ELASTIC AND INELASTIC SCATTERING OF PROTONS ON ATOMIC NUCLEI

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In the frameworks of distorted wave HEA, on the base of three-dimensional formulation the expression for scattering amplitude of protons of high energies on atomic nuclei was obtained in the analytical form. As a consequence of short range character of a proton-nucleon interaction, the scattering of protons on nuclei was presented as a sequence of unitary scattering. With a help of the developed mathematical method the recurrent formula was received what allowed to express the form-factor in the distorted wave in Born,s ones and its derivatives .As a result of the analysis of experimental cross sections of elastic scattering of protons with energy 1 GeV, the parameters of distribution of protons and neutrons in spherical nuclei ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{208}Pb as well as a width of a surface layer of nucleons, root-mean-square radii of protons and neutrons were determined. Fermi-function was used for the distribution of nucleons density.

Development of experimental engineering last decade allowed tocarry out numerous experiments on scattering of protons on nuclei in the range of intermediate energies of striking particles. This area of energies is of a particular interest, because checking behavior of amplitude of particles scattering on nuclei becomes possible. Different methods for getting obvious expressions of the amplitude for electrons scattering on nuclei are listed in [1]. The most successful expression of the amplitude was presented in[2]. Further, this theory was developed in [1], where opportunities and good accuracy for performance of quantitative studies were shown. Later in [3,4] this method was advanced for elastic and inelastic scattering of electrons on spherical nuclei and the good results were received. What concerns proton scattering, it appears possible to receive the amplitudes [5] fair in the range of small angles. Recently, in [6] on the base of three-dimensional quasiclassics within the limits of high-energy approach (HEA), the amplitude of protons scattering on nuclei was obtained in the range of small angles of scattering as well as of large ones. Obviously, this method will find it's wide application.

The purpose of the present study is to receive the amplitude of scattering in an analytical form to connect it with the theory of multy-scattering and to develop a method of it's calculation. Let's write down a differential section of the process in general form:

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\left(\frac{k}{E}\right)^2\frac{2J_f+1}{2J_i+1}\sum_{\sigma_i\sigma_f}\sum_{M_iM_f}\left|\ f_{if}\left(k_i,k_f\right)\ \right|^2 \tag{1}$$

Wave functions of relative motion of the striking and scattered nucleons as the solutions of Schredinger equation produce the following form:

$$\psi^{(\pm)}(\mathbf{k},\mathbf{r})=\exp\ \left\{ i\ \left[\ \mathbf{kr}\mp\Phi^{(\pm)}(\mathbf{k},\mathbf{r})\right.\right\} \tag{2}$$

where the distorted member is

$$\Phi^{(\pm)}(\mathbf{k},\mathbf{r})=\frac{m}{\hbar^2 k}\int_0^\infty V\left(\mathbf{r}\mp\hat{k}s\right)\ ds \tag{3}$$

Using property of spherical symmetry of nuclear potential from the static equation we get:

$$\nabla^2V(r)-k_0^2V(r)=4\pi\gamma\rho(r) \tag{4}$$

The coefficient of expansion of potential is received in form

$$a=\left(\frac{4\pi}{3k^3}\right)\gamma\rho\ \ (0)+\frac{k_0^2\ V(0)}{3k^3}, \tag{5}$$

where γ=0,08.

Here γ corresponds to the irrationalized constant of connection, defined from the experiment on nucleon-nucleon scattering [7].

It is possible to neglect the change of nucleons location in the nucleus during a flight of a fast proton through it. The scattering occurs basically forward on small angles. Scattered nucleon consistently interacts with several nucleons of a nucleus, which are met on it's way. Therefore, as a consequence of short-range nucleon-nucleon interaction, the scattering of nucleon may be written as a sequence of unitary scattering. Taking it into account, nuclear potential may be represented as the sum of component interactions of the striking particle with the nucleons of the target nucleus:

$$V(\mathbf{r}\xi) = \int \upsilon(|\mathbf{r}-\mathbf{x}|)\rho(\mathbf{x}\xi)d\mathbf{x} \tag{6}$$

