ALLOY DISORDER SCATTERING-LIMITED ELECTRON MOBILITY OF THE SEMICONDUCTOR THIN WIRES

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We study the effect of the alloy disorder scattering on the electron transport in a quasi one-dimensional semiconductor. It is shown that the alloy disorder gives a significant contribution to the electron scattering. It is found that the one-dimensional mobility is significantly greater than two-dimensional mobility. It is shown that the alloy disorder scattering-limited electron mobility increases with growth of the wire radius. We compare our results with different scattering mechanisms for one-dimensional system. Results are also included for the alloy composition dependence of the mobility.

1. INTRODUCTION

Recently the interest increases in the study of properties of ultrathin semiconducting wires, also called as quantum well wires with submicron dimensions. Electrons in a semiconductor quantum wire can be viewed as a quasi-one-dimensional (Q1D) electron gas [1-5]. There are Q1D structures where carriers are confined to move along the length of the wire and the motion is quantized in the transverse directions. The physical properties of low-dimensional semiconducting structures such as electronic transport characteristics differ from properties of bulk semiconductors because the translational symmetry is broken [6]. Sakaki [1] suggested to use semiconductor quantum wires as a basis for very fast transport processes. Reduction of phase space in Q1D leads to a decrease in the scattering process, therefore, the mobility of carriers has to increases in comparison with to the bulk value. These high mobilities could be utilized in high-speed device applications [1,7]. The mobility of the electrons in the Q1D electron gas is limited by various scattering processes, which have large importance at different ranges of temperature and carrier concentration. There have been calculations of electron scattering by acoustic phonon [8], impurity-limited mobility [2], phonon-limited mobility [3], the mobility of electrons scattered by impurities, by acoustic and polar optical phonons [4,9]. Alloy disorder scattering is an important scattering mechanism when the confining quantum well consists of ternary semiconductor. Alloy disorder scattering in ternary compound semiconductors and quantum well structures has been the subject of many theoretical and experimental investigations [10-20]. In its conventional theory, the constituent type A and type B atom pairs are assumed to be distributed randomly within the volume of the crystal.

The main purpose of this paper is to derive approximate analytical expressions for the momentum relaxation time associated with the electron alloy disorder interactions, and the calculation of the mobility of Q1D electron gas. The outline of this paper is as follows. In Sec.II we give the general framework of the paper. In Sec.III we present the calculation of the momentum relaxation time for the alloy disorder scattering. In Sec.IV the quantum theory of the alloy disorder-limited electron mobility of thin semiconduting wires is presented. In Sec.V some numerical results of our calculation are presented for $In_{1-x}Ga_xAs$. Finally, a brief discussion about our numerical analysis is given.

2. GENERAL FRAMEWORK

We assume that the electrons are confined to move in a cylindrical wire of radius R and are free to move along the axis of the wire, which has the length L. Within the framework of the effective-mass approximation the electron wave function in quantum wire is

$$\varphi(r) = L^{-1/2} \exp(iKz)\varphi_n(\rho), \qquad (1)$$

where $\rho=(x,y)$ and $r=(\rho,z)$ we take the wire axis as the z direction. The electron spectrum is parabolic and isotropic

$$E_{nK} = E_n + \hbar^2 K^2/2m^* \qquad (2)$$

We make the same approximation as in [2,3] i.e., we take

$$\varphi_0(\rho) = \left(\pi R^2\right)^{-1/2} Y(R - \rho) \qquad (3)$$

which means that the envelope function is constant inside a cylindrical wire of radius R and zero outside. $Y(X)$ is step function.

When the confining quantum well consists of a ternary semiconductor (like $Ga_{1-x}In_xAs$), in the virtual crystal approximation alloy disorder scattering potential has the form [17-20]:

$$H_{dis} = \delta V\left\{(1-x)\sum_{rIn} Y_{\Omega_0}(r - r_{In}) - \sum_{rICa} Y_{\Omega_0}(r - r_{Ca})\right\}, \qquad (4)$$

where $Y_{\Omega_0}(r_a - r_b) = 1/\Omega_0$ when r_a and r_b are inside the same unity cell and vanishes elsewhere, and the summations run over all the unit cells, Ω_0 is the volume of the unit cell.

