THE SOLUTION OF KANE'S EQUATIONS IN MAGNETIC FIELD IN JANNUSSIS FUNCTIONS REPRESENTATION

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The solution of Kane's equations in uniform magnetic field in the representation of Jannussis functions is found. The results can be used in calculations in different models of nanostructures with participating of Kane's semiconductors in magnetic field.

INTRODUCTION

The solution of the Kane's equations in uniform magnetic field for the first time w6.1(a)s given by Bowers and Yafet [1]. In [2] Jannussis obtained the solution of Schrodinger equation for the lattice electrons in uniform magnetic fields using the special form of functions, which he called "schrauben" (screw) functions. Later Jannussis obtained the solution of the Dirac equation in uniform magnetic field in the form represented through the screw functions. The solution of Dirac-Pauli equation which takes into account the anomalous magnetic momentum of electron in uniform magnetic field was obtained in [4] by generalization of the method of Jannussis. In this work the solutions of Kane`s equations in uniform magnetic field are given by the use of Jannussis screw functions. As it is known the Kane`s equations describe the spectra of conduction band electrons, light and spin-orbital splitting valence bands holes in A^3B^5 semiconductors such as InSb, InAs, GaAs and others.

THE SOLUTION OF THE KANE'S EQUATION IN MAGNETIC FIELD BY THE USE OF JANNUSSIUS FUNCTIONS

In eight-bands Kane's Hamiltonian the interaction of conduction band with the valence band is taken into account by single Kane's parameter *P*. The system of Kane's equations including the non dispersive heavy hole band have the form [5]:

$$(\vec{\alpha}\cdot\vec{k}\cdot P+\beta\cdot G-E)\Psi=0 \qquad (1)$$

where

$$\alpha_x=\begin{pmatrix} 0 & 0 & \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{6}} & 0 & 0 & \frac{1}{\sqrt{3}} \\ 0 & 0 & 0 & \frac{-1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (2)$$

$$\alpha_y=\begin{pmatrix} 0 & 0 & \frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & \frac{i}{\sqrt{3}} \\ 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{3}} & 0 \\ \frac{-i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (3)$$

$$\alpha_z=\begin{pmatrix} 0 & 0 & 0 & \sqrt{\frac{2}{3}} & 0 & 0 & \sqrt{\frac{1}{3}} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\frac{2}{3}} & 0 & 0 & -\sqrt{\frac{1}{3}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\frac{1}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{1}{3}} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (4)$$

$$G=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -E_g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -E_g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -E_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -E_g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -E_g-\Delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -E_g-\Delta \end{pmatrix} \qquad (5)$$

$$\beta=\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&0&-1\end{pmatrix},\quad \Psi=\begin{pmatrix}\Psi_1\\\Psi_2\\\Psi_3\\\Psi_4\\\Psi_5\\\Psi_6\\\Psi_7\\\Psi_8\end{pmatrix} \tag{6}$$

Here P is the Kane's parameter, E_g and Δ are values of forbidden gap and spin-orbital interaction, respectively and

$$\vec{k}=-\imath\vec{\nabla}-\frac{e}{c}\vec{A} \tag{7}$$

The vector potential is chosen in the symmetric gauge

$$\vec{A}=\frac{1}{2}[\vec{H}\times\vec{r}] \tag{8}$$

and magnetic field intensity $\vec{H}=H\vec{e}_z$ is directed along the z-axis.

As it is known the solution of equation (1) in the cylindrical coordinates with the symmetric form of vector potential $\vec{A}$ can be expressed in terms of Laguerre's functions. We will show that the solution of equation (1) with the symmetric gauge for $\vec{A}$ in rectangular coordinate system can be obtained through the Jannussis screw functions. We search for the solution of Kane's equation (1) in the form

$$\Psi=\sum_n \Psi_{k,n}\begin{pmatrix}C_{1,n}u_1\\C_{2,n}u_2\\C_{3,n}u_3\\C_{4,n}u_4\\C_{5,n}u_5\\C_{6,n}u_6\\C_{7,n}u_7\\C_{8,n}u_8\end{pmatrix} \tag{9}$$

where u_1- u_8-s being the periodic part of wave functions are defined as in [1] through the combinations of spinors (s=1/2) and s,p-like band functions $|S\rangle$, $|X\rangle$, $|Y\rangle$, $|Z\rangle$. However in contrast to [1] we take $\Psi_{k,n}$ in the form of Jannussis functions (cf.[1])

$$\Psi_{k,n}=\sqrt{\frac{\lambda_H}{2\pi}\left(\frac{2}{\lambda_H}\right)^n\frac{1}{n!}}exp\left(\left(-\frac{1}{\lambda_H}\right)\left(K_x^2+K_y^2\right)+ikr\right)\left(-K_y-iK_x\right)^n \tag{10}$$

