MESONS INFRARED RENORMALON CORRECTED DISTRIBUTION AMPLITUDES AND THE $\eta\gamma$, $\eta'\gamma$ TRANSITION FORM FACTORS

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The pseudoscalar η and η' mesons electromagnetic transition form factors $F_{\eta\gamma}(Q^2)$ and $F_{\eta'\gamma}(Q^2)$ are calculated using the frozen coupling constant approximation and the mesons' infrared renormalon corrected distribution amplitudes. In calculations the usual η-η' mixing scheme is employed. As input parameters the phenomenological values of the octet-singlet mixing angle θ = -15.4° and of the decay constants $f_1 \approx 0.108$ GeV and $f_8 \approx 0.116$ GeV are used. Comparison is made with the CLEO Collaboration data.

1. The meson-photon electromagnetic transition form factor (FF) $F_{M\gamma}(Q^2)$ is the simplest exclusive process for investigation of which the perturbative QCD (PQCD) methods can be used [1-4]. Within the PQCD standard hard scattering approach (HSA) the form factor $F_{M\gamma}(Q^2)$ can be expressed as the convolution of the hard scattering amplitude $T_H\left(x, Q^2; \mu_F^2, \mu_R^2\right)$ and the process-independent distribution amplitude (DA) $\Phi_M\left(x, \mu_F^2\right)$ of a corresponding meson. The hard scattering amplitude $T_H(x, Q^2; \mu_F^2, \mu_R^2)$ is calculable within PQCD and is known with $O(\alpha_S)$ order accuracy [5-7]. The mesons DA's $\Phi_M(x, \mu_F^2)$ are universal nonperturbative functions characterizing the mesons themselves. An information on their shape has to be deduced either from experimental data or from nonperturbative QCD computations.

Recently, the CLEO experimental data related with the $\eta\gamma$ and $\eta'\gamma$ transition FF's appeared [8]. These results together with the CELLO Collaboration data [9] form basis for theoretical investigations of the $\eta\gamma$ and $\eta'\gamma$ transition form factors and the η-η' mixing problem.

It is known that the physical η and η' states consist dominantly of a flavor $SU_f(3)$ octet η_8 and singlet η_1, respectively. In the usual η-η' mixing scheme we have

$$|\eta\rangle = \cos\theta|\eta_8\rangle - \sin\theta|\eta_1\rangle\ ,$$
$$|\eta'\rangle = \sin\theta|\eta_8\rangle + \cos\theta|\eta_1\rangle\ . \qquad (1)$$

The values of the mixing angle θ were extracted from the experimental data and were evaluated in various theoretical works. The transition FF's $\eta\gamma$ and $\eta'\gamma$ in the light-cone perturbation theory and conventional mixing scheme were computed [10] retaining the dependence of $T_H(x, k_\perp, Q^2)$ on the quark transverse momentum $k_\perp$ and using the wave function $\psi(x, k_\perp)$ modeled in accordance with the Brodsky-Huang-Lepage (BHL) prescription [11]. The numerical results show that there exists a gap between the data and the light-cone perturbation computation with BHL wave function.

In the modified HSA, in which the transverse degrees of freedom and the Sudakov FF are taken into account, the $\eta\gamma$ and $\eta'\gamma$ FF's were calculated in Ref.[12] and the value θ = -18°±2° was obtained. The more general mixing scheme with two mixing angles θ_1, θ_8 were also investigated [13, 14]. In these works the values of the parameters $\theta_1, \theta_8, f_1, f_8$ as well as the value of the mixing angle of the particle states θ = -15.4° were found [14]. The results of Refs.[12] and [13] are in good agreement with the data [8].

The $\eta\gamma$ and $\eta'\gamma$ electromagnetic transition FF's in the standard HSA and the conventional mixing scheme using the running coupling constant method [15,16] have been investigated recently [17]. In this paper the power suppressed corrections proportional to $\left(1/Q^2\right)^p$, $p = 1,2,3\ldots$ to $Q^2F_{\eta\gamma}(Q^2)$ and $Q^2F_{\eta'\gamma}(Q^2)$ have been evaluated and the agreement with CLEO data [8] has been obtained.

