THE KANE OSCILLATOR

A.M. BABAEV
Institute of Physics of Azerbaijan National Academy of Sciences
H.Javid, av., 33, Baku, 370143

The energy spectrum and wave functions for Kane oscillator describing the spectra of electrons, light hole and spin-orbit-splitting bands in a quantum dot with harmonic lateral confinement is found.

1. INTRODUCTION

As shown in [1], for description of an energy spectrum of quantum dots either "infinite potential barries", or a model of parabolic potential confinement can be used. It has been established, that the model of parabolic potential is realistic enough for describtion of not too large quantum dots. Within the framework of this model a number of problems of quantum dots physics were considered for the standard dispersion of electrons, including quantum crystallizations in an external magnetic field [2].

The semiconductor compounds (InAs, GaAs, InSb etc.) on which quantum dots are created now, have a complex energy spectrum described by multiband Hamiltonian. In particular, nonparabolicity of a spectrum is possible to take into account within the framework of eight-bands Kane model [3].However due to complexity of the obtained equation in [3], the analysis of its solution was carried out in the framework of rather special approximation.

The complexity of the equation mentioned above arises in a standard way of introduction of parabolic confinement potential through scalar potential. If the external potential is introduced by minimal substituting [4]:

$$\vec{p} \to \vec{p} - i\beta\lambda\vec{r} \tag{1}$$

then for Dirac Hamiltonian the oscillator equation would be obtained with additional constant term originated from spin-orbital coupling as shown [4-7].

In the present paper we have shown that if one introduces the external potential into the equations invariant under the rotational group [8,9] by way like (1) it would be transformed to the oscillator equation. Note that method [8] directly gives a system of equations for radial functions for any number of considered bands.

We have called the obtained equation Kane oscillator, by an analogy to Dirac oscillator . To obtain the Kane spectrum from the system of equations (18) in [8] we consider the values (j=1/2, τ=0) for the conduction, heavy and light hole (j=3/2, τ=1), and spin-orbit-splitting (j=1/2, τ=1) bands. The first index characterizes the weight of an irreducible representation and a second one indicates the subspace with the same weight. We have chosen indices for the states which clearly show that they are created from corresponding s, p states. In order to give a physical meaning to the equations we consider the coupling coefficients s, and p, correspondingly, τ=0, τ=1 to be nonzero, where τ is a label of the subspace.

Substituting

$$\frac{d}{dr} \to \frac{d}{dr} + \beta\lambda r \tag{2}$$

(where λ is a parameter characterizes a steepness of a well, β is a diagonal matrix with elements $(-1)^{\left(\tau+\frac{1}{2}-|j_z|\right)}$, j_z is a magnetic quantum number) the system of the equations , including also the dispersionless band of heavy hole take a form [8,9]:

$$\frac{-ia}{2(E-E_g)}\left[\frac{d}{dr}+\lambda r+\frac{1\mp(l_0+\frac{1}{2})}{r}\right]f_3^{\mp}-\frac{i\sqrt{2}b}{E-E_g}\left\{\left[\frac{d}{dr}+\lambda r+\frac{5\pm(l_0+\frac{1}{2})}{r}\right]f_2^{\pm}+\frac{\alpha}{r}f_2^{\pm}\right\}+f_1^{\pm}=0 \tag{3}$$

$$\frac{i\sqrt{2}b\alpha}{r}f_0^{\pm} - f_1^{\pm} = 0 \tag{4}$$

$$\lambda - \frac{i\sqrt{2}b}{E}\left[\frac{d}{dr} - \lambda r - \frac{1 \pm (l_0 + \frac{1}{2})}{2r}\right]f_0^{\pm} + f_2^{\pm} = 0 \tag{5}$$

$$\frac{-ia}{2(E - E_g)} \left[\frac{d}{dr} - \lambda r + \frac{1 \mp \left(l_0 + \frac{1}{2} \right)}{r} \right] \cdot f_0^{\mp} + f_3^{\pm} = 0 \tag{6}$$

$$\alpha = \frac{\sqrt{3}}{2} \sqrt{\left(l_0 - \frac{1}{2} \right) \cdot \left(l_0 + \frac{3}{2} \right)}$$