Here

$$K_x=k_x+\frac{1}{2}\lambda_H\cdot y,\ K_y=k_y-\frac{1}{2}\lambda_H\cdot x, \tag{11}$$

where $\lambda_H=\dfrac{eH}{c\hbar}$ is the square of reciprocal magnetic length. The functions $\Psi_{k,n}$ are normalized and obey the relations

$$\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}+\frac{1}{2}\lambda_H(x+iy)\right)\Psi_{k,n}=\sqrt{2\lambda_H n}\,\Psi_{k,n-1}, \tag{12}$$

$$\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}-\frac{1}{2}\lambda_H(x-iy)\right)\Psi_{k,n}=-\sqrt{2\lambda_H(n+1)}\,\Psi_{k,n+1}. \tag{13}$$

Substituting the wave function (9) into equation (1) and taking into account the relations (12-13) we obtain the following system of equations for the wave function coefficients C_{in}:

$$-EC_{1,n}-iP\sqrt{n\lambda_H}C_{3,n-1}+Pk_z\sqrt{\frac{2}{3}}C_{4,n}-iP\sqrt{\frac{n+1}{3}\lambda_H}{}_{5,n+1}+Pk_z\sqrt{\frac{1}{3}}C_{7,n}-iP\sqrt{\frac{2(n+1)}{3}\lambda_H}C_{8,n+1}=0 \tag{14}$$

$$-EC_{2,n+1} - iP\sqrt{\frac{n+1}{3}\lambda_H}C_{4,n} + Pk_z\sqrt{\frac{2}{3}}C_{5,n+1} - iP\sqrt{(n+2)\lambda_H}C_{6,n+2} + P\sqrt{\frac{2(n+1)}{3}\lambda_H}C_{7,n} - Pk_z\sqrt{\frac{1}{3}}C_{8,n} = 0 \tag{15}$$

$$P\sqrt{n\lambda_H}C_{1,n} - (E+E_g)C_{3,n-1} = 0 \tag{16}$$

$$Pk_z\sqrt{\frac{2}{3}}C_{1,n} + iP\sqrt{\frac{n+1}{3}\lambda_H}C_{2,n} - (E-E_g)C_{4,n} = 0 \tag{17}$$

$$Pk_z\sqrt{\frac{2}{3}}C_{2,n+1} + iP\sqrt{\frac{n+1}{3}\lambda_H}C_{1,n} - (E+E_g)C_{5,n+1} = 0 \tag{18}$$

$$iP\sqrt{(n+2)\lambda_H}C_{2,n+1} - (E-E_g)C_{6,n+2} = 0 \tag{19}$$

$$Pk_z\sqrt{\frac{1}{3}}C_{1,n} - iP\sqrt{\frac{2(n+1)}{3}\lambda_H}C_{2,n+1} - (E+E_g+\Delta)C_{7,n} = 0 \tag{20}$$

$$iP\sqrt{\frac{2(n+1)}{3}\lambda_H}C_{1,n} - \sqrt{\frac{1}{3}}Pk_zC_{2,n+1} - (E+E_g+\Delta)C_{8,n+1} = 0 \tag{21}$$

