ENERGY SPECTRUM OF SUPERLATTICE IN MAGNETIC FIELD

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An expression defining energy of electron in superlattice with thin conducting layers in magnetic field is solved. The magnetic field is in layers plane. A potential of the superlattice is defined as δ-functions. Influence of weak magnetic field on minizone spectrum is investigated. It was shown that the influence of magnetic field leads to slight shift of minizones.

Many theoretical and experimental investigations of electron spectrum of heterostructures, in which a magnetic field was directed along semiconductor layers containing two-dimensional electron gas, have been fulfilled. For example, a spectrum of quantum wells systems in magnetic field was investigated theoretically in works [1,2]. An electron mass, connected with movement along the axes of GaAs-$Al_xGa_{1-x}As$ superlattice, was experimentally investigated in work [3] using the cyclotron resonance method.

The modern technology allows to make superlattices with thickness of layers of a few or even of one inter-atomic distance. In this case one can use the model of potential of superlattice being chosen as δ-functions.

The aim of the present paper is to investigate influence of magnetic field directed along the layers of superlattice, on the electron minizone spectrum within the framework of the δ-potential model.

The axes of the superlattice is chosen along x-axes. The magnetic field in layers plane is chosen along y-axes. The vector potential is chosen as follows A=(0, 0,-Bx). In the approximation of effective mass the Schrodinger equation is the following:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi+\frac{1}{2m}\left(i\hbar\frac{\partial}{\partial z}+\frac{|e|Bx}{c}\right)^2\psi+u(x)=E\psi, \qquad (1)$$

where $u(x)=(\hbar^2\Omega/m)\cdot\delta(x-n\cdot d)$, d is the period of the superlattice, Ω is potential power, n= 0, ±1, ±2,… is the number of a dielectric layer.

The wave function may be written in the following form:

$$\psi=e^{ik_z z+ik_y y}\varphi(x)\ . \qquad (2)$$

Substituting (2) into equation (1) we obtain the following equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2}+\frac{\hbar^2}{2ml^4}(x-x)^2\varphi+\left[u(x)-\left(e-\frac{\hbar^2k_y^2}{2m}\right)\right]\varphi=0, \qquad (3)$$

where $l=\{c\hbar/|e|B\}^{1/2}$ is the magnetic length, $x=l^2k_z$ is the center of electron orbit in the magnetic field.

We shall make the following replacement of variable:

$$x=\frac{1}{2^{1/2}}\left(\xi+\bar{\xi}\right),\quad \bar{\xi}=2^{1/2}x/l \qquad (4)$$

As a result the equation (2) will obtain the following form:

$$\left[\frac{\partial^2}{\partial\xi^2}-\frac{\xi^2}{4}+\varepsilon-U(\xi)\right]\Phi(\xi)=0, \qquad (5)$$

where $\varepsilon=\left(E-\frac{\hbar^2k_y^2}{2m}\right)\frac{ml^2}{\hbar^2}$,

$$U(\xi)=2^{1/2}\,\Omega l\,\delta\left(\xi+\bar{\xi}-\frac{2^{1/2}dn}{l}\right).$$

By $\xi\neq-\bar{\xi}+\frac{2^{1/2}dn}{l}$ the equation (5) has the following form:

$$\frac{\partial^2}{\partial\xi^2}\Phi+\left(p+\frac{1}{2}-\frac{\xi^2}{4}\right)\Phi=0 \qquad (6)$$

where $p=\varepsilon-½$.

It is well known equation for the parabolic cylinder function and it has the following common solution:

$$\Phi_n=A_nD_p(\xi)+B_nD(-\xi), \qquad (7)$$

where D_p is the parabolic cylinder function, n is the number of conducting layer which is defined by the following inequalities:

$$-\bar{\xi}+\frac{2^{1/2}dn}{l}\le\xi\le-\bar{\xi}+\frac{2^{1/2}d(n+1)}{l} \qquad (8)$$

The border conditions for the functions Φ_n are defined by the system of the following equations:

$$\Phi_n(\xi_n+0)=\Phi_{n-1}(\xi_n-0) \qquad (9)$$

$$\Phi'_n(\xi_n+0)=\Phi'_{n-1}(\xi_n-0)+\lambda\Phi_{n-1}(\xi_n)$$

where $\xi_n=-\bar{\xi}+n\eta$, $\lambda=2^{1/2}\Omega$, $\eta=2^{1/2}d/l$.

In x-axes ξ_n corresponds to coordinates of the dielectric layers $x=nd$.

Subscript n at the function $\Phi_n(\xi)$ is defined by the inequalities (8), and corresponds to the conducting layer $nd\le x\le(n+1)d$.

We have from the Bloch theorem:

$$\Phi_0(\xi)=A_0D_p(\xi)+B_0D_p(-\xi) \qquad (10)$$

$$\Phi_1(\xi)=e^{ik_xd}\,[A_0D_p(\xi-\eta)+B_0D_p(-\xi+\eta)$$

Substituting (10) into the border conditions (9) one can obtain an equation for the electron spectrum taking into account that

$D_p(x)\, D_p^{\prime}(-x) - D_p(-x)\, D_p^{\prime}(x) = 2p/G$, where G is gamma function

$$cosk_x d = \frac{1}{2}\left[D_p(x_1)D_p^{'}(-x_1+h) - D_p(-x_1)D_p^{'}(x_1-h)\right] + \frac{1}{2}\left[D_p(x_1-h)D_p^{'}(-x_1) - D_p(-x_1+h)D_p^{'}(x_1)\right] + \frac{l}{2D}\left[D_p(x_1-h)D_p^{'}(-x_1) - D_p(-x_1+h)D_p^{'}(x_1)\right] \quad (11)$$

where $D=(2p)^{1/2}/G(-p)$.

