ELECTRON TRANSPORT PHENOMENA IN SIZE-QUANTIZED n-Ge AND n-Si FILMS

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The behavior of some kinetic coefficients depending on *n*-Ge and *n*-Si films surface orientation is investigated in the nonquantized magnetic field. The general solution of transport equation and expressions for the relaxation time at various electron scattering cases are obtained. Also the expressions for halvanomagnetic and thermomagnetic tensors are obtained at the arbitrary degeneration of electron gas. The Hall constant and thermopower of *n*-Ge and *n*-Si films in a strong and weak transverse magnetic field are calculated.

1. Introduction

At present thin semiconducting films are intensively investigated in the size-quantized conditions connected with microelectronics development. When the specimen sizes are of de Broglie wavelength of the current carriers' the quantum-sized effects occur and the wave functions form changes. Some thermodynamic and kinetic properties of conducting films with standard zone (simple isotropic model) have been considered in the works [1-4]. In the work [5] the electron states in anisotropic size-quantized *n*-Ge and *n*-Si films have been considered and the so-called size-quantized anisotropy (dependence of physical values on film surface orientation) was predicted. Evidently, such dependence can be observed in kinetic properties of *n*-Ge and *n*-Si films.

2. Transport equation

The energy spectrum of electrons in the size-quantized *n*-Ge and *n*-Si films takes a form:

$$\varepsilon_s\left(n_s,k_x,k_y\right)=\frac{\hbar^2}{2m_{II}}\left(\frac{\pi}{d}\right)^2\varphi_s^{\,2}(\alpha)n_s^2+\frac{\hbar^2}{2m_{\perp}}\left[\varphi_s^{-2}(\alpha)k_x^2+k_y^2\right], \tag{1}$$

where $m_{||}$ and $m_{\perp}$ are the longitudinal and transverse effective electron masses, respectively; d is the film thickness, s is the ellipsoid number, n_s=1, 2, 3,… is the sized quantum number, α is the angle of rotation of a normal to the [001] film surface around one of the crystallographic axes, $\varphi_s(\alpha)$ are anisotropy functions characterizing ellipsoids orientation as regard to the system of reference (see [5]).

To consider some kinetic properties of the system it is necessary to solve the Boltzmann transport equation in the external nonquantized magnetic field. The solution of the same equation for *n*-Ge and *n*-Si bulk specimens was obtained in the works [6,7]. In our case of size-quantized *n*-Ge and *n*-Si films we have to solve the two-dimensional transport equation in film's plane.

If one represents the nonequilibrium distribution function of electrons in the form:

$$f_1=\left(-\frac{\partial f_0}{\partial\varepsilon_s}\right)\left(\vec{v}\vec{P}\right), \tag{2}$$

and assumes that the external nonquantized magnetic field is directed along a normal to the film surface (transverse field) then the following solution of equation is obtained:

$$\vec{P}=\frac{1}{1+\nu^2}\left\{\hat{\tau}\vec{\Phi}_0+\frac{e}{c}\hat{\tau}\left[\vec{H}\hat{M}^{-1}\left(\hat{\tau}\vec{\Phi}_0\right)\right]\right\}, \tag{3}$$

where f_0 is the Fermi-Dirac function, $\nu=\dfrac{e\left|\hat{\tau}\right|^{1/2}}{c\left|\hat{M}\right|^{1/2}}H$,

$\vec{\Phi}_0=-e\vec{E}-\dfrac{\varepsilon_s-\xi}{k_0T}k_0\vec{\nabla}T$, $\hat{\tau}$ is the tensor of relaxation time, $\hat{M}^{-1}$ is the inverse tensor of electron effective masses, $\left|\hat{\tau}\right|$ and $\left|\hat{M}\right|$ are determinants of the relaxation time and effective masses tensors, respectively.

As we see from (2) and (3) $\vec{P}$ like nonequilibrium function depends on components of the relaxation time tensor. Therefore, to calculate the kinetic coefficients it is necessary to obtain the relaxation time expression for different electron scattering cases. In *n*-Ge and *n*-Si films plane $\vec{P}$ has two components depending on only one relaxation time parameter τ_s.