Relations for energy levels can be obtained in two ways. The first one is to demand that the determinant of matrix of coefficients of $C_{1,n} - C_{8,n+1}$ in the set of equations (14-21) to be zero. The second by substituting the coefficients $C_{3,n-1} - C_{8,n+1}$ from the equations (16-21) into the (14) and (15). So we would have the following set of equations for $C_{1,n}$ and C_{2n+1}

$$C_{1,n}\cdot\left\{-E + \frac{P^2k_z^2}{3}\left(\frac{2}{E+E_g} + \frac{1}{E+E_g+\Delta}\right) + \frac{P^2\lambda_H(2n+1)}{3}\left(\frac{2}{E+E_g} + \frac{1}{E+E_g+\Delta}\right) - \right.$$
$$\left. - \frac{P^2\lambda_H}{3}\left(\frac{1}{E+E_g} - \frac{1}{E+E_g+\Delta}\right)\right\} = 0 \tag{22}$$

$$C_{2,n+1}\cdot\left\{-E + \frac{P^2k_z^2}{3}\left(\frac{2}{E+E_g} + \frac{1}{E+E_g+\Delta}\right) + \frac{P^2\lambda_H(2n+1)}{3}\left(\frac{2}{E+E_g} + \frac{1}{E+E_g+\Delta}\right) - \right.$$
$$\left. - \frac{P^2\lambda_H}{3}\left(\frac{1}{E+E_g} - \frac{1}{E+E_g+\Delta}\right)\right\} = 0 \tag{23}$$

If to demand that $C_{1,n} \neq 0$ or $C_{2,n+1} \neq 0$ we will have two expressions for the light carriers spectra correspondingly:

$$-E + \frac{P^2k_z^2}{3}\left(\frac{2}{E+E_g} + \frac{1}{E+E_g+\Delta}\right) + \frac{P^2\lambda_H(2n+1)}{3}\left(\frac{2}{E+E_g} + \frac{1}{E+E_g+\Delta}\right) -$$
$$- \frac{P^2\lambda_H}{3}\left(\frac{1}{E+E_g} - \frac{1}{E+E_g+\Delta}\right) = 0 \tag{24}$$

$$-E+\frac{P^2k_z^2}{3}\left(\frac{2}{E+E_g}+\frac{1}{E+E_g+\Delta}\right)+\frac{P^2\lambda_H(2n+1)}{3}\left(\frac{2}{E+E_g}+\frac{1}{E+E_g+\Delta}\right)+$$
$$+\frac{P^2\lambda_H}{3}\left(\frac{1}{E+E_g}-\frac{1}{E+E_g+\Delta}\right)=0 \tag{25}$$

One pair of solutions of cubic equations (22) and (23) gives the dispersion relation for conduction band spin-up and spin-down states correspondingly, but the other two pairs correspond to light and spin-orbital splitting hole bands with total momentum projection *M* along (*M*=1/2) and opposite (*M*=-1/2) to the magnetic field direction.

To obtain the light carriers wave functions $\Psi_{E,J=1/2,M=+1/2}$ and $\Psi_{E,J=1/2,M=-1/2}$ we must put $C_{1,n}\neq 0, C_{2,n+1}=0$ and $C_{2,n+1}\neq 0, C_{1,n}=0$ in (9) correspondingly, expressing all other coefficients $C_{3,n}-C_{8,n}$ through them. So the wave functions for the light carriers have been obtained in the following forms

$$\Psi_{E,J=1/2,M=+1/2}=\frac{1}{\sqrt{N_1}}\cdot\begin{pmatrix}\Psi_{k,n}\\ 0\\ \dfrac{iP\sqrt{n\lambda_H}}{E+E_g}\Psi_{k,n-1}\\ \dfrac{Pk_z\sqrt{\frac{2}{3}}}{E+E_g}\Psi_{k,n}\\ \dfrac{iP\sqrt{\frac{n+1}{3}\lambda_H}}{E+E_g}\Psi_{k,n+1}\\ 0\\ \dfrac{Pk_z\sqrt{\frac{1}{3}}}{E+E_g+\Delta}\Psi_{k,n}\\ \dfrac{iP\sqrt{\frac{2(n+1)}{3}\lambda_H}}{E+E_g+\Delta}\Psi_{k,n+1}\end{pmatrix},\qquad \Psi_{E,J=1/2,M=-1/2}=\frac{1}{\sqrt{N_2}}\cdot\begin{pmatrix}0\\ \Psi_{k,n}\\ 0\\ \dfrac{iP\sqrt{\frac{n}{3}\lambda_H}}{E+E_g}\Psi_{k,n-1}\\ \dfrac{Pk_z\sqrt{\frac{2}{3}}}{E+E_g}\Psi_{k,n}\\ \dfrac{iP\sqrt{\frac{n+1}{3}\lambda_H}}{E+E_g}\Psi_{k,n+1}\\ -\dfrac{iP\sqrt{\frac{2n}{3}\lambda_H}}{E+E_g+\Delta}\Psi_{k,n-1}\\ -\dfrac{Pk_z\sqrt{\frac{1}{3}}}{E+E_g+\Delta}\Psi_{k,n}\end{pmatrix} \tag{26}$$

