NONLINEAR WAVE INTERACTION UNDER COHERENT POPULATION TRAPPING CONDITIONS

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The four-wave interaction under coherent population trapping conditions when two of three waves incident on the medium are strong has been investigated in the constant field approximation where, unless the constant field approximation, the reverse reaction of the excited waves on the exciting wave phase is taken into account. Moreover, the constant intensity approximation allows to account both the phase mismatch and the damping of all the interacting waves simultaneously. It has been shown that with certain parameter values of the problem considered, the use of the constant intensity approximation, rather than the constant field approximation, is more reasonable.

One of the perspective direction of the further development of a superhigh resolution spectroscopy, the creation of specific scheme of atomic cooling, amplification and generation of the light beams in the lack of population inversion and so on is due to coherent population trapping (CPT) phenomenon [1-4].

CPT is a special state of multilevel system, when the excitation of the upper levels because of absence of interaction of system with optical fields is impossible. In this state population is distributed between the bottom levels, occurs so-called "trapping" or capture in a coherent superposition basic and final excited state, and the intermediate state is not populated [5]. At CPT the system can not absorb or radiate resonant photons, the significant reduction of absorption of strong pump fields is observed at their propagation in the medium while the nonlinear polarization in this area is great enough. This effect is of interest in terms of increase in a nonlinear frequency mixing efficiency at the two- photon resonance in a multilevel system due to medium transparency for the strong pumping fields under certain conditions.

The three-frequency interaction [5-6] and four wave interaction [3] when two of three waves incident on the medium are strong have been considered for the case of CPT. To analyze the nonlinear frequency conversion efficiency, the authors investigated the effects of the wave propagation in a nonlinear medium in the constant pumping field approximation, i.e. without account of the change of interacting wave phases. In addition, the analysis of wave interaction in the constant intensity approximation [7] allows to take into account the reverse reaction of the excited waves on the exciting wave phases.

In the present paper a theoretical analysis of the four-wave interaction in a nonlinear medium with a simultaneous account both of the phase change and the damping of all interacting waves has been carried out in the constant intensity approximation.

Consider the Raman scattering of three laser radiations in a four-level system. In this case, two of four incident electromagnetic waves at frequencies ω_1 and ω_2 are assumed to be strong, and the third one at the frequency ω_3 is considered to be weak (Fig.1). As a result of a nonlinear interaction in atomic system, the fourth wave at the frequency $\omega_4=\omega_1-\omega_2+\omega_3$ is generated. It is assumed that the ground state is occupied initially, and the transition 2→0 is a dipole- forbidden transition. This is attributed to the fact that the coherent trapping is observed only in the case of two-photon excitation of a metastable level.

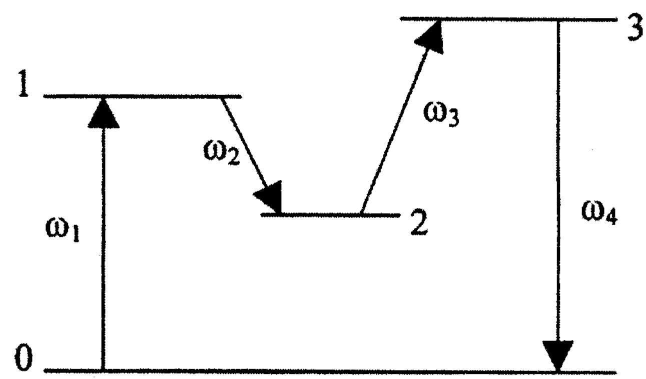

Fig.1. The scheme of transitions for the Raman scattering process under coherent population trapping conditions.

The reduced equations for the field amplitudes at a four-wave interaction of the plane waves propagating along the z-axis have the following form in a steady-state conditions [8]:

$$
\begin{aligned}
\frac{dA_1}{dz} + \delta_1 A_1 &= i\gamma_1 A_2 A_3^* A_4\, e^{-i\,\Delta z}, \\
\frac{dA_2}{dz} + \delta_2 A_2 &= i\gamma_2 A_1 A_3 A_4^* e^{i\,\Delta z}, \\
\frac{dA_3}{dz} + \delta_3 A_3 &= i\gamma_3 A_1^* A_2 A_4 e^{-i\,\Delta z}, \\
\frac{dA_4}{dz} + \delta_4 A_4 &= i\gamma_4 A_1 A_2^* A_3 e^{i\,\Delta z},
\end{aligned}
\qquad (1)
$$

where $A_{1,2}$ are the complex amplitudes of pumping waves at the frequencies $\omega_{1,2}$; A_3 is a complex amplitude of a weak laser wave at the frequency ω_3; A_4 is a complex amplitude of a weak wave generated in a nonlinear medium; δ_j are the absorption coefficients; $\gamma_j = 2\pi k_j\chi_j^{NL}$ are the nonlinear wave coupling coefficients; $\chi_j^{NL}(\omega_j)$ is the atomic nonlinear susceptibility responsible for generation at the frequency ω_j; $\Delta=k_1-k_2+k_3-k_4$ is the phase mismatch of interacting waves.