In this work we calculate the $\eta\gamma$ and $\eta'\gamma$ transition FF's in the standard HSA using the ordinary η-η' mixing scheme, the frozen coupling constant approximation and the η_1, η_8 mesons infrared (IR) renormalon corrected DA's [18]. The similar consideration of the $\pi^0\gamma$ FF has been fulfilled in our work [19].

2. To find the $\eta\gamma$ and $\eta'\gamma$ transition FF's we have to calculate the $SU_f(3)$ singlet η_1 and octet η_8 mesons transition FF's. Unlike the η_8 meson, the $SU_f(3)$ singlet η_1 contains a two-gluon valence Fock state [20]. The gluonic state at the leading order does not contribute to the form factor $F_{\eta_1\gamma}$. Due to the quark-gluon mixing, the gluonic component of the meson DA has an influence also on the evolution of the quark component of the distribution amplitude. But in this work we neglect the gluonic part of the meson η_1 DA, treating the η_1 and η_8 mesons on the same footing, i.e., as the mesons consisting only on quark valence Fock states.

In the framework of the PQCD the meson-photon electromagnetic transition form factor $F_{M\gamma}(Q^2)$ is given by the expression [1]

$$F_{M\gamma}(Q^2) = \int_0^1 dx\Phi_M(x, \mu_F^2)T_H(x, Q^2; \mu_F^2, \mu_R^2), \qquad (2)$$

where $Q^2=-q^2>0$ and q is the four-momentum of the virtual photon. Here $T_H(x, Q^2; \mu_F^2, \mu_R^2)$ is the hard scattering amplitude of the subprocess $\gamma^*+\gamma\to q+\overline{q}$, $\Phi_M(x, \mu_F^2)$ is the meson DA. In Eq.(2) μ_F^2 and μ_R^2 are the factorization and renormalization scales, respectively.

The hard scattering amplitude T_H with $O(\alpha_S)$ order accuracy is given by the formula [5-7]

$$T_H(x, Q^2, \alpha_S) = \frac{N}{Q^2}\frac{1}{x}\left\{1 + C_F\frac{\alpha_S(Q^2)}{4\pi}\left[\ln^2 x - \frac{x\ln x}{1 - x} - 9\right]\right\} + [x \to (1 - x)] \quad . \quad (3)$$

In Eq.(3) $C_F = \frac{4}{3}$ is the color factor and the scales μ_F^2, μ_R^2 are taken equal to Q^2 ($\mu_R^2 = Q^2$ only in the frozen coupling approximation, see Ref.[17]).

The normalization constants N_1 and N_8 for the mesons η_1 and η_8 are

$$N_1 = 2\sqrt{2}\left(e_u^2 + e_d^2 + e_s^2\right), \quad N_8 = 2\left(e_u^2 + e_d^2 - 2e_s^2\right) \quad , \quad (4)$$

where e_q is the charge of the quark q.

3. The next ingredient to be chosen in Eq.(2) is the meson DA $\Phi_M\left(x, Q^2\right)$. In Ref.[18] the evolution equation for the pseudoscalar meson distribution amplitude, that takes into account the IR renormalon effects was found. As a result, the meson IR renormalon corrected DA was predicted. This distribution amplitude can be expanded in Gegenbauer polynomials $\left\{C_n^{3/2+\alpha}(2x - 1)\right\}$

$$\Phi_M\left(x, Q^2\right) = f_M[x(1 - x)]^{1+\alpha}\sum_{n=0}^{\infty} b_n\left(Q^2\right)A_n(\alpha_S)C_n^{3/2+\alpha}(2x - 1) \quad . \quad (5)$$