The τ_s expression for electrons scattering on acoustical and non-polar optical phonons, point defects and ionized impurities in films plane takes a form:

$$\tau_s^{-1}(\varepsilon_s)=\sum_{\beta_s'} W_{\beta_s'\beta_s}\left(1-\frac{k_x'k_x+\varphi_s^4(\alpha)k_y'k_y}{k_x^2+\varphi_s^4(\alpha)k_y^2}\right), \quad (4)$$

where $W_{\beta_s'\beta_s}$ is the probability of electron transition from a $\beta_s=(n_s,k_x,k_y)$ state to a $\beta_s'=(n_s',k_x',k_y')$ state and back.

The exact calculations in the cases of electron scattering on phonons and point defects give us:

$$\tau_s(\varepsilon_s)=\tau_0\varphi_s^{-1}(\alpha)\left(\overline{n_s}+\frac{1}{2}\right)^{-1}, \quad (5)$$

where $\overline{n_s}=\left[\sqrt{\frac{\varepsilon_s}{\varepsilon_{1s}}}\right]$ is an integer part of $\sqrt{\frac{\varepsilon_s}{\varepsilon_{1s}}}$ (the average number of film subbands below energy ε_s), $\varepsilon_{ls}=\varepsilon_s$ ($n_s=1$, $k_x=k_y=0$), τ_0 is the multiplier that doesn't depend on energy but proportional to the film thickness.

As it is seen from (5) τ_0 depends on energy only through $\overline{n_s}$. Moreover, τ_s essentially depends on *n*-Ge and *n*-Si films surface orientation and this dependence vanishes only at the $\overline{n_s}>>1$ limit when the result for bulk specimen is obtained.

In the case of electrons scattering on ionized impurities the τ_s analytical expression for arbitrary $\overline{n_s}$ values is n't obtained. However, if we assume that the scattering on ionized impurities occurs without the transition between film subbands then the following result for relaxation time is obtained:

$$\tau_s(\varepsilon_s)=\tau_0\varphi_s^{-1}(\alpha)\left(\overline{n_s}+\frac{1}{2}\right)^{-1}\left(\frac{\varepsilon_s-\varepsilon_{ns}}{k_0T}\right)^2, \quad (6)$$

where ε_{ns} is the discrete part of the energy spectrum (1).

3. Hall constant and thermopower

Having knowledge of the transport equation solution and expression for the relaxation time one can calculate the current and the energy stream densities and then we can determine components of kinetic tensors. For the σ_{ik} and β_{ik} ($i,k=1, 2,; i\le k$) tensors connecting the current and the energy stream densities with the electric field and the temperature gradient the following expressions are obtained:

$$\sigma_{ik}=\frac{e^2m_\perp}{\pi d\hbar^2}\left(-\frac{eH}{c}\right)^{k-i}\sum_{s=1}^{N}\left\{\varphi_s(\alpha)\sum_{ns}\int_{\varepsilon_{ns}}^{\infty}\frac{(\varepsilon_s-\varepsilon_{ns})\left(-\frac{\partial f_0}{\partial\varepsilon_s}\right)K_k(\varepsilon_s)K_{k-i}(\varepsilon_s)d\varepsilon_s}{1+\nu^2(\varepsilon_s)}\right\}, \quad (7)$$

$$\beta_{ik}=\frac{em_\perp}{\pi d\hbar^2T}\left(-\frac{eH}{c}\right)^{k-i}\sum_{s=1}^{N}\left\{\varphi_s(\alpha)\sum_{ns}\int_{\varepsilon_{ns}}^{\infty}\frac{(\varepsilon_s-\varepsilon_{ns})(\varepsilon_s-\xi)\left(-\frac{\partial f_0}{\partial\varepsilon_s}\right)K_k(\varepsilon_s)K_{k-i}(\varepsilon_s)d\varepsilon_s}{1+\nu^2(\varepsilon_s)}\right\}, \quad (8)$$

where

$$K_0=1, K_1(\varepsilon_s)=\frac{\tau_s(\varepsilon_s)\varphi_s^{-2}(\alpha)}{m_\perp}, K_2(\varepsilon_s)=\frac{\tau_s(\varepsilon_s)}{m_\perp},$$

N=4(*n*-Ge) and N=6(*n*-Si).

