

**SIMULATION OF THRESHOLD PROPERTIES OF FERROELECTRIC LIQUID CRYSTAL**

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The one-dimensional model of thin planar ferroelectric liquid crystal cell has been considered. The dependences of threshold voltage both on some material parameters of ferroelectric liquid crystal (spontaneous polarization, anisotropy of dielectric permittivity, elastic constants) and on some external parameters (cell thickness, anchoring energies) have been obtained. The qualitative explanation of obtained results has been proposed.

One of the basic problems by utilizing of liquid crystal displays is to reduce the operating voltages as much as possible. The later can be essentially reduced in the case of ferroelectric liquid crystals (FLC) [1]. The first reason of so high sensitivity to an external electric field is the strong coupling of the electric field with spontaneous polarization. Except of spontaneous polarization, the threshold voltage of electrooptical effect is influenced by elastic constants, anisotropy of dielectric permittivity of FLC, surface conditions etc. In this paper the attempt was made to study influence of each of these factors on the FLC switching threshold on the basis of numerical calculations.

The description of electrooptic properties of FLC-cell can be reduced to determination of the director field  $\vec{n}(\vec{r}, t)$ . The later is given in the given point of space by two angles: the tilt angle  $\theta$  and the azimuthal angle  $\phi$  (fig. 1):

$$\vec{n} = \vec{n}(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \quad (1)$$

The tilt angle depends mainly on temperature and the electric field essentially does not change its value. But the azimuthal angle varies spatially in absence of an electric field, and additionally in time under the field action. In thick samples, where the spirally twisted structure takes place, the angle  $\phi$  varies both in a direction, perpendicular to layers, and in layers plane ( $\phi = \phi(x, z)$ ), but in thin samples, only in a layer plane [2]. Therefore, for not too high fields the task of finding of a director field  $\vec{n}(\vec{r}, t)$  is reduced to the task of searching of azimuthal angle  $\phi(x, t)$  distribution, what is determined by the competition of elastic and surface forces, and also by the electric field force.

It is not difficult to deduce the balance torque equation allowing us to determine the equilibrium configuration of the system  $\phi(x)$  (the thickness is less than pitch of spiral structure:  $d \ll L$ ):

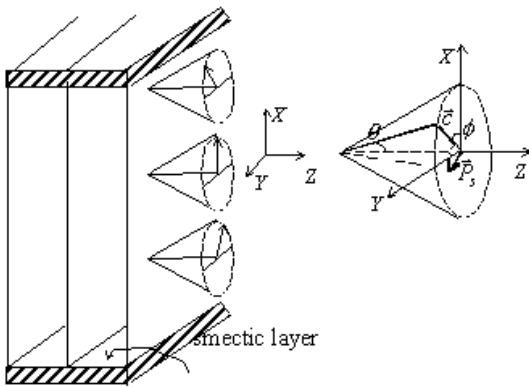


Fig.1. The cell geometry

$$G\theta^2 \frac{d^2 \phi}{dx^2} + P_s E \cos \phi + \left( \frac{P_s^2}{2\chi_{\perp} \epsilon_0} + \frac{\Delta \epsilon \epsilon_0 \theta^2 E^2}{2} \right) \sin 2\phi = 0 \quad (2)$$

under boundary conditions

$$G \frac{d\phi}{dx} \Big|_{\pm d/2} = (W_1 \cos \phi \pm W_2 \sin 2\phi)_{\pm d/2} \quad (3)$$

$$0 \leq x \leq d/2, \quad -\pi/2 \leq \phi \leq \pi/2$$

The following notations are used:  $G$  is an elastic constant,  $P_s$  is a spontaneous polarization,  $E$  is an electric field strength,  $\Delta \epsilon = \epsilon_{||} - \epsilon_{\perp}$  is an anisotropy of dielectric permittivity,  $\chi_{\perp}$  is transversal component of the dielectric susceptibility,  $\epsilon_0 = 8.85 \text{ pf/m}$  is electric constant.

The first term in the right hand side of equation (3) expresses the polar interaction of molecules with the

substrate surface. The polar interaction tends to orient the spontaneous polarization toward, or out of the surface. It is equivalent to a condition  $\phi(d/2) = -\phi(-d/2) = \pi/2$  for our geometry. The second term is a dispersion part of the surface energy: the dispersion interaction is responsible for planar orientation of smectic A phase and required  $\phi(d/2) = \phi(-d/2) = \pi/2$ . The appropriate anchoring energies are denoted by  $W_1$  and  $W_2$ . The signs «-» and «+» concern to top and bottom surfaces respectively.