In (26) E is the root of equation (24) or (25) for M=1/2 or M=-1/2, correspondingly and three roots of each of these equations correspond to conduction, light and spin-orbital splitting holes bands. The factors N_1 and N_2 are obtained from the normalization conditions of the wave functions and have the form

$$\frac{\qquad}{\qquad}\quad\frac{\qquad}{\qquad}\quad\frac{\qquad}{\qquad}\quad\frac{\qquad}{(E\quad E\qquad)},$$

$$E + E_g = 0, \qquad (27)$$

i.e. the heavy hole mass is infinite in this model. To obtain the heavy hole wave functions $\Psi_{E=-E_g, J=3/2, M_j=3/2}$ and $\Psi_{E=-E_g, J=3/2, M_j=-3/2}$ is necessary to put $C_{1,n} = 0$ and $C_{2,n+1} = 0$ in (14) and (15). Then if one takes into account (27) the only nonzero coefficients in the set of equations (14-15) are $C_{3,n-1}$, $C_{4,n}$, $C_{5,n+1}$ and $C_{6,n+2}$. Taken by turns $C_{4,n} = 0$ and $C_{5,n+1} = 0$ we can obtain for the heavy hole wave functions the following expressions:

$$\Psi_{E=-E_g, J=3/2, M_j=3/2} = \frac{1}{\sqrt{N_3}} \begin{pmatrix} 0 \\ 0 \\ \sqrt{\frac{(n+1)(n+2)}{3}}\Psi_{k,n-1} \\ 0 \\ -\sqrt{n(n+2)}\Psi_{k,n+1} \\ -i\sqrt{\frac{2}{3}}nk_z\Psi_{k,n+2} \\ 0 \\ 0 \end{pmatrix},$$

$$\Psi_{E=-E_g, J=3/2, M_j=-3/2} = \frac{1}{\sqrt{N_4}} \begin{pmatrix} 0 \\ 0 \\ -ik_z\sqrt{\frac{2(n+2)}{3}}\Psi_{k,n-1} \\ \sqrt{n(n+2)}\Psi_{k,n} \\ 0 \\ \sqrt{\frac{n(n+1)}{3}}\Psi_{k,n+2} \\ 0 \\ 0 \end{pmatrix}, \qquad (28)$$

where the normalization factors N_3 and N_4 have the form

$$N_3 = \frac{(4n+1)(n+2)}{3} + \frac{2}{3}nk_z^2, \quad N_4 = \frac{(4n+7)n}{3} + \frac{2}{3}(n+2)k_z^2,$$

Conclusion

The solutions for the three band Kane's model in magnetic field in the representation of Jannussis function are obtained. These solutions can be useful for calculations of some physical quantities of A^3B^5 semiconductors and their different structures such as quantum wells, quantum dots and wires in magnetic field. As seen from the properties (12) and (13) these functions also facilitate the construction of coherent states for Kane's model in magnetic field.

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Bircins maqnit sahəsində Keyn tənliklərinin həlli Yannussis funksiyaları vasitəsi ilə tapılmışdır. Alınan nəticələr Keyn tipli yarımkeçirici nanostrukturların müxtəlif parametrlərinin maqnit sahəsində hesablanmasında istifadə oluna bilər.

А.М. Бабаев, О.З. Алекперов

РЕШЕНИЕ УРАВНЕНИЙ КЕЙНА В МАГНИТНОМ ПОЛЕ В ПРЕДСТАВЛЕНИИ ФУНКЦИИ ЯННУССИСА

Получено решение уравнений Кейна в однородном магнитном поле через функции Яннуссиса. Полученные результаты могут быть использованы при расчетах различных параметров наноструктур на основе Кейновских полупроводников в однородном магнитном поле.

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