the indices p and b denote the scattering by SW phonons and crystal boundaries, ρ and L are the density and the minimum size of specimen, respectively. When LW phonons are scattered by electrons, $\beta(q)$ depends on the spectrum of electrons and for the spectrum (1) we obtain:

$$\beta_e(q) = \left(\frac{m_n s_0^2}{8\pi T_e}\right)^{1/2}\frac{NW_0}{T_e}\left(1 + \frac{2T_e}{\varepsilon_g}\right)^2\left(1 + \frac{3T_e}{2\varepsilon_g}\right)^{-3/2}q^t\ , \quad (10)$$

where N is the concentration of electrons.

Solving the coupled equations (5) - (6) by the same way as in [5] it is easy to calculate the electric current density of electrons [4]

$$\vec{j} = -\frac{e}{3\pi^2\hbar^3}\int_0^{\infty}\vec{f}_1(\varepsilon)p^2(\varepsilon)d\varepsilon\ . \quad (11)$$

Let us direct external electric field along the x axis, and the gradient of lattice temperature (or the gradient of external electric field) along the z axis. Under this conditions from equation $j_z=0$ we obtain the following expressions for the electron (α_e) and phonon (α_p) parts of the thermoelectromotive force (α):

$$\alpha = \alpha_e + \alpha_p;\ \alpha_e = -\frac{\beta_{11}^{(e)}}{\sigma_{11}};\ \alpha_p = -\frac{\beta_{11}^{(p)}}{\sigma_{11}}\ , \quad (12)$$

where

$$\sigma_{11} = \int_0^{\infty}a(x)[1 + b(x)]dx\ , \quad x = \frac{\varepsilon}{T_e} \quad (13)$$

$$\beta_{11}^{(e)} = \frac{1}{e}\int_0^{\infty}a(x)\left\{x - \frac{\zeta(T_e)}{T_e} + \left[1 - \frac{\zeta(T_e)}{T_e}\right]b(x)\right\}dx\ , \quad (14)$$

$$\beta_{11}^{(p)} = \frac{1}{e}\int_0^{\infty}a(x)\{\lambda(x) + \lambda(\vartheta_e)b(x)\}dx,\quad x = \frac{\varepsilon}{T_e},\ \vartheta_e = \frac{T_e}{T},\ \vartheta_p = \frac{T_p}{T}\ . \quad (15)$$

Here $\zeta(T_e)$ is the chemical potential of hot electrons,

$$a(x) = \frac{e^2}{3\pi^2\hbar^3}\frac{p^3(x)}{m(x)\nu(x)}\exp\left[\frac{\zeta(T_e)}{T_e} - x\right], \quad (16)$$

$$b(x) = \frac{\gamma(x)}{1 - \gamma(\vartheta_e)}\frac{m(x)}{m(\vartheta_e)}\frac{\nu(x)}{\nu(\vartheta_e)}\ , \quad (17)$$

$$\gamma(x) = \frac{3 + t}{(2p)^{3+t}}\frac{\nu_p(x)}{\nu(x)}\int_0^{2p}\frac{\beta_e(q)}{\beta(q)}q^{2+t}dq\ , \quad (18)$$

$$\lambda(x) = \frac{3+t}{(2p)^{3+t}}\frac{m(x)s_0^2}{T_p}\nu_p(x)\int_0^{2p}\frac{1}{\beta(q)}q^{2+t}dq\,, \quad (19)$$

$\nu_p(x)$ is the electron scattering frequency by phonons. The coefficient $\lambda(x)$ characterizes the efficiency of the thermal drag, whereas $\gamma(x)$ describes the same for the mutual drag.