The boundary conditions in this case are

$$A_j(z=0)=A_{j0}\,(j=1-3)\ ;\ A_4(z=0)=0. \quad (2)$$

It is assumed that at the boundary of a nonlinear medium the coherent population trapping effect takes place.

Solving the system (1) in the constant intensity approximation with the boundary conditions (2), we obtain

$$A_3(z) = \frac{A_{30}}{4\lambda}\cdot e^{-\frac{\delta+2i\Delta-2\lambda}{4}z}\left[\delta - 2\delta_3 + 2\lambda + 2i\Delta + \left(2\delta_3 - \delta + 2\lambda - 2i\Delta\right)e^{-\lambda z}\right], \quad (3)$$

$$A_4(z) = i\frac{\gamma_4}{\lambda}A_{10}A_{20}A_{30}\cdot 2\sinh\frac{\lambda}{2}z\cdot e^{-(\frac{\delta}{4}+i\frac{\Delta}{2})z}, \quad (4)$$

where

$$\delta = \sum_1^4 \delta_j\ ;\ I_j = A_j\cdot A_j^*\ ;\ \lambda = \sqrt{\left(\frac{\delta-2\delta_4}{2}-i\Delta\right)^2 - 4\gamma_4(\gamma_1 I_{20}I_{30} - \gamma_2 I_{10}I_{30} + \gamma_3 I_{10}I_{20})}$$

Then, the formula for conversion efficiency $\eta=I_4/I_{30}$ at the output of a nonlinear medium of the length l is obtained:

$$\eta = 4\gamma_4^2 I_{10}I_{20}\cdot l^2\cdot e^{-\frac{\delta}{2}l}\left|\frac{\sinh\frac{\lambda l}{2}}{\lambda l}\right|^2. \quad (5)$$

For $\gamma_1=\gamma_2=0$, from (5) we obtain the result for the constant field approximation, ($\delta_{1,2}$=0) [3], where

$$\lambda' = \sqrt{\left(\frac{\delta_3-\delta_4}{2}-i\Delta\right)^2 - 4\gamma_3\gamma_4 I_{10}I_{20}}\,.$$

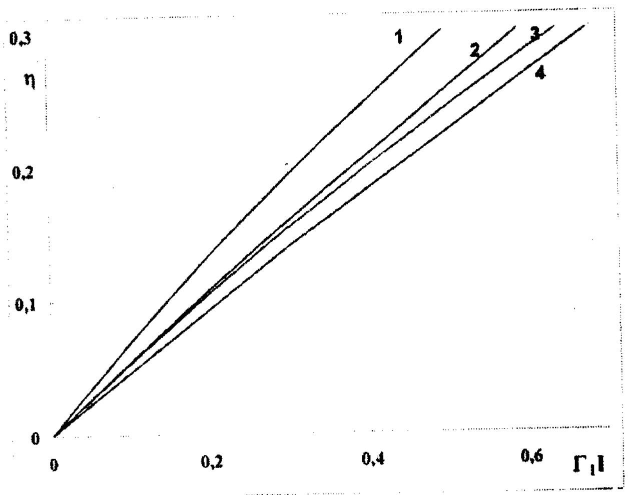

Fig.2. The dependence of conversion efficiency η on pumping intensity for Δl=1.5; δ_j = 0; γ = 0.7; $\Gamma_2 l$ = 0.9 and $\Gamma_3 l$ = 0.8 (1,2); $\Gamma_2 l$ = 0.7 and $\Gamma_3 l$ = 0.6 (3,4):
1, 3- in the constant field approximation;
2, 4- in the constant intensity approximation.

With a certain relationship of the frequencies ω_j, the values $4\gamma_4(\gamma_1 I_{20}I_{30}-\gamma_3 I_{10}I_{30})$ and $4\gamma_4\gamma_3 I_{10}I_{20}$ can be of the same order. In this case, in Eq. (5) the term $4\gamma_4(\gamma_1 I_{20}I_{30}-\gamma_3 I_{10}I_{30})$ in the parameter λ cannot be ignored, and therefore the use of the constant intensity approximation is more reasonable.