As the binding energy of a nucleus is small in comparison with the energy of a striking proton, the nucleons binding may be neglected and hence, the potential of nucleon-nucleon interaction can be expressed in the amplitudes of scattering on free nucleons defined from the solution of Schredinger equation

$$f_{NN}(\mathbf{k}',\mathbf{k}) = -\frac{\mu_0}{2\pi\ \hbar^2}\int e^{-i\mathbf{k}'\mathbf{r}'}\upsilon(\mathbf{r}')\ \psi_{\mathbf{k}}(\mathbf{r}')\ d\mathbf{r}', \tag{7}$$

where $\upsilon^{иск}(\mathrm{r}) = \upsilon(r)\exp\left\{-\frac{im}{\hbar^2 k}\int_{-\infty}^{z}\upsilon dz'\right\}$ is a distorted nucleon-nucleon potential, μ_0 - equivalent mass, q' - momentum of the particle striking on a nucleon target. Taking into account Fourier transformation in (6), the nuclear potential is received :

$$V(\mathbf{r}\xi) = -\frac{\hbar^2}{(2\pi)^2\mu_0}\int e^{i\mathbf{q}'(\mathbf{r}-\mathbf{x})}f_{NN}(\mathbf{q}')\rho(\mathbf{x}\xi)d\mathbf{q}'d\mathbf{x} \tag{8}$$

.

Here for nucleon-nucleon amplitude a following parameterization is chosen:

$$f_{NN}(q') = \frac{ik\sigma}{4\pi}(1-i\varepsilon_0)\ e^{-\beta^2 q'^2}/2 \tag{9}$$

After integration the following expression for differential section is received:

$$\frac{d\sigma}{d\Omega} = \left(\frac{k^2}{4\pi\mathrm{E}}\right)^2 \frac{1}{2}\frac{2J_f+1}{2J_i+1}\sum_{LM}\frac{1}{2L+1}\left|\ F_{LM}(q)\ \right|^2, \tag{10}$$

where the form-factor is

$$F_{LM}(q) = \int e^{i[\mathbf{q}\mathbf{x}+\Phi(x)]}f_{NN}(\mathbf{q}_{эф})\rho_L(x)\ Y^*_{LM}(\hat{x})\ dx \tag{11}$$

For derivation of this form-factor we obtained the following recurrent formula:

$$(m+1)\ \mathrm{F}_{L,m+1}(q) = \sum_{n=0}^{6}\alpha_n \mathrm{F}_{L,m}^{(n)}(q) + \sum_{n=1}^{6}\beta_m \mathrm{F}_{L,m-1}^{(n)}(q)\,, \quad m=1,2,3,\ldots.. \tag{12}$$

$$\mathrm{F}_{L,1}(q) = \sum_{n=0}^{4}\alpha_n \mathrm{F}_{L,0}^{(n)}(q)\ , \qquad \mathrm{F}_{L,m}^{(n)}(\mathrm{q}) = \frac{\partial^n \mathrm{F}_{L,m}(\mathrm{q})}{\partial \mathrm{q}^n} \tag{13}$$

This recurrent formula allows to express the form factor $F_L(q,\gamma)$ (11)Born's form-factor and it's derivatives.

ELASTIC SCATTERING OF PROTONS ON SPHERICAL NUCLEI.

Analysis of the cross-sections with a help of multy scattering theory of protons of intermediate energies allows to obtain a quiet exact information about nucleon distribution in nuclei. It is known, that the fast protons have the same sensitivity as protons and neutrons of a nucleus. Therefore, the data on scattering of protons on nuclei makes it possible to get the information about izoscalar density, i.e. about the sum of neutron and proton densities.

$$\rho\ (r) = \rho_\rho(r) + \rho_n(r) \tag{14}$$

Distribution of protons and neutrons densities is chosen as Fermi-function

$$\rho_i(r) = \rho_{oi}\left(1 + e^{\frac{r-c}{b_i}}\right)^{-1} = \rho_{oi}\tilde{\rho}(r|b_i), \quad i = p;n \tag{15}$$

Experimental data on elastic scattering of protons with energy ~ 1GeV on nuclei ^{40}Ca, ^{48}Ca, ^{90}Zr and ^{208}Pb [6] are analyzed within the framework HEA with a use of probe function (14). The best consent with the theoretical cross sections is achieved at the certain sets of protons and neutrons as it is shown on fig. 1 and 2. The parameters itself are presented in the table.

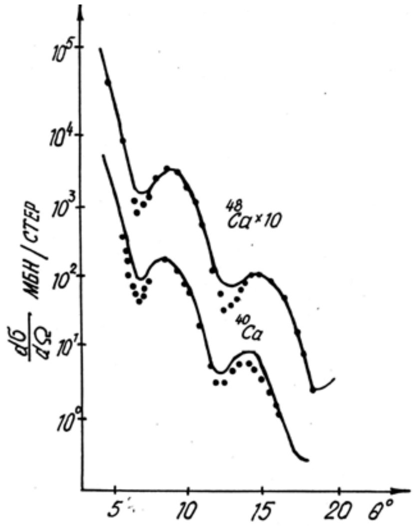

Fig.1. Differential sections of elastic scattering of protons with energy 1 GeV on ^{40}Ca and ^{48}Ca. Points- experimental data, solid lines- cross sections, derived by a method of distorted waves.