Because of the complexity of general analysis of expressions (12) - (15), later we examine the dependence of electron momentum on its energy in the form

$$p(\varepsilon) = (2m_n\varepsilon_g)^{1/2}\left(\frac{\varepsilon}{\varepsilon_g}\right)^s \quad , \qquad (20)$$

which for the spectrum (1) corresponds to the parabolic ($T_e \ll \varepsilon_g, s=1/2$) and strongly nonparabolic ($T_e \gg \varepsilon_g, s=1$) cases, respectively. In these cases $m(\varepsilon)$, $\nu(\varepsilon)$ and $\beta(q)$ may be presented in the form:

$$m(\varepsilon) = 2sm_n\left(\frac{\varepsilon}{\varepsilon_g}\right)^s \quad , \qquad (21)$$

$$\nu(\varepsilon) = 2s\nu_0(T)\vartheta_p^l\left(\frac{\varepsilon}{\varepsilon_g}\right)^{(2s-1)(1-r)}\left(\frac{\varepsilon}{T}\right)^{-r}, \qquad (22)$$

$$\beta(q) = \beta(T)\vartheta_e^{n(s-2)}\left(\frac{T}{\varepsilon_g}\right)^{n\left(s-\frac{1}{2}\right)}\left(\frac{s_0q}{T}\right)^k, \qquad (23)$$

where $n=1, k=t$ for scattering of LW phonons by electrons, $n=0, k=0$ for the scattering by crystal boundaries and $n=0, k=1$ when LW phonons are scattered by SW phonons.

For the spectrum (20) from the expressions (12) - (19) we obtain:

$$\alpha_e = -\frac{1}{e}\left(1 + C_1\frac{\gamma_0}{1-\gamma_0}\right)^{-1}\left\{3 - s + 2sr - \frac{\zeta(T_e)}{T_e} + \left[1 - \frac{\zeta(T_e)}{T_e}\right]C_1\frac{\gamma_0}{1-\gamma_0}\right\} \quad , \qquad (24)$$

$$\alpha_p = -\frac{1}{e}\frac{C_2 + (C_1 - C_2)\gamma_0}{1 + (C_1 - 1)\gamma_0}\frac{(3+t)2^{2-\frac{3k}{2}}s^2}{3+t-k}\left(\frac{m_ns_0^2}{T}\right)^{1-\frac{k}{2}}\left(\frac{T\vartheta_e}{\varepsilon_g}\right)^{\left(s-\frac{1}{2}\right)(4+t-k-n)}\vartheta_e^{\frac{3n+t-k}{2}}\frac{\nu_{\varepsilon 0}(T)}{\beta(T)}, \qquad (25)$$

where

$$C_1 = \frac{\Gamma(1 + 3s + 2sr + 2st - sk)}{\Gamma(3 - s + 2sr)}, \quad C_2 = \frac{\Gamma(1 + 3s + 2sr + st - sk)}{\Gamma(3 - s + 2sr)}, \qquad (26)$$

$$\gamma_0 = \frac{(3+t)2^{\frac{3(t-k)}{2}}}{3+2t-k}\left(\frac{m_ns_0^2}{T}\right)^{\frac{t-k}{2}}\left(\frac{T\vartheta_e}{\varepsilon_g}\right)^{\left(s-\frac{1}{2}\right)(2r+2t-k-n+1)}\vartheta_e^{r+t+\frac{3n-3-k}{2}}\vartheta_p^{l-1}\frac{\beta_e(T)}{\beta(T)}\frac{\nu_{\varepsilon 0}(T)}{\nu_0(T)} \quad , \qquad (27)$$

For the spectrum (20) the chemical potential of non-degenerate electrons takes the form:

$$\zeta(T_e) = T_e\ ln\frac{3\pi^2\hbar^3N}{\Gamma(1+3s)(2m_nT)^{3/2}}\left(\frac{T}{\varepsilon_g}\right)^{-3\left(s-\frac{1}{2}\right)}\vartheta_e^{-3s} \quad . \qquad (28)$$

It is seen from (24) and (25) the nonparabolicity of electron spectrum significantly influences on quantity of thermoelectromotive force of hot carriers and leads to the change of its dependence on the electron temperature. For the all real scattering mechanisms $4+t-k-n>0$, consequently the nonparabolicity of spectrum leads to a more rapid growth of phonon part of thermoelectromotive force α_p with increasing of T_e.