Let us consider a weak magnetic field so that $d<<1$.

Having chosen the center of the electron's orbit $\bar{x}$ inside the area $(0,d)$ we shall obtain:

$$x_1, x << 1 . \quad (12)$$

The parabolic cylinder function we can represent in the form of the following combination of degenerated hypergeometric functions:

$$D_p(z) = 2^{p/2} exp\left(-z^2/4\right)\left[\frac{p^{1/2}}{G((1-p)/2)}F\left(-p/2, 1/2, z^2/2\right) - \frac{(2p)^{1/2} z}{G(-p/2)}F\left((1-p)/2, 3/2, z^2/2\right)\right], \quad (13)$$

where the hypergeometric function is represented by the following row:

$$F(a, b; z) = 1 + \frac{a}{g}\frac{z}{1!} + \frac{a(a+1)z^2}{g(g+1)}\frac{z^2}{2!} + \ldots$$

Taking into account (12), representing $p=e\text{-}1/2$ and assuming that $x_1\, e^{1/2}$, $\bar{x}\,\varepsilon^{1/2}$ @ 1 while calculating the expression for the spectrum we shall keep all members proportional to the quantities $(x_1 e^{1/2})^k$, $(x e^{1/2})^k$, where k=1, 2, 3,…

After a long calculation we shall obtain:

$$\cos k_x d = \cos e^{1/2}h + \left(l/2e^{1/2}\right)\sin e^{1/2}h + K\left(e^{1/2}\bar{x}, e^{1/2}\bar{x}_1\right)\bar{x}^4 + L\left(e^{1/2}x, e^{1/2}x_1\right)c + \left(l/2e^{1/2}\right)M\left(e^{1/2}x_1, e^{1/2}\bar{x}\right)x_1^4 + \left(l/2e^{1/2}\right)M\left(e^{1/2}x, e^{1/2}x_1\right)\bar{x}^4 \quad (14)$$

where K, L and M unwieldy expressions of the order of unit.

We shall define $c=(2m)^{1/2}\,(E\text{-}h^2k_y^2/2m)^{1/2}/h$ and take into account that $e^{1/2}h=cd$, $l/2e^{1/2}=W c$. By neglect of the members x_1^4 and $\bar{x}^{-4}$ in the expression (14) at the limit $B\to 0$, we obtain the standard expression of the electron spectrum in the Dirac comb:

$$cosk_x d = cos\,cd + (W/c) sin\,cd \quad (15)$$

Assuming that the electron spectrum doesn't depend on the position of the center of electron's orbit within the layer (0, d), in order to estimate influence of the weak magnetic field on the spectrum, we can assume that $\bar{x}$ =0.

The equation (14) will be reduced to the form:

$$cosk_x d = cos\,cd + (W/c) sin\,cd + [K(cd) + (W/c)M(cd)]\, h^4, \quad (16)$$

where $K(cd) = K(e^{1/2}x, e^{1/2}x_1)$, $M(cd)] = M(e^{1/2}x_1, e^{1/2}x)$ by $\bar{x}$ =0.

We shall search a solution of the equation (16) in the following form $c = c_0 + \bar{c}$, where $\bar{c} << c_0$. c_0 is the solution of (15) by k_x=0

$$tgx = Wd/2x, \quad x = c_0 d/2 . \quad (17)$$

Keeping the member $\bar{a}\ \bar{c}$, it is easy to obtain:

$$E = \frac{h^2 k_y^2}{2m} + \frac{h^2 c_0^2}{2m} + D\left(1 - \cos k_x d\right) + 4D\left[K\left(c_0 d\right) + \left(W/c_0\right)M\left(c_0 d\right)\right]\left(d/1\right)^4 \quad (18)$$

where the width of minizone is

$$D = \frac{h^2 c_0}{md \sin c_0 d\left[1 + W/dc_0^2 - \left(W/c_0\right)ctg c_0 d\right]}.$$

So, the influence of weak magnetic field leadscomes to the slight shift of minizones of the order of

$$4D(W/c_0)\,(d/l)^4.$$

Let us make estimation of the following quantities: $c_0 d$, D, $h^2 c_0^2/2m$.

We shall consider the superlattice GaAs – $Al_x\, Ga_{1-x}$ As with x=0.3, d= 200Å and the width of barrier a = 50Å . For such parameters the distance between conduction zones of GaAs and $Al_x\, Ga_{1-x}$ As is DE_c=100meV [4]. One can estimate the power of potential from the following expression: $h^2 W m = \Delta E_c a_0$

So, one can define the quantity $c_0 d$=2.5 from the equation (17) for the first minizone. According to this value we shall obtain position of the lowest minizone $h^2 c_0^2 / 2m$ =13.8meV. The width of the minizone is Δ=3meV.

Using these quantities one can estimate that the lowest minizone's shift is

$$36(d/l)^4 meV \quad . \tag{19}$$

Though, the expression (19) has been obtained at $d \ll l$, one can suppose that at $d \leq l$ the minizone's shift may be of the order of minizone's width.

[1] G. Gumbs. Phys.Rev. B 54, 16, 11354, 1996.
[2] V.I. Belyavski, Yu.V. Kopaev, S.V. Shevtsov. FTT, 40, 9, 1719, 1998.
[3] T. Duffield, R. Bhat, H. Koza, M.C. Tamargo, J.R. Harbison, F. DeRoza, D.H. Hwang, P. Grabl, K.M. Rush, S.J. Allen. In: Proc. 18th Int. Conf. On the Physics of Semiconductors, Stockholm, Sweden, 1, 239, 1986.
[4] A.P. Silin. UFN , 143, 3, 485, 1985.

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