For FLCs, used in electrooptical cells the typical values of  $P_s$ ,  $G$ ,  $\theta$ ,  $\chi_{\perp}$  and  $\Delta \epsilon$  have the order of  $10^{-4} \text{ cm}^{-2}$ ,  $10^{-11} \text{ N}$ ,  $0.4$ ,  $10$ ,  $-3$ , respectively. Other external parameters  $W_1$ ,  $W_2$ ,  $d$  and  $E$  have the order of

$$10^{-4} \text{ J} \cdot \text{m}^{-2}, \quad 10^{-5} \text{ J} \cdot \text{m}^{-2}, \quad 5 \cdot 10^{-6} \text{ m}, \quad 10^6 \frac{\text{V}}{\text{m}},$$

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respectively. Furthermore, we represent the polar part of anchoring energy as

$$W_l = W_{l0} + \alpha \cdot P_s \quad (4)$$

where  $W_{l0}$  is the permanent (independent of spontaneous

polarization) component of polar anchoring energy, the second term is the contribution due to the spontaneous polarization. It is reasonable to take  $W_l \cong 10^{-5} J \cdot m^{-2}$  and  $\alpha \cong 0.5 V$ .

The dynamics of the switching process can be described by the equation similar to (2):

$$\gamma\theta^2 \frac{\partial \phi}{\partial t} = G\theta^2 \frac{\partial^2 \phi}{\partial x^2} + P_s E \cos \phi + \left( \frac{P_s^2}{2\chi_{\perp}\epsilon_0} + \frac{\Delta\epsilon\epsilon_0\theta^2 E^2}{2} \right) \sin 2\phi \quad (5)$$

under the same boundary conditions.  $\gamma_{\phi}$  is a rotational viscosity that has the order  $0.1 Pa \cdot s$ .

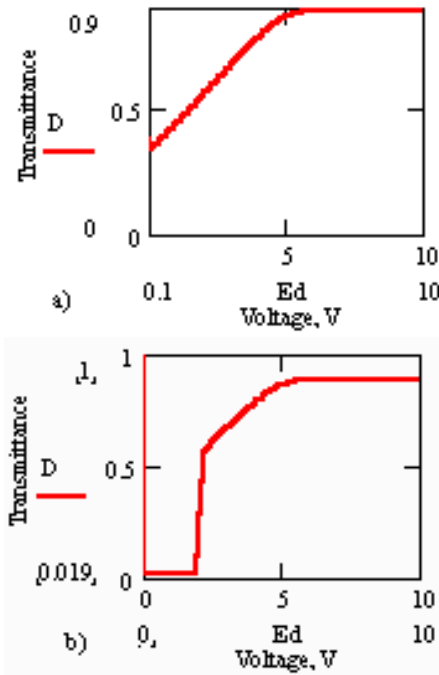


Fig. 2. Volt-contrast characteristics of FLC cell: a) the initial state is twist; b) the initial state is uniform

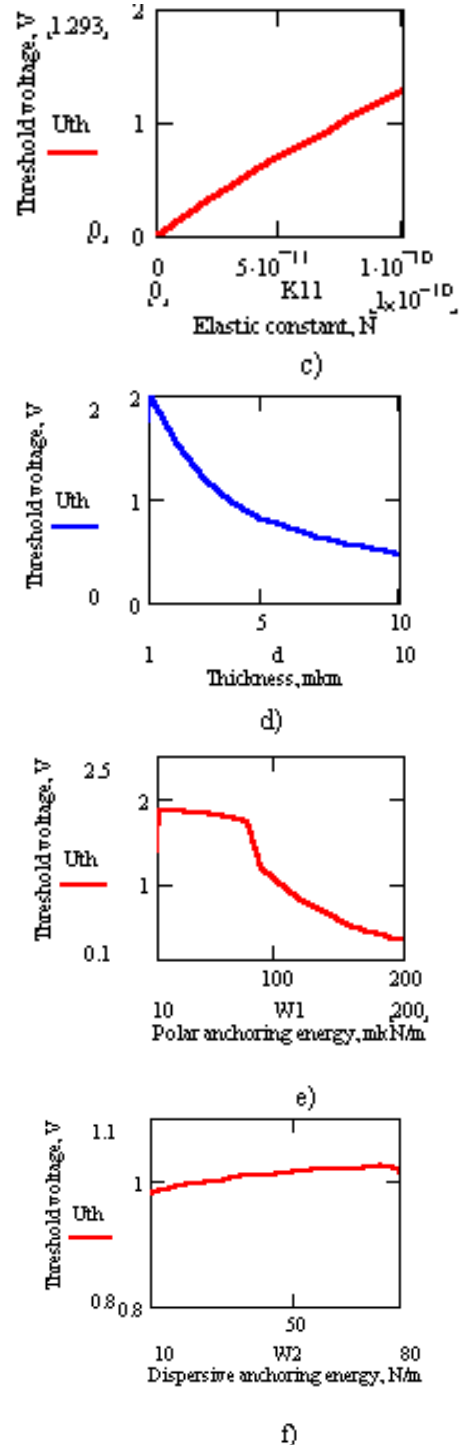
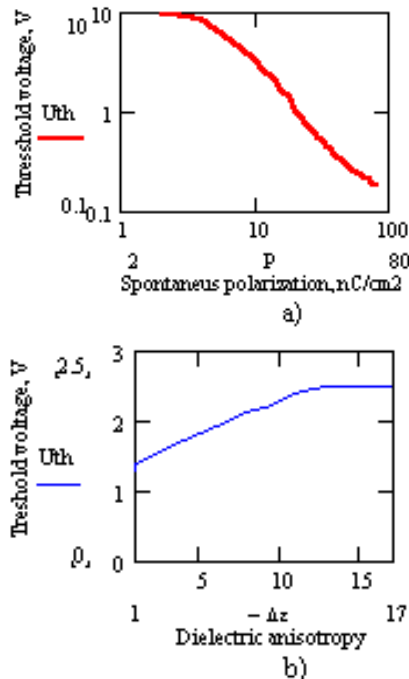