Consider the limiting cases $\gamma_0 \ll 1$ and $\gamma_0 \to 1$. Under conditions of weak mutual drag from (24) and (25) we have

$$\alpha_e = -\frac{1}{e}\left\{3 - s + 2sr - \frac{\zeta(T_e)}{T_e} - C_1(2 - s + 2sr)\gamma_0\right\} \quad , \qquad (29)$$

$$\alpha_p = -\frac{1}{e}\left[C_2 + C_1(1 - C_2)\gamma_0\right]\frac{(3+t)2^{2-\frac{3k}{2}}s^2}{3+t-k}\left(\frac{m_n s_0^2}{T}\right)^{1-\frac{k}{2}}\left(\frac{T\vartheta_e}{\varepsilon_g}\right)^{\left(s-\frac{1}{2}\right)(4+t-k-n)}\vartheta_e^{\frac{3n+t-k}{2}}\frac{v_{f0}(T)}{\beta(T)}, \quad (30)$$

Since $C_1>0$ and $2-s+2sr\geq 0$ for the all real scattering mechanisms and for each spectrum of electrons with $s\geq 1/2$, from (29) we obtain that the mutual drag leads to the decrease of α_e both in the parabolic and nonparabolic cases.

Let us consider now the thermoelectromotive force under conditions of the strong mutual electron-phonon drag. This takes place when the electrons and phonons are scattered mainly by each other ($k=t, n=1, r=-t/2, l=1$). In this case $\vartheta_p=\vartheta_e$ and from (27) we obtain $\gamma_0 = \frac{\beta_e(T)}{\beta(T)}\frac{v_{f0}(T)}{v_0(T)} \to 1$.

Consequently (24) and (25) takes the forms:

$$\alpha_e = -\frac{1}{e}\left\{1 - \frac{\zeta(T_e)}{T_e}\right\}, \quad (31)$$

$$\alpha_p = -\frac{1}{e}\frac{4\sqrt{2}(2s)^2}{3\pi^{3/2}}\left(\frac{T}{\varepsilon_g}\right)^{3\left(s-\frac{1}{2}\right)}\frac{(m_n T)^{3/2}}{\hbar^3 N}\vartheta_e^{3s}. \quad (32)$$

The decrease of electron part of thermoelectromotive force [illegible] fluence of mutual drag [illegible] can see also from the comparison (31) with (29). As it follows from (28) for the nondegenerate electrons

$$\frac{(m_n T)^{3/2}}{\hbar^3 N}\left(\frac{T}{\varepsilon_g}\right)^{3\left(s-\frac{1}{2}\right)} \approx \exp\left[-\frac{\zeta(T)}{T}\right] >> 1, \quad (33)$$

and from the comparison (31) with (32) it is seen that under the conditions of strong mutual drag $\alpha_p >> \alpha_e$, i.e. the thermoelectromotive force mainly consists of the phonon part.

As it follows from (32) the nonparabolicity of electron spectrum strongly changes the dependence of thermoelectromotive force on the electron temperature. Under the conditions of strong mutual drag thermoelectromotive force increases as $\alpha \sim T_e^{3/2}$ for the parabolic, and as $\alpha \sim T_e^3$ for the strong nonparabolic cases.