For both η_1 and η_8 mesons owing to C-invariance the sum in Eq.(5) runs over even n (n=0,2,4...). In other words, the η_1 and η_8 mesons DA's are symmetric under $x \leftrightarrow 1-x$ replacement. Here, f_M is the meson decay constant. In accordance with our normalization of f_M (for the pion, $f_\pi = 0.0923$ GeV), which differs from that of Ref.[18], $A_n(\alpha_S)$ is given by the expression

$$A_n(\alpha_S) = \frac{\Gamma(3 + 2\alpha)}{\sqrt{3}\Gamma(1 + \alpha)\Gamma(2 + \alpha)}\frac{n!}{(2 + 2\alpha)_n}\frac{3 + 2\alpha + 2n}{2 + 2\alpha + n} \quad , \quad (6)$$

where $\Gamma(z)$ is the Gamma function, $(\alpha)_n$ is the Pochhammer symbol, $(\alpha)_n = \Gamma(\alpha+n)/\Gamma(\alpha)$. In Eq. (6) $\alpha = \beta_0\alpha_S(Q^2)/4\pi$, $\alpha_S(Q^2)$ is the one-loop QCD coupling constant, β_0 is the QCD beta function one-loop coefficient

$$\alpha_S\left(Q^2\right) = \frac{4\pi}{\beta_0 \ln\left(Q^2/\Lambda^2\right)}, \beta_0 = 11 - \frac{2}{3}n_f \ . \quad (7)$$

Here n_f is the number of the quark flavors (in our case n_f=3), Λ is the QCD scale parameter Λ=0.2 GeV.

In this work we neglect the dependence of $\Phi_M\left(x, Q^2\right)$ on the factorization scale Q^2, therefore do not write down the expression for $b_n(Q^2)$. For our purposes it is convenient to expand the meson DA in powers of x

$$\Phi_M\left(x, Q^2\right) = f_M[x(1 - x)]^{1+\alpha}\sum_{n=0}^{\infty} K_n(\alpha_S)x^n \ . \ (8)$$

The new coefficients K_n in Eq.(8) can be found using Eq.(5) and known expressions for $\left\{C_n^{\lambda}(\xi)\right\}$ [21].

The calculation of the $M\gamma$ transition form factor using Eqs.(2), (3) and (8) yields

$$\begin{aligned} Q^2F_{M\gamma}\left(Q^2\right) = 2Nf_M\sum_{n=0}^{\infty} K_n\Big\{B(1 + \alpha + n,\ 2 + \alpha)\{1 + \frac{C_F\alpha_S(Q^2)}{4\pi}[\,(\psi(1 + \alpha + n) - \\ - \psi(3 + 2\alpha + n))^2 + \psi'(1 + \alpha + n) - \psi'(3 + 2\alpha + n) - 9]\,\} + \\ + \frac{C_F\alpha_S(Q^2)}{4\pi}B(2 + \alpha + n,\ 1 + \alpha) \times [(\psi(2 + \alpha + n) - \psi(3 + 2\alpha + n)]\Big\} \end{aligned} \quad . \quad (9)$$

Here $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the Beta function and $\psi'(z) = d\ln\Gamma(z)/dz$. In deriving of the expression (9) we take into account that because of the symmetry of the η_1 and η_8 mesons DA's and T_H under interchange $x \leftrightarrow 1-x$, the second term in Eq.(3) after integration over x leads to the same contribution to $Q^2F_{M\gamma}(Q^2)$ as the first one.

4. In the framework of the usual η-η' mixing scheme the relations between $SU_f(3)$ basis states η_1, η_8 and the physi-

cal ones η, η' [Eq.(1)] lead to the similar relations between the physical $\eta\gamma, \eta'\gamma$ and the $\eta_1\gamma$, $\eta_8\gamma$ transition form factors

$$F_{\eta\gamma}(Q^2) = \cos\theta F_{\eta_8\gamma}(Q^2) - \sin\theta F_{\eta_1\gamma}(Q^2),$$
$$F_{\eta'\gamma}(Q^2) = \sin\theta F_{\eta_8\gamma}(Q^2) + \cos\theta F_{\eta_1\gamma}(Q^2). \quad (10)$$