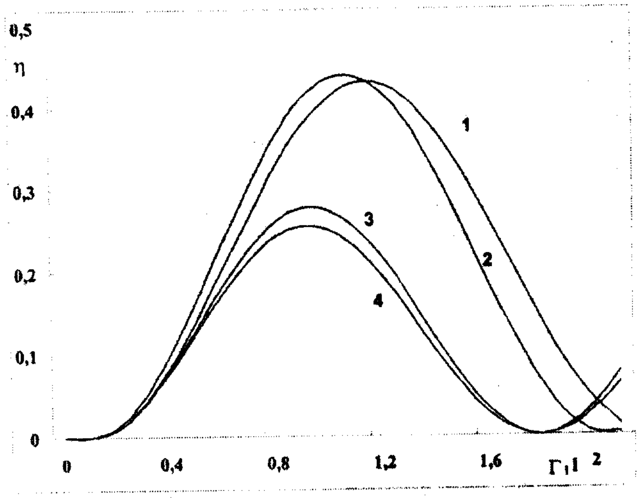

Fig.3. The dependence of conversion efficiency η on the reduced length of the nonlinear medium Γl in the constant intensity approximation for $\delta_{1,2}$ = 0; $\delta_3 = \delta_4$ γ=0,8; Γ_3/Γ_1=1; Δ/Γ_1=2 (1, 2), 3 (3, 4); δ_3/Γ_1=0.2 (1-3), 0.6 (4); Γ_2/Γ_1=1 (1,3,4), 1.2(2).

From (5) follows, that the efficiency of conversion is periodic function of length of nonlinear medium. Optimal length l^{opt}, at which the efficiency reaches the maximal value is determined by expression ($\delta_1=\delta_2=0$; $\delta_3=\delta_4$)

$$l^{opt} = \frac{1}{\lambda''}\,arctg\,\frac{2\lambda''}{\delta_3}, \quad (6)$$

where

$$\lambda'' = \sqrt{\frac{\Delta^2}{4} + \gamma_4\left[\left(\gamma_3 I_{10} + \gamma_1 I_{30}\right)I_{20} - \gamma_2 I_{10} I_{30}\right]}\,.$$

According to (6) as against result in the constant field approximation ($\gamma_{1,2}$=0) in the constant intensity approximation optimal length of a nonlinear medium depends besides intensity of waves pumping on intensity of a wave on frequency ω_3.

The dependences of conversion efficiency on pumping intensity at different interacting wave intensity values are shown in Fig.2. As would be expected, the account of the phase effects at four-wave interaction leads to decrease in frequency conversion efficiency as compared to the efficiency value in the constant field approximation. Such behaviour of curves received in the constant intensity approximation, is connected as against constant field approximation by reverse pumping of power of a excited wave in exciting.

The dependences of conversion efficiency on the reduced length of the nonlinear medium are presented in Fig.3. As seen in Fig.3, the curves have a distinct maximum. With increase of the phase mismatch Δ and the losses in the medium, the conversion efficiency decreases (curves 1,3 and 3,4). Moreover, as the value Δ, δ increases, the optimal length value in the nonlinear medium corresponding to the maximum frequency conversion decreases.

As seen from the above dependences, with increase of the phase mismatch Δ and the γ parameter, the difference in the results obtained in the constant intensity and constant field approximation decreases. Hence, at low γ and Δ the use of the constant intensity approximation is more reasonable. By choice of optimal value l^{opt} it is possible to achieve essential increase of efficiency of frequency conversion.

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ENERJİ SƏVİYYƏLƏRİNİN KOHERENT DOLDURULDUĞU ŞƏRAİTDƏ DALĞALARIN QEYRİ-XƏTTİ QARŞILIQLI TƏ'SİRİ

Enerji səviyyələrinin koherent doldurulduğu şəraitdə dörd dalğanın qarşılıqlı tə'siri araşdırılmışdır.Burada mühitə düşən üç dalğadan ikisinin güclü olduğu nəzərdə tutulur. Sabit amplitud yaxınlaşmasından fərqli olaraq, generasiya olunan dalğanın əsas dalğanın fazasına göstərdiyi əks tə'siri də nəzərə alan sabit intensivlik yaxınlaşmasında baxılmışdır.

Sabit intensivlik yaxınlaşmasında qarşılıqlı tə'sirdə olan dalğaların udulması və fazalar fərqinin qarşılıqlı tə'sir prosesinə tə`siri nəzərə alınmışdır.

Müəyyən edilmişdir ki, belə prosesləri araşdırmaq üçün sabit amplitud yaxınlaşması ilə müqayisədə sabit intensivlik yaxınlaşmasından istifadə etmək daha məqsədəuyğundur.

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НЕЛИНЕЙНОЕ ВЗАИМОДЕЙСТВИЕ ВОЛН ПРИ КОГЕРЕНТНОМ ПЛЕНЕНИИ НАСЕЛЕННОСТЕЙ

Проведено исследование четырех-волнового взаимодействия в условиях когерентного пленения населенности, когда два из трех падающих на среду волн являются сильными. Рассмотрение ведется в приближении заданной интенсивности, которая в отличие от приближения заданного поля учитывает обратную реакцию возбуждаемых волн на фазу возбуждающих волн. Кромс того даннос приближение позволяет одновременно учесть как фазовое рассогласование, так и затухание всех взаимодействующих волн.

Получено что, при определенных значениях параметров задачи целесообразно работать в приближении заданной интенсивности, чем в приближении заданного поля.

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