The momentum relaxation time τ of electrons in a Q1D system due to the scattering potential H_{dis} is given by the relaxation rate [21]

$$\tau^{-1} = \frac{2\pi}{\hbar} \sum_{f} |< f|H_{dis}|i >|^2 (1 - \cos\theta)\, \delta\left(E_f - E_i\right) \tag{5}$$

Here, i and f represent the initial and final states and θ is the angle between the incident and scattered wave vectors of electrons along the axis of the wire. Because of the Q1D nature of the electron gas in a thin wire in the quantum limit, the scattering angle θ is limited by two values 0 and π. Thus the value $(1-\cos\theta)$ in Eq.(5) will be 2. The total wave function of the state "f" is $\varphi fT=\varphi T u_c(r_e)$, where $u_c(r_e)$, are Bloch functions corresponding to the extrema of the conduction band.

In order to evalute the matrix elements $<f|H_{dis}|i>$ in Eq.(5) we replace $r_e \rightarrow r_e + r_{In(Ca)}$ and use relations: $u_c(r_e+r_{In(Ca)}) = u_c(r_e)$ and $f_{\Omega_0} d^3 r |u_c(r)|^2 = \Omega_0$. The scattering rate induced by the alloy disorder is calculated using, the statistical average of products of two different matrix elements.

The mobility of electrons confined by Q2D quantum well and free to move along the Z axis of the thin semiconducting wire is given in the relaxation time approximation by

$$\mu = e\left[\sum_K \left(\frac{\hbar K}{m^*}\right) \tau\left(E_K\right) \frac{\partial f_0(E_K)}{\partial E_K}\right] \cdot \left(\sum f_0(E_K)\right)^{-1} \tag{6}$$

where $f_0(E_K)$ is the electron distribution function for carriers in the wire.

3. MOMENTUM SCATTERING RATE FOR ALLOY DISORDER SCATTERING

Using the wave functions given by Eq.(1) and (3) , matrix elements induced by the alloy disorder become

$$<f|H_{dis}|i> = \frac{\delta V}{L} \sum_{r_p} L_0^2(\rho) e^{i\Delta K Z_p} \left[(1-x)\delta_{p,In} - x\delta_{p,ICa}\right] \tag{7}$$

where $\Delta K = K - K'$. The summation on r_p is over all the crystal unit cells and $\delta_{p,In}=1\ (=0)$ if the cell at r_p contains one In (respectively, Ga) atom.

A straightforward calculation of the square of the matrix element in (7) gives

$$|< f|H_{dis}|i>|^2 = \frac{\Omega_0 (\delta V)^2 x(1-x)}{\pi L R^2} \tag{8}$$

When (5) and (8) are combined, the momentum relaxation rate for the electron alloy disorder scattering is obtained:

$$\frac{1}{\tau} = \frac{2\Omega_0 (\delta V)^2 x(1-x) m^*}{\pi \hbar^3 R^2 K} \tag{9}$$

which combined with $E = \hbar^2 K^2 / 2m^*$ gives $\tau(E)$ explicity as a function of the energy E.

Eq. (9) shows that the scattering rate increases and the momentum relaxation time decreases as the radius of the wire decreases for scattering from the alloy disorder.

The formula similar to Eq.(9) has been obtained by Ando [19] and Bastard [18] for Q2D electron gas.

4. MOBILITY

In the general case, without approximations for using Eq. (9) in Eq.(6) the mobility can be written as

$$\mu = \frac{2e\hbar R^2 K_b T F_1(\eta)}{m^* \Omega_0 (\delta V)^2 x(1-x) n_{1D}} \tag{10}$$

Here $n_{1D} = 2\sqrt{2K_B T m^*} F_{1/2}(\eta)/\pi\hbar$ is the density of electrons per unit length of the wire, $F_n(\eta)$ is the Fermi integral of the argument $\eta=\zeta/KT$, and ζ is the chemical potential. It can be seen that in Eq.(10) the mobility increases as the wire radius increases.

For comparison we also give the result obtained for the acoustic phonon scattering limited mobility of the thin semiconducting wire [8]

$$\mu_{ac} = \frac{e\hbar}{\pi^{1/2}} \left(K_B T\right)^{1/2} \frac{p_1 u^2 A}{E_{ac}} \tag{11}$$

Here A is the cross-sectional area of the wire, p_1 is the mass density of the semiconductor, u is the sound velocity, and E_{ac} is the deformation potential. In this case, the mobility is suppressed as the cross-sectional area A of the wire decreases.