On a base of the general expressions (7) and (8) for kinetic tensors we can calculate all kinetic effects in various conditions. Let us show some of them in a strong ($\nu>>1$) and weak ($\nu<<1$) transverse magnetic fields.

So, for the Hall constant in the strong magnetic field we obtain:

$$R_f=-\frac{1}{n_{ef}ec}, \quad (9)$$

where n_{ef} is the concentration of electrons in film. Therefore, in this case R_f doesn't depend on the film surface orientation, degree of electron gas degeneration and electron scattering mechanisms. But unlike the bulk specimen n_{ef} depends on film thickness and this dependence characterizes the quantum-sized effect.

For the thermopower in this case we have:

$$\alpha_f=-\frac{k_0}{e}\frac{B(\alpha)}{A(\alpha)}, \quad (10)$$

where

$$A(\alpha)=\sum_{s=1}^{N}\left\{\varphi_s(\alpha)\sum_{ns}F_1(\eta_{ns})\right\},$$

$$B(\alpha)=\sum_{s=1}^{N}\left\{\varphi_s(\alpha)\sum_{ns}\left[F_2(\eta_{ns})-\eta_{ns}F_1(\eta_{ns})\right]\right\}.$$

Here $F_t(\eta_{ns})=\int_0^{\infty}x_s^t\left(-\frac{\partial f_0}{\partial x_s}\right)dx_s$ are the uniparametric Fermi integrals of t index,

$$x_s=\frac{\varepsilon_s-\varepsilon_{ns}}{k_0T},\eta_{ns}=\frac{\xi-\varepsilon_{ns}}{k_0T}.$$

The analysis of expression (10) for α_f shows us that in this case thermopower doesn't depend on scattering mechanisms and for degenerated electron gas we obtain:

$$\alpha_f=-\frac{(\pi k_0)^2T}{3en_{ef}}g_f(\alpha), \qquad (11)$$

where $g_f(\alpha)=\frac{m_\perp}{\pi d\hbar^2}\sum_{s=1}^{N}\left\{\varphi_s(\alpha)\overline{n_s}\right\}$ is the density of electron states in *n*-Ge and *n*-Si films [5].

Therefore, the α_f behavior in this case is the same like for the density of electron states near the Fermi energy at the fixed n_{ef}. Otherwise, the thermopower depends on the film thickness as *1/d* until the film subband coincides with the Fermi energy. In this case α_f has a leap and is equal to the thermopower value in bulk specimen. Therefore, the thermopower dependence on film thickness has a saw-toothed character.

In another case of nondegenerated electron gas for α_f we have:

$$\alpha_f=-\frac{k_0}{e}\left\{2-ln\frac{\pi d\hbar^2n_{ef}}{m_\perp k_0TC(\alpha)}+\frac{D(\alpha)}{C(\alpha)}\right\}, \qquad (12)$$

where

$$C(\alpha)=\sum_{s=1}^{N}\left\{\varphi_s(\alpha)\sum_{ns}exp(-x_{ns})\right\},$$

$$D(\alpha)=\sum_{s=1}^{N}\left\{\varphi_s(\alpha)\sum_{ns}x_{ns}\,exp(-x_{ns})\right\},\ x_{ns}=\frac{\varepsilon_{ns}}{k_0T}.$$

The α_f behavior on film thickness in this case differs from the one for degenerated electrons. The analysis of expression (12) shows us that for fixed n_{ef}. Otherwise, thermopower in this case also is a nonmonotonous function of film thickness. When $\overline{n_s}>>1$ limit the result for bulk specimen is obtained that doesn't depend on a film thickness.

