# ABOUT THE THEORY OF THE NON-HEISENBERG MAGNETS <br> A.M. SULEIMANOV, M.B. HUSEINOV <br> The Institute of Physics, National Azerbaijan Academy of Sciences, H.Javid ave. 33, Baku 

The non-Heisenberg crystal has been investigated when the atom states multiplicity is taken into account. The additional branches in the spectrum of spin excitations have been obtained. The curves of the frequency dependence on the wave vector and the external magnetic field have been constructed.

In non-Heisenberg magnetics the terms of the higher order have been taken into consideration side in addition to the bilinear exchange [1,2].

In these works it is usually assumed, that the exchange parameters of the higher order are much smaller than parameters of the bilinear exchange.

In the present work the case, when parameters of bilinear and biquadratic exchange are equal has been investigated.

It is known, that the atom with the spin $S$ has $2 S+1$ energy states (the atom states multiplicity). The method of standard basic operators (SBO) [3], which allows to consider the atom states multiplicity has been chosen for the mathematical apparatus.

For the spin $S=1$ case, which we have considered, the spin operators are expressed through SBO by the following way:

$$
\begin{align*}
& S_{i}^{+}=\sqrt{2}\left(L_{12}^{i}+L_{23}^{i}\right) \\
& S_{i}^{-}=\sqrt{2}\left(L_{21}^{i}+L_{32}^{i}\right) \\
& \left.S_{i}^{z}=L_{11}^{i}+L_{33}^{i}\right) \tag{1}
\end{align*}
$$

$L_{\alpha \beta}^{i}$ are SBO, characterizing the spin transition between $\alpha$ and $\beta_{\text {states }}$ in the site $i$.

The Hamiltonian for the case under consideration has a form:

$$
\begin{equation*}
H=-J \sum_{i, j} S_{i} S_{j}-B \sum_{i, j}\left(S_{i} S_{j}\right)^{2}-V \sum_{i}\left(S_{i}^{z}\right)^{2}-h \sum_{i} S_{i}^{z} \tag{2}
\end{equation*}
$$

where $J$ and $B$ are parameters of bilinear and biquadratic exchange, respectively, $V$ is the single-ion anisotropy;
$h=g \mu_{B} H ; H$ is an external magnetic field, $g$ is the Lande factor; $\mu_{B}$ is the Bohr magneton. If we assume that $J=B$, then

$$
\begin{equation*}
H=-J\left[\sum_{i, j} S_{i} S_{j}+\sum_{i, j}\left(S_{i} S_{j}\right)^{2}\right]-V \sum_{i}\left(S_{i}^{z}\right)^{2}-h \sum_{i} S_{i}^{z} \tag{3}
\end{equation*}
$$

From (1), (3) and the equation of motion:

$$
\begin{equation*}
i \frac{d L_{\alpha \beta}^{j}}{d t}=\left[L_{\alpha \beta}^{j}, H\right] \tag{4}
\end{equation*}
$$

we will calculate for spin excitation frequencies:
$\omega_{l}(\vec{k})=2 J(0)\left(D_{13}-D_{23}\right)-2 J(\vec{k}) D_{12}+V+h$ $\omega_{2}(\vec{k})=2 J(0)\left(D_{13}-D_{23}\right)-2 J(\vec{k}) D_{23}-V+h$

$$
\begin{equation*}
\omega_{3}(\vec{k})=2[J(0)-J(\vec{k})] D_{13}+2 h \tag{5}
\end{equation*}
$$

$D_{\alpha \beta}=D_{\alpha}-D_{\beta}, D_{\alpha}$ is the occupation probability of the level $\alpha$

Frequencies $\omega_{1}$ and $\omega_{2}$ correspond to the excitations with the change of $S_{z}$ by $l\left(\left|\Delta S_{z}\right|=1\right)$ and frequency $\omega_{s}$ corresponds to the excitation with the change of $S_{z}$ by $2\left(\left|\Delta S_{z}\right|=2\right)$.

The energy gap $(k=0, h=0)$ looks as :

$$
\begin{align*}
& \omega_{1}(0)=V \\
& \omega_{2}(0)=-V  \tag{7}\\
& \omega_{3}(0)=0
\end{align*}
$$

For the simple cubic crystal the frequency dependence on the wave vector $\vec{k}$ reads:
$\omega_{l}(k)=V+h+\frac{1}{3} J(0) \hat{e}^{2}$ (does not depend on k )
$\omega_{2}(\hat{e})=V+h$
$\omega_{3}(\hat{e})=z h+\frac{1}{3} J(0) k^{2}$
Schemes of frequencies dependence on $\vec{k}$ in the isotropic case $(V=0)$ are presented on fig.1.


Fig.1. Dependence $\omega(k)$ on the wave vector $k$ for the isotropic crystal.

In the case of $k=0$ the frequencies dependence on the external field is presented on fig. 2 .


Fig.2. Field dependence of frequencies for the case $k=0(h l=V)$.

As it is seen from the figure when $h=V, \omega_{1}=\omega_{3}, \omega_{2}=0$. It means that levels with $S_{z}=0$ and $S_{z}=-1$ coincide when $h=V$, and therefore $\omega_{2}=0, \omega_{1}=\omega_{3}$.

Let us note that $\omega_{1}$ corresponds to the transition between
states $S_{z}=0$ and $S_{z}=1, \omega_{2}-$ between $S_{z}=0$ and $S_{z}=-1$, and $\omega_{3}-$ between $S_{z}=1$ and $S_{z}=-1$.

We have for the probability of levels filling:

$$
\begin{aligned}
D_{1} & =\frac{1}{\Phi}\left(l+f_{1}\right)\left(l+f_{2}\right) \\
D_{2} & =\frac{1}{\Phi} f_{1}\left(l+f_{2}\right) \\
D_{3} & =\frac{1}{\Phi} f_{1} f_{2}
\end{aligned}
$$

$$
\Phi=1+2 f_{1}+f_{2}+3 f_{1} f_{2} ; \quad f_{1}=\left(e^{\frac{\omega_{i}}{k T}}-1\right) \quad \text { the }
$$ function of Bose-Einstein distribution.

In the region of low temperatures parameters of dipole and quadrupole ordering

$$
\begin{align*}
\tau & =1-\frac{1}{N} \sum_{n} f\left(\omega_{1}\right)  \tag{10}\\
Z & =1-\frac{3}{N} \sum_{k} f\left(\omega_{2}\right) \tag{11}
\end{align*}
$$

In this temperature area $\tau$ and $Z$ depend on the temperature according to the law $T^{3 / 2}$. At high temperatures:

$$
\begin{align*}
& \sigma=\frac{4}{J(0)}\left(\theta_{c}-\theta\right)^{1 / 2}  \tag{12}\\
& Z=\frac{3}{J(0)}\left(\theta_{c}-\theta\right) \tag{13}
\end{align*}
$$

where $\boldsymbol{\theta}_{c}=\frac{4 J(0)}{3}$ is the Curie temperature, $\boldsymbol{\theta}=\frac{T}{J(0)}$.
[3] S.B.Haley and Erdos P. Phys.Rev.B.1972, v. 5, p.11061119.k
[1] E.L.Nagayev. UFN; 1982; 136; issue 1; p. 61-103.
[2] Y.M.Seyidov, M.B.Huseynov, N.G.Huseynov. FMM, 1985,59, issue 2, p. 253-260.