For the case of non degenerate carrier statistics, $f_0(E)$ is given by the Maxwell-Boltzmann distribution and mobility is given by

$$\mu_{n.deg.} = \frac{\pi^{1/2} e\hbar^2 R^2 \left(K_B T\right)^{1/2}}{\sqrt{2m^{*3/2}} \Omega_0 (\delta V)^2 x(1-x)} \tag{12}$$

When carriers are degenerate, at low temperature, $f_0(E)$ is given by a Fermi-Dirac distribution and mobility is $e\tau(EF)/m^*$:

$$\mu_{deg.} = \frac{\pi e\hbar^3 R^2 K_B}{2m^{*2} \Omega_0 (\delta V)^2 x(1-x)} \tag{13}$$

where the Fermi energy E_F of electrons in a degenerate 1D electron gas is $\hbar^2 K_F^2 / 2m^*$ with $K_F=\pi n_{1D}/2$.

It is noteworthy that the mobility due to the alloy disorder scattering in 3- and 2-dimensional system has the functional dependence on the temperature according to $T^{-1/2}$ [10] and T^{0} [13], respectively.

Eqs. (12)-(13) show that μ varies as T^{0} at the low temperature and as $T^{1/2}$ at the high temperature. It occurs because the density of states in a 3-, 2-, and 1-dimensional system has the functional dependence on energy according to $E^{1/2}$, E^{0} and $E^{-1/2}$ respectively.

5. NUMERICAL ANALYSIS AND DISCUSSION

As a numerical example, we consider a $Ga_xIn_{1-x}As$ quantum wire. Certain material parameters such as the effective mass m^* and lattice parameter occur in various formulas for mobility (relaxation time),. The lattice parameter has been assumed to follow Vegard's "law"

$$\alpha = \alpha_A x + \alpha_B(1 - x),$$

where α_A is the lattice parameter of the component A and α_B is the lattice parameter of the component B.

The electron effective mass m^* has been taken as

$$\frac{1}{m^*} = \frac{x}{m_A^*} + \frac{1-x}{m_B^*},$$

where m_A^* is the effective mass in the pure A component and m_B^* is the effective mass in the pure B component.

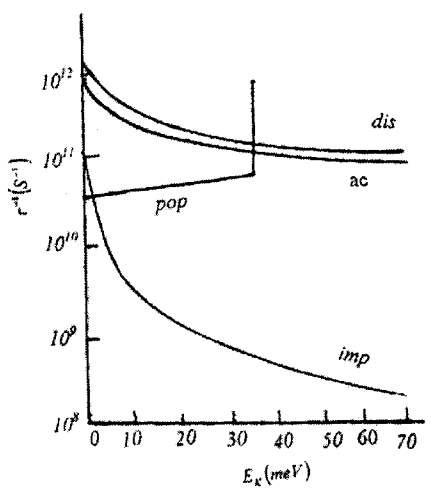

Fig.1. For a 1D $Ga_{0.8}In_{0.2}As$ system, the momentum relaxation rate is plotted as function of the electron energy E_K for alloy disorder scattering. Scattering rates for scattering from background impurity, acoustic phonon and polar optic phonon are shown for 1D GaAs system [4].

The momentum relaxation rate, τ^{-1} as a function of the longitudinal energy E_K in the $Ga_{0.8}In_{0.2}As$ is shown in fig.1. These numerical results are plotted from Eq.(9) for scattering alloy disorders. In order to compare the momentum relaxation rate for the alloy disorder scattering with momentum relaxation rate for various scattering mechanisms, values τ^{-1} for GaAs thin wire structures were found in the literature [4] and are plotted in fig.1. It is shown that for impurity, alloy disorder and acoustic phonon scattering the momentum relaxation rate diverge E_K as approaches to zero.

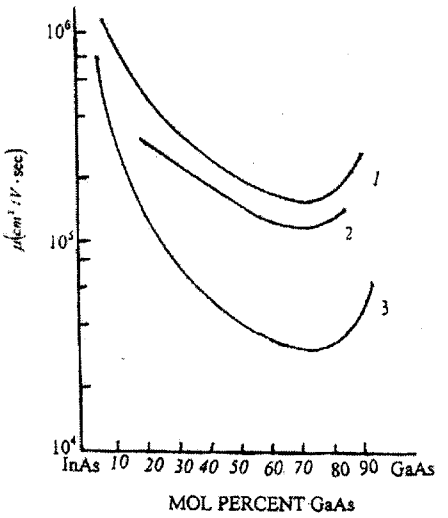

Fig.2. Dependence of the alloy disorder scattering limited electron mobility on the alloy composition for the case of $Ga_xIn_{1-x}As$ quantum wire (curve 1), quantum well structure (curve 2, [11], and 3D systems (curve 3, [10].