Fig. 3. The dependence of threshold voltage on a) spontaneous polarization b) dielectric anisotropy; c) elastic constant; d) cell thickness; e) polar anchoring energy; f) azimuthal anchoring energy

The equation (5) is a nonlinear heat conduction equation with boundary conditions (2) of general form and can be solved by the sweep method [3].

To know the dependence  $\phi(x, t)$ , it is not difficult to find light transmittance of FLC cell in the crossed polarizers, for example, by Johnes' (2x2) retardation matrix method [4]. To study the threshold characteristics of the FLC cell the slowly rising electric field (with the rate of order  $1 \frac{V}{s}$ ) has been applied. As seen from a fig. 2a, the transition from twist state to uniform state has not threshold character. But if as the initial state to take the uniform state ( $\phi = -\pi/2$ ), the transition to other uniform state ( $\phi = \pi/2$ ) has clearly expressed threshold character (fig. 2b).

The results of numerical calculations are shown in fig.3, as dependences of a threshold voltage on spontaneous polarization, dielectric anisotropy, cell thickness, elastic constant and anchoring energies.

As expected, the threshold voltage is proportional to spontaneous polarization (fig. 3a). A small deviation from this low in the range of weak spontaneous polarizations is related, on our opinion, to polar anchoring of molecules with a surface, which appears as a destabilizing factor. For upper surface there is an excess of free energy of density about  $\frac{W_1}{d}$  that is comparable with field energy  $P_s E$ .

The dielectric permittivity also influences on the value of the threshold. As, the interaction of a field with negative

dielectric anisotropy appears as the stabilizing factor (appropriate density of energy has the order  $\frac{-\Delta\epsilon\epsilon_0 U^2}{2d^2}$ ),

with increasing of  $|\Delta\epsilon|$  the threshold voltage slowly rises (fig. 3b).

The transition from one homogeneous state to another occurs by creation of intermediate twist state of energy density about  $\frac{G}{d^2}$ . Therefore the threshold voltage increases with rising of an elastic coefficient. This dependence has the form of direct proportionality with surprising accuracy (fig.3c).

The increasing of cell thickness leads at first to the diminishing of the threshold voltage, however, in further this tendency loosened (fig.3d). A reason of it also is weakening of polar anchoring energy density.

The destabilizing role of polar anchoring is expressed also in following: the threshold voltage diminishes with increase of polar anchoring energy  $W_1$  (fig. 3e). The breaks observed in a graphics hint about qualitatively different ways of switching at different intervals of values  $W_1$ .

As the dispersion interaction of molecules with a surface acts as the stabilizing factor, the threshold voltage slowly rises with increase of dispersive part of anchoring energy  $W_2$  (fig. 3f).

Note that the proposed model is one – dimensional where the switching by domain wall motion is not under consideration.

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## **SEQNETOELEKTRİK MAYE KRİSTALIN ELEKTROOPTİK XASSƏLƏRİNİN MODELƏŞDİRİLMƏSİ**

Birölçülü hal üçün nazik planar seqnetoelektrik maye kristal nümunəsinin riyazi modeli qurulmuşdur. Astana gərginliyinin həm maye kristalın maddi parametrlərindən (spontan polyarizasiya, dielektrik nüfuzluğunun anizotropiyası, elastik sabit), həm də xarici parametrlərdən (yuvaciğın qalınlığı, ilişmə enerjiləri) asılılığı müəyyən olunmuş və bu asılılıqların keyfiyyətə izahı verilmişdir.

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## **МОДЕЛИРОВАНИЕ ЭЛЕКТРООПТИЧЕСКИХ СВОЙСТВ СЕГНЕТОЭЛЕКТРИЧЕСКОГО ЖИДКОГО КРИСТАЛЛА**

В данной работе представлена одномерная модель тонкой планарной ячейки сегнетоэлектрического жидкого кристалла. Установлена зависимость порогового напряжения, как от материальных параметров жидкого кристалла (спонтанной поляризации, анизотропии диэлектрической проницаемости, упругой постоянной), так и от внешних параметров (толщины ячейки, энергий сцепления). Приведена качественная интерпретация полученных результатов.

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