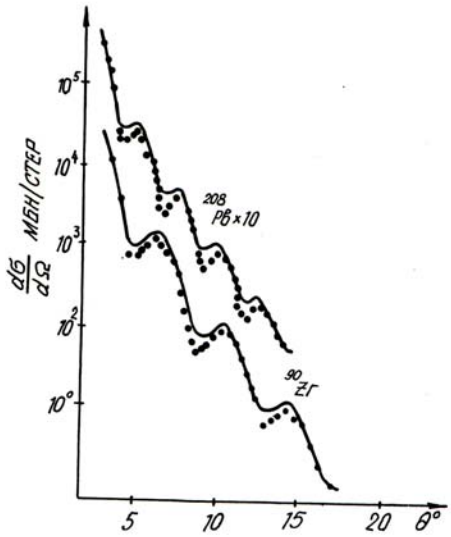

Fig.2. Differential sections of elastic scattering of protons with energy 1 GeV on nuclei ^{90}Zr and ^{208}Pb. Points- experimental data. Solid lines- cross sections derived by the method of distorted waves.

As it is seen from the table, the good agreement of the cross sections is received at α = α =α, i.e., thickness of a surface layer of protons and neutrons in the spherical nuclei are not different. It proves once again that the fast protons on the sur`faces of spherical nuclei are not sensitive to a thin structure. As it is known, the fine structure in distributions of protons density appears at the account of three-parameter Fermi –functions in elastic scattering of electrons on nuclei.

TABLE.

Parameters describing distribution of density of protons, neutrons and nucleons.

^{40}Ca	0.60	0.6	2.260	3.920	2.662	3.70	1
^{48}Ca	0.64	0.4	2.480	3.590	2.754	3.18	1.023
^{90}Zr	0.40	0.30	2.306	4.308	4.207	4.22	1.040
^{208}Pb	o.60	0.3	1.710	5.482	4.982	5.26	1.048

To satisfy condition (14) we chose distribution of proton and neutron density in the following general form:

$$\rho_{p(n)}(r)=\frac{1}{2}\left[1\mp\beta(r)\right]\ \tilde{\rho}(r),\ \ \beta(r)=\alpha_0-\alpha_1\frac{r^2}{c^2}, \tag{16}$$

Thus the distribution of protons and neutrons density in nuclei accept the following form:

$$\rho_{p(n)}(r)=\rho^{0}_{0p(n)}\left(1\pm W_{p(n)}\frac{r^2}{2c^2}\right)\tilde{\rho}(r)\ , \tag{17}$$

where $W_{p(n)}$ - parameters, describing the fine structure in distribution of protons and neutrons density, are connected with each others.

All calculations were carried out using the parameters of elementary amplitudes, according to the data on elastic nucleon-nucleon scattering [10,11].

Comparing the derived cross sections with experimental ones it can be seen that on the right slopes of diffractional peaks the consent is good, while on the left slopes and in the area of diffractional minimum some excess of the derived values is observed.

On fig.3. the diagrams of distribution of protons and neutrons density are presented. Parameters of these distributions are obtained from the combined analysis of experimental cross sections in the distorted wave HEA of proton and electron scattering on the appropriate nuclei [9].

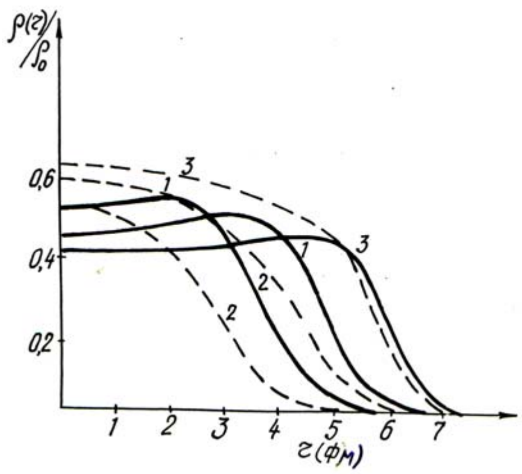

Fig. 3. Distributions of density of protons (solid lines), neutrons (dotted): 1- ^{90} Zr, 2- ^{48} Ca, 3- ^{208}Pb

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