In fig.2 the composition dependence of the electron mobility is shown for the $In_{1-x}Ga_xAs$ quantum well wire with the radius $R=100Å$. Curves 2,3 in fig.2 show the composition dependence of the electron mobility for the $In_{1-x}Ga_xAs$ quantum well [11] and bulk $In_{1-x}Ga_xAs$ [10], respectively. In three cases bulk density of electrons is the same $n_{3D}=2\cdot10^{17}cm^{-3}$. The strength of the alloy disorder potential δV is chosen $0.53eV$.

The nature of the composite dependence of the electron mobility limited by alloy disorder scattering being $[x(1-x)m^*(x)\alpha_0(x)3]^{-2}$. Here $\alpha_0(x)$ is the lattice parameter and $m^*(x)$ is the electron effective mass at the composition x.

The minimum in mobility at about 10 per cent InAs seems are in agreement with other works [10,12]. This could be result of the influence of band structure on the scattering in the alloys.

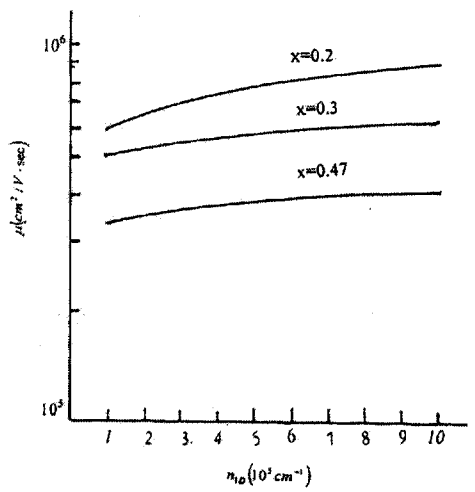

Fig.3. Variation of the mobility versus the linear density of electron for different alloy compositions.

In fig.3. the variation of electron mobility limited only by the alloy disorder scattering is shown as a function of the electron density for $Ga_{0.2}In_{0.8}As$ and $Ga_{0.47}In_{0.53}As$ cylindrical wires with the radius $R=100$Å. Thus the alloy disorder scattering is important at low concentrations especially for systems with a small Ga content x.

In conclusion, we have developed the theory for mobility assuming scattering of electrons by alloy disorder, and have found that this scattering is the dominant mechanism in ternary-based quantum wires.

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KVANT NAQİLLƏRDƏ ELEKTRONLARIN ƏRİNTİNİN NİZAMSIZLIĞINDAN SƏPİLMƏSİ HALINDA YÜRÜKLÜYÜ

Kvazi birölçülü yarımkeçiricilərdə ərintinin nizamsızlığından səpilməsinin elektronların daşınmasına təsiri öyrənilmişdir. Müəyyən olunmuşdur ki, birölçülü sistemlərdə yürüklük ikiölçülü sistemlərdəki yürüklükdən çoxdur. Göstərilmişdir ki, elektronların ərintinin nizamsızlığından səpilməsi halında yürüklük kvant naqilin radiusunun artması ilə artır. Nəticələr birölçülü sistemlər üçün olan müxtəlif səpilmə mexanizmləri ilə müqayisə olunmuşdur. Yürüklüyün tərkibdən asılılığı müəyyən edilmişdir.

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ПОДВИЖНОСТЬ ЭЛЕКТРОНОВ В КВАНТОВОЙ ПРОВОЛОКЕ, ОГРАНИЧЕННАЯ РАССЕЯНИЕМ НА СПЛАВНОМ БЕСПОРЯДКЕ

В квазиодномерном полупроводнике изучается эффект сплавного рассеяния на перенос заряда. Показано, что сплавной беспорядок дает значительный вклад в электронное рассеяние. Найдено, что одномерная подвижность намного больше, чем двумерная подвижность. Показано, что ограниченное сплавное рассеяние увеличивается с ростом радиуса проволоки. Наши результаты были сравнены с различными механизмами рассеяния для одномерных систем. Изучались также зависимости подвижности от состава сплава.

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