Here the following notations:

$$\begin{aligned} f_0^{\pm} &= f^{l_0}_{\frac{1}{2},\frac{1}{2},0} \pm f^{l_0}_{\frac{1}{2},-\frac{1}{2},0}, \\ f_1^{\pm} &= f^{l_0}_{\frac{3}{2},\frac{3}{2},1} \pm f^{l_0}_{\frac{3}{2},-\frac{3}{2},1} \\ f_0^{\pm} &= f^{l_0}_{\frac{3}{2},\frac{1}{2},1} \pm f^{l_0}_{\frac{3}{2},-\frac{1}{2},1}, \\ f_0^{\pm} &= f^{l_0}_{\frac{1}{2},\frac{1}{2},1} \pm f^{l_0}_{\frac{1}{2},-\frac{1}{2},1} \end{aligned} \tag{7}$$

are used.
As well as:

$$\frac{C^{0,1}_{1/2,1/2}}{i\chi} = \frac{ia}{E-E_g}, \quad \frac{C^{1,0}_{1/2,1/2}}{i\chi} = \frac{ia}{E+\Delta}, \quad \frac{C^{0,1}_{1/2,3/2}}{i\chi} = \frac{ib}{E-E_g}, \quad \frac{C^{1,0}_{3/2,1/2}}{i\chi} = \frac{ib}{E} \tag{8}$$

where Eg is the energy of the bottom of conduction band, Δ is the spin-orbit-splitting energy. The parameters a,b are matrix elements of coupling between the conduction and valence bands. The quantities like $C^{0,1}_{1/2,1/2}$, $f^{l_0}_{\frac{1}{2},\frac{1}{2},0}$ etc. and χ are determined in Gelfand et al [8]. The system of equations (1)-(6) are rewritten so in order to separate the independent solutions ("even" and "odd").

2. THE ENERGY SPECTRUM

Substituting (4)-(6) in (3) we have obtained:

$$\left\{ \left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{(l_0+\frac{1}{2})(l_0+\frac{1}{2}\pm 1)}{r^2} - \lambda^2 r^2 - 3\lambda \right] \left[\frac{a^2}{4(E-E_g)(E+\Delta)} + \frac{2b^2}{E(E-E_g)} \right] + 1 \right.$$
$$\left. \pm \left[\frac{a^2}{2(E-E_g)(E+\Delta)} - \frac{2b^2}{E(E-E_g)} \right] \lambda \left(l_0 + \frac{1}{2} \pm 1 \right) \right\} f_0^{\pm} = 0 \tag{9}$$

Energy spectrum is given by:

$$\varphi(E) = 2\lambda \left(N + \frac{3}{2} \right) \tag{10}$$

where

$$\varphi(E) = \frac{4E(E - E_g)(E + \Delta)}{a^2 E + 8b^2(E + \Delta)} \pm \lambda \left(l_0 + \frac{1}{2} \pm 1 \right) \frac{2a^2 E - 8b^2(E + \Delta)}{a^2 E + 8b^2(E + \Delta)} - 3\lambda \tag{11}$$

and the corresponding eigenfunctions read:

$$f^{\pm}_{0,n} = A_{n l_0 \pm \frac{1}{2}} r^{l_0 \pm \frac{1}{2}} \exp\left(-\frac{\lambda r^2}{2} \right) L_n^{l_0 \pm \frac{1}{2} + \frac{1}{2}} \left(\lambda r^2 \right) \tag{12}$$

where $L_n^{l_0 \pm \frac{1}{2} + \frac{1}{2}}(\lambda r^2)$ is an associated Laguerre polynomial, $N=2n+l_0\pm 1/2$,n=0,1,2,..is a principal quantum number. The normalization constants are:

$$A_{nl_0\pm\frac{1}{2}}=\left[\frac{2\lambda^{l_0\pm\frac{1}{2}+\frac{1}{2}}n!}{\Gamma\cdot\left(n+l_0\pm\frac{1}{2}+\frac{3}{2}\right)}\right]^{\frac{1}{2}},\ \lambda=\frac{m_n\omega}{\hbar} \qquad (13)$$