As we have emphasized above, in the literature the different values for θ were predicted. For example, in the chiral perturbation theory the value $\theta = -20°$ [22], in Ref.[12] $\theta = -18°\pm2°$ were obtained. In this work we rely on the recent phenomenological analysis carried out in Ref.[14] and use in numerical calculations the following values of f_1, f_8 and θ

$$f_1 = 1.17f_\pi = 0.108\ GeV,\ f_8 = 1.26f_\pi = 0.116\ GeV,\ \theta = -15.4^0. \quad (11)$$

The obtained result (9) is valid for all DA's of the mesons η_1, η_8. But we are going to consider in our numerical calculations only small admixture of the Gegenbauer polynomials $C_2^{3/2+\alpha}(2x-1)$ and $C_4^{3/2+\alpha}(2x-1)$ to the η_1, η_8 mesons DA's. Then in Eqs.(8) and (9) $n = 0\div4$. The explicit expressions of the coefficients $K_n, n = 0\div4$ can be found in Appendix B of our work [19].

The $\eta\gamma$ and $\eta'\gamma$ transition form factors calculated employing the η_1 and η_8 mesons asymptotic DA's (b_0=1, b_n=0 for $n\neq0$) and the various values of the mixing angle (at fixed f_1, f_8) are shown in Fig.1. As is seen the $\eta\gamma$ form factor for all θ lies above the corresponding data [Fig.1(a)] excluding some points. At the same time the $\eta'\gamma$ FF can be considered as describing the data [Fig. 1(b)]. By varying the value of θ at fixed f_1, f_8 we observe that at θ smaller than $\theta = -15.4°$ the situation with $F_{\eta'\gamma}(Q^2)$ becomes better, whereas $F_{\eta\gamma}(Q^2)$ increases with decreasing of θ. At θ larger than $\theta = -15.4°$ we find the opposing picture (it is not shown in Fig. 1).