The same kinetic coefficients in the weak magnetic field take forms:

$$R_f=-\frac{1}{n_{ef}ec}\frac{A(\alpha)C'(\alpha)}{A'^2(\alpha)}, \qquad (13)$$

$$\alpha_f=-\frac{k_0}{e}\left\{\frac{D'(\alpha)}{A'(\alpha)}+\nu_0^2\left[\frac{F'(\alpha)C'(\alpha)+B'(\alpha)D'(\alpha)}{A'^2(\alpha)}-\frac{D'(\alpha)C'^2(\alpha)}{A'^3(\alpha)}-\frac{E'(\alpha)}{A'(\alpha)}\right]\right\}, \qquad (14)$$

where

$$\nu_0=\frac{e\tau_0}{m_\perp c}H,\ A'(\alpha)=\sum_{s=1}^{N}\left\{\frac{\left[\varphi_s^{-2}(\alpha)+1\right]}{2}\left(\overline{n_s}+\frac{1}{2}\right)^{-1}\sum_{ns}F_{r+1}(\eta_{ns})\right\},$$

$$B'(\alpha)=\sum_{s=1}^{N}\left\{\frac{\left[\varphi_s^{-4}(\alpha)+\varphi_s^{-6}(\alpha)\right]}{2}\left(\overline{n_s}+\frac{1}{2}\right)^{-3}\sum_{ns}F_{3r+1}(\eta_{ns})\right\},$$

,

$$D'(\alpha)=\sum_{s=1}^{N}\left\{\frac{\left[\varphi_s^{-2}(\alpha)+1\right]}{2}\left(\overline{n_s}+\frac{1}{2}\right)^{-1}\sum_{ns}\left[F_{r+2}(\eta_{ns})-\eta_{ns}F_{r+1}(\eta_{ns})\right]\right\},$$

$$E'(\alpha)=\sum_{s=1}^{N}\left\{\frac{\left[\varphi_s^{-4}(\alpha)+\varphi_s^{-6}(\alpha)\right]}{2}\left(\overline{n_s}+\frac{1}{2}\right)^{-3}\sum_{ns}\left[F_{3r+2}(\eta_{ns})-\eta_{ns}F_{3r+1}(\eta_{ns})\right]\right\},$$

$$F'(\alpha)=\sum_{s=1}^{N}\left\{\varphi_s^{-3}(\alpha)\left(\overline{n_s}+\frac{1}{2}\right)^{-2}\sum_{ns}\left[F_{2r+2}(\eta_{ns})-\eta_{ns}F_{2r+1}(\eta_{ns})\right]\right\},$$

r=0 for phonons and point defects and r=2 for ionized impurities.

Therefore, in this case unlike the strong field R_f depends on film surface orientation. Thermopower α_f depends on magnetic field like small correction proportional ν_0^2.

In conclusion, we note that kinetic coefficients in size-quantized n-Ge and n-Si films essentially depend on film surface orientation. Therefore, they possess the so-called size-quantized anisotropy. It is the most particular feature of kinetic quantum-sized effects in the anisotropic n-Ge and n-Si films.

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n-Ge VƏ *n*-Si ÜÇÜN KVANTLANMIŞ NAZİK TƏBƏQƏLƏRDƏ ELEKTRON KEÇİRMƏ HADİSƏLƏRİ

n-Ge və *n*-Si nazik təbəqələrin səthindən asılı olaraq kvantlanmamış maqnit sahəsində kinetik əmsalların xassələri təhlil olunur. Kinetik tənliyin ümumi həlli alınmışdır və müxtəlif elektron səpilmələri üçün relaksasiya zamanı hesablanmışdır. Həmçinin ixtiyari cırlaşmış elektron qazının qalvanomaqnit və termomaqnit tenzorlar üçün ifadələr alınmışdır. Və nazik təbəqələrdə güclü və zəif eninə maqnit sahəsində Holl əmsalı və termol-EHQ hesablanmışdır.

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ЭЛЕКТРОННЫЕ ЯВЛЕНИЯ ПЕРЕНОСА В РАЗМЕРНО-КВАНТОВАННЫХ ПЛЕНКАХ *n*-Ge И *n*-Si

Исследовано поведение некоторых кинетических коэффициентов для размерно-квантованных пленок n-Ge и n-Si в неквантованном магнитном поле в зависимости от ориентации их поверхности относительно кристаллографических осей. Получено общее решение для кинетического уравнения и выражения для времени релаксации при различных механизмах рассеяния. Также получены выражения для компонент гальваномагнитных и термомагнитных тензоров при произвольном вырождении электронного газа. Вычислены коэффициент Холла и термо-ЭДС для пленок n-Ge и n-Si в сильных и слабых поперечных магнитных полях.

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