The parameters a and b are related to the effective mass as follows [11]:

$$\frac{\hbar^2}{2m_n}=\frac{2b^2}{E_g}+\frac{1}{4}\frac{a^2}{E_g+\Delta}\ ;\ \frac{\hbar^2}{2m_{sh}}=\frac{1}{4}\frac{a^2}{E_g+\Delta};$$

$$\frac{\hbar^2}{2m_{1h}}=\frac{2b^2}{E_g} \qquad (14)$$

where m_n, m_{1h} and m_{sh} are the effective masses of electron, light hole and spin-orbit splitting hole, correspondingly.

A case $a=\frac{2}{\sqrt{3}}P$, $b=\frac{1}{\sqrt{3}}P$ corresponds to one parameters Kane model and using (14) we find:

$$\varphi(E)=\frac{2mn}{\hbar^2}\cdot\left(\frac{E(E-E_g)(E+\Delta)}{E_g(E_g+\Delta)}\frac{E_g+\frac{2}{3}\Delta}{E+\frac{2}{3}\Delta}\mp\frac{\frac{2}{3}\Delta}{E+\frac{2}{3}\Delta}\frac{\hbar\omega}{2}\left(l_0+\frac{1}{2}\pm1\right)-3\frac{\hbar\omega}{2}\right) \qquad (15)$$

Using (12) in (4), (5) and (6), we get $f_1^{\pm}$, $f_2^{\pm}$ and $f_3^{\pm}$:

$$f_{1,n}^{\pm}=\frac{i\sqrt{2}b}{E}\frac{\alpha}{r}f_{0,n}^{\pm} \qquad (16)$$

$$f_{2,n}^{\pm}=\frac{i\sqrt{2}b}{E}\left[\left(\frac{4n-1+2l_0\pm1\mp(l_0+1/2)}{4r}-\lambda r\right)f_{0,n}^{\pm}-\frac{\sqrt{n(n+1/2+l_0\pm1/2)}}{r}f_{0,n-1}^{\pm}\right] \qquad (17)$$

$$f_{3,n}^{\pm}=-\frac{ia}{E+\Delta}\left[\left(\frac{2n\mp1+l_0(1\mp1)}{2r}-\lambda r\right)f_{0,n}^{\mp}-\frac{\sqrt{n(n+1/2+l_0\pm1/2)}}{r}f_{0,n-1}^{\mp}\right] \qquad (18)$$

The equations (15) describes the spectrum of electrons, light and spin-orbit splitting hole bands .

As well as in a case Dirac of oscillators in ground state energy appears twice more, than for isotropic oscillator of the standard Schrodinger equation .

The equations (15) might be useful in analysis of an influence of the nonparabolicity on a energy spectrum of electrons in a quantum dot.

CONCLUSION

The oscillator equation is obtained from a system of the equations for multiband Hamiltonian describing spectrum of electrons, light and spin-orbit-splitting hole bands in Kane semiconductors by the method of a minimal interaction.The solution of this equation allows to describe the influence of nonparabolicity on a spectrum of electrons in a quantum dot.

Recently this problem has been considered in [10,11] within the framework of the infinite potential barrier model. The advantage of our consideration consists of obtaining of the analytical expressions and the possibility of their analysis in comparision with numerical calculations [10,11].

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A.M. Babayev

KEYN OSSİLYATORU

Parabolik saxlayıcı potensiala malik kvant noqtələrində elektronların, yüngül və spin-orbital parçalanmis deşiklərin spektrlərini xarakterizə edən Keyn ossılyatorunun enerji spektləri və dalğa funksiyaları tapılmışdır.

А.М. Бабаев

КЕЙНОВСКИЙ ОСЦИЛЛЯТОР

Найден энергетический спектр и волновые функции кейновского осциллятора, описывающего спектр энергии электронов, легких дырок и спин-орбитально отщепленной зоны дырок в квантовой точке с параболическим удерживающим потенциалом.

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