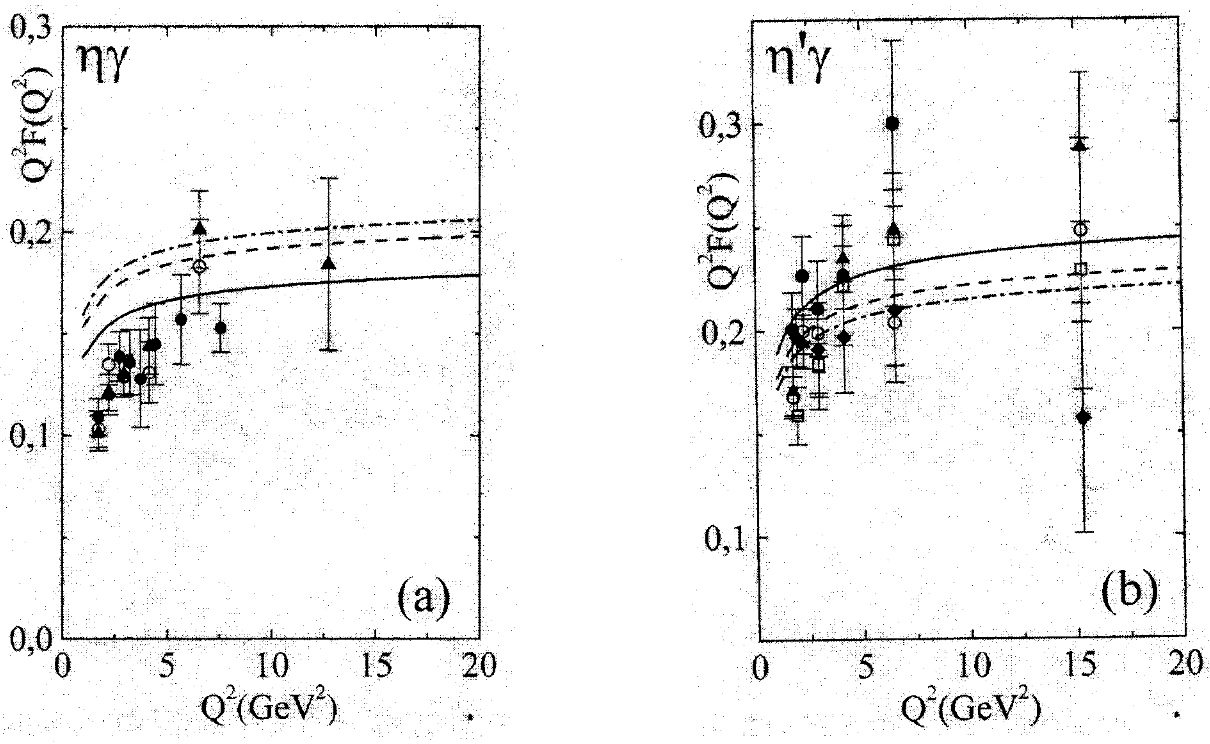

Fig.1. The $\eta\gamma$(a) and $\eta'\gamma$(b) electromagnetic transition form factors vs Q^2. All curves are obtained employing the asymptotic DA's for both the η_1 and η_8 mesons. The solid curves correspond to the octet-singlet mixing angle $\theta = -15.4°$, the dashed curves to $\theta = -20°$, the dot-dashed curves to $\theta = -22°$. The data are taken from Ref.[8].

The admixture of the Gegenbauer polynomials $C_2^{3/2+\alpha}, C_4^{3/2+\alpha}$ to the mesons DA's changes the situation. The results found using the η_1, η_8 mesons model DA's are depicted in Fig. 2. As is seen DA's with $b_2=b_4=-0.2$ for both η_1 and η_8 can be considered as DA's describing the data within the scheme employed in this work.

To reveal the effect of the IR renormalons on the transition form factors we have also computed $F^0_{\eta\gamma}(Q^2)$, $F^0_{\eta'\gamma}(Q^2)$ using the ordinary ($\alpha\equiv0$ in Eq.(5)) DA's [Fig. 3]. This effect is significant for the asymptotic DA's and amounts to 14 % at Q^2 = 1 GeV^2 and to 6 % at Q^2 = 20 GeV^2 of the ordinary FF's $F^0_{\eta\gamma}(Q^2)$, $F^0_{\eta'\gamma}(Q^2)$.

Let us compare the obtained in this work results with ones from Ref.[17], where the same FF's have been computed applying the running coupling constant method and the ordinary DA's. This method allows one to evaluate the power suppressed corrections to the form factors. From the comparison it is evident that these corrections play the important role in explaining the CLEO data, mainly in the region of small Q^2.