The dependence of ϑ_e on electric field intensity in the absence of mutual drag ($\gamma_0 \to 0$) were considered elsewhere [8]. Here we investigate the E-dependence of ϑ_e under the conditions of the strong mutual drag ($\gamma_0 \to 1$). In this case $\vartheta_p=\vartheta_e$ and the electron temperature is determined from the energy balance equation

$$\sigma_{11}(\vartheta_e)E^2 = W_{pp}(\vartheta_e), \quad (34)$$

[illegible] the power transferred by the LW phonons to the "thermal reservoir" of the SW phonons. Here we consider the following limiting cases:

$$1)\ \frac{\beta_p+\beta_b}{\beta_e} << \frac{v_i}{v_p};\quad 2)\ \beta_p >> \beta_b,\ \frac{\beta_p}{\beta_e} >> \frac{v_i}{v_p};\quad 3)\ \beta_p << \beta_b,\ \frac{\beta_b}{\beta_e} >> \frac{v_i}{v_p}. \quad (35)$$

The obtained results at $\vartheta_p=\vartheta_e>>1$ are shown in the Table. As it seen from Table the nonparabolicity of electon spectrum strongly changes the E - dependence of electron temperature.

Using the Table one can easily obtain the dependence of themoelectric power on the heated electric field in the considered cases.For instance if in (35) inequality 1) is satisified then $\alpha_p \sim E^2$ in the parabolic, and $\alpha_p \sim E^{3/2}$ in the strong nonparabolic cases.

Table. Dependences of ϑ_e on E in the conditions $\gamma_0 \to 1$

	$s = 1/2$	$s = 1$
Case 1)	$\vartheta_e \sim E^{4/3}$	$\vartheta_e \sim E^{1/2}$
Case 2)	$\vartheta_e \sim E^{1/3}$	$\vartheta_e \sim E^{1/5}$
Case 3)	$\vartheta_e \sim E^{4/11}$	$\vartheta_e \sim E^{2/9}$

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M.M. Babayev, T.M. Qasım

GÜCLÜ ELEKTRİK SAHƏSİNDƏ YERLƏŞDİRİLMİŞ CIRLAŞMAMIŞ KEYN YARIMKEÇİRİCİLƏRİNİN QARŞILIQLI ELEKTRON-FONON SÖVQÜ ŞƏRAİTİNDƏ TERMOELEKTRİK HƏRƏKƏT QÜVVƏSİ

Elektron və fononların qızması və qarşılıqlı sövqü şəraitində cırlaşmamış Keyn yarımkeçiricilərinin termoelektrik hərəkət qüvvəsi tədqiq edilmişdir. Elektronların spektri ikizonalı Keyn formasında götürülmüşdür. Göstərilmişdir ki, elektron spektrinin geyri-

parabolikliyi termoelektrik hərəkət qüvvəsinin qiymətinə güclü tə'sir edir və onun elektron temperaturu T_e-dən, eləcə də qızdırıcı elektrik sahəsinin intensivliyindən asılılıqlarını dəyişdirir. Güclü elektron-fonon sövqü şəraitində termoelektrik hərəkət qüvvəsi, əsasən, fonon hissədən təşkil olunur. Bu şəraitdə parabolik halda termoelektrik hərəkət qüvvəsi $\sim T_e^{3/2}$ kimi, güclü qeyri-parabolik halda isə $\sim T_e^{3}$ kimi artır.

М.М. Бабаев, Т.М. Гасым

ТЕРМОЭДС В НЕВЫРОЖДЕННЫХ КЕЙНОВСКИХ ПОЛУПРОВОДНИКАХ, НАХОДЯЩИХСЯ В СИЛЬНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЕ В УСЛОВИЯХ ВЗАИМНОГО УВЛЕЧЕНИЯ ЭЛЕКТРОНОВ И ФОНОНОВ

Исследована термоэдс в невырожденных Кейновских полупроводниках с учетом разогрева электронов и фононов, а также их взаимного увлечения. Спектр электронов предполагается Кейновским в двухзонном приближении. Показано, что непараболичность спектра значительно влияет на величину термоэдс и изменяет зависимости термоэдс от электронной температуры T_e, а также от греющего электрического поля. В условиях сильного взаимного увлечения термоэдс, в основном, состоит из фононной части. В этих условиях в параболическом случае термоэдс $\sim T_e^{3/2}$, а в сильно непараболическом случае $\sim T_e^{3}$.

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