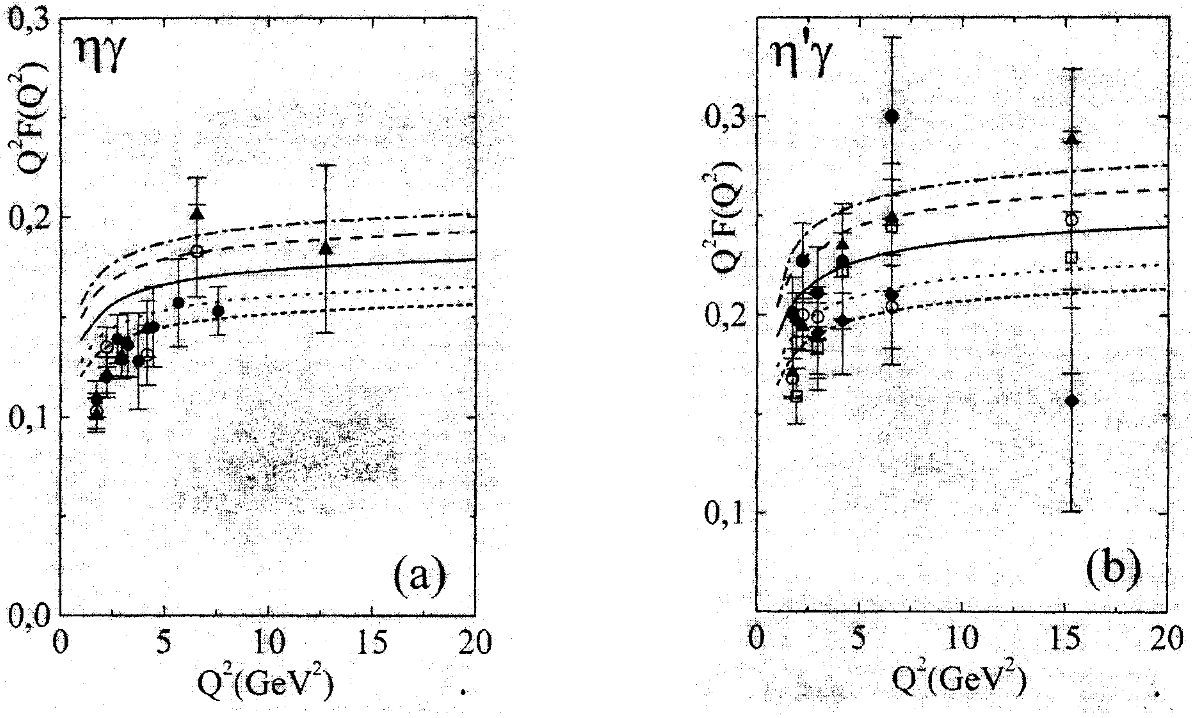

Fig.2. The $\eta\gamma$ (a) and $\eta'\gamma$(b) electromagnetic transition form factors as functions of Q^2. The octet-singlet mixing angle is θ= -15.4°.The solid curves are found using the asymptotic DA's for both the η_1 and η_8 mesons. The correspondence between the other curves and the parameters of the η_1 and η_8 mesons model DA's is: the dashed curves $b_2(\eta_1)=b_2(\eta_8)$=0.2; the dot-dashed curves $b_2(\eta_1)=b_2(\eta_8)$=0.2, $b_4(\eta_1)=b_4(\eta_8)$=0.2; the dotted curves $b_2(\eta_1)=b_2(\eta_8)$=-0.2; the short-dashed curves $b_2(\eta_1)=b_2(\eta_8)$=-0.2; $b_4(\eta_1)=b_4(\eta_8)$=-0.2.

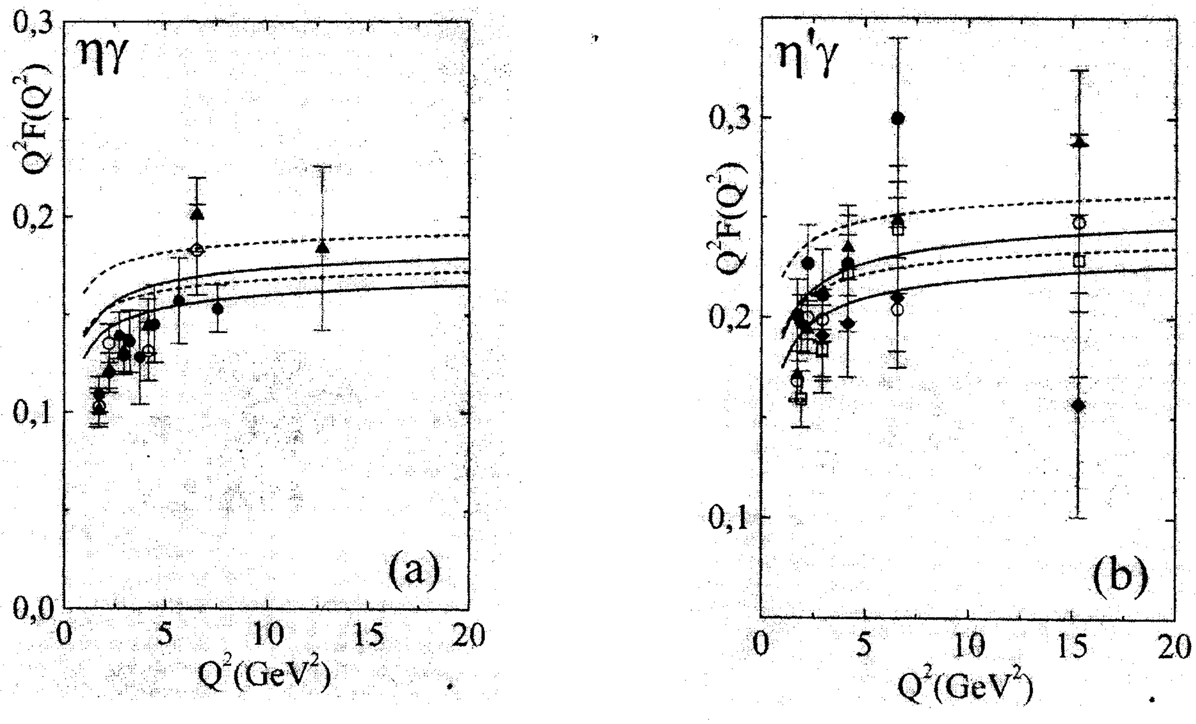

Fig.3. The $\eta\gamma$(a) and $\eta'\gamma$(b) transition form factors. The mixing angle is θ = -15.4°. The solid curves are obtained by means of the IR renormalon corrected DA's [Eq.(5)], whereas the dashed curves are computed using the η_1,η_8 mesons ordinary ($\alpha \equiv 0$ in Eq.(5)) DA's. The correspondence between the curves and the mesons DA's parameters is; the upper solid and dashed curves - asymptotic DA's for both the η_1 and η_8 ; the lower solid and dashed curves $b_2(\eta_1)=b_2(\eta_8)$=-0.2.

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MEZONLARIN İNFRAQIRMIZI RENORMALON DÜZƏLİŞLİ PAYLANMA FUNKSİYALARI VƏ $\eta\gamma$, $\eta'\gamma$ KEÇİD FORMFAKTORLARI

Psevdoskalyar η və η'mezonlarının elektromaqnit keçid formfaktorları $F_{\eta\gamma}(Q^2)$, $F_{\eta'\gamma}(Q^2)$ "donmuş" qarşılıqlı tə'sir sabiti yaxınlaşmasında, mezonların infraqırmızı renormalon düzəlişli paylanma funksiyalarından istifadə etməklə hesablanmışdır. Hesablamalarda adi η-η' qarışma sxemindən istifadə edilir. Oktet-sinqlet qarışma bucağının θ= -15.4° və parçalanma sabitlərinin $f_1 \approx 0.108$ QeV və $f_8 \approx 0.116$ QeV fenomenoloji qiymətləri məsələnin parametrləri kimi seçilmişdir. CLEO kollaborasiyasının nəticələri ilə müqayisə aparılır.

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КОРРЕКТИРОВАННЫЕ ИНФРАКРАСНЫМИ РЕНОРМАЛОНАМИ ФУНКЦИИ РАСПРЕДЕЛЕНИЯ МЕЗОНОВ И ПЕРЕХОДНЫЕ $\eta\gamma$, $\eta'\gamma$ ФОРМФАКТОРЫ

Электромагнитные переходные формфакторы $F_{\eta\gamma}(Q^2)$, $F_{\eta'\gamma}(Q^2)$ псевдоскалярных мезонов η и η' вычислены в приближении "замороженной" константы связи с использованием функций распределений мезонов, корректированных инфракрасными ренормалонами. При вычислениях применяется обычная схема η-η' смешивания. В виде входных данных используются феноменологические значения угла октет-синглетного смешивания θ= -15.4° и постоянных распада $f_1 \approx 0.108$ ГэВ и $f_8 \approx 0.116$ ГэВ. Проводится сравнение с данными коллаборации CLEO.

Дата поступления:23.05.01 *Редактор: И.Г. Джафаров*