

THE TRANSIENT RADIATION OF THE NON-INVARIANT SOURCE IN THE PLANE-LAYERED MEDIUM

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The process of the transient radiation of the non-invariant relativistic source of the electromagnetic field, in particularly, the magnetic dipole moment in the plane-layered medium is considered. The general expressions, describing the radiation field and change of the own field are obtained. The analysis of the obtained formulas for the ultrarelativistic velocity of the magnetic moment is done.

1. INTRODUCTION

The investigation of the transient radiation of the non-invariant relativistic source of the electromagnetic field of charge had been carried out firstly half an age ago in the work of Ginsburg and Frank [1], in which it was shown, that the so-called transient radiation appears at the charge motion through the plane boundary of the separation of two isotropic mediums with the different physical properties, if the charge have the constant velocity, which is less, than the phase velocity of radiation in medium. The radiation is mainly directed along the charge motion at the high charge velocities.

The transient radiation has been the subject of the intensive investigations during the last decades. The many investigations were carried out for the creation of the practical systems, using the transient radiation for the identification of the relativistic particles, which are one of the more important problems in the high energy physics.

The investigation of the transient radiation of the non-invariant sources of the electromagnetic field, in particular, the dipole moment was considered in the ref [2-5]. The question about the transient radiation as invariant so non-invariant sources on the blurred boundary of the separation of the mediums was considered in the ref [4-7]. The present paper, deals to the transient radiation of the magnetic moment in the weakly nonhomogeneous plane-layered medium.

2. EQUATIONS FOR THE HERTIZIAN VECTORS IN THE NONHOMOGENEOUS MEDIUM AND THEIR FOURIER TRANSFORMATIONS

Let's consider the non-magnetic ($\mu = 1$) non-homogeneous medium, the dielectric constant of which depends on the coordinations: $\varepsilon = \varepsilon(x, y, z)$. In addition, for the magnetic Hertzian vector \vec{I}_m as in the case of the isotropic medium we obtain the following nonhomogeneous wave equation:

$$\square \vec{I}_m = -4\pi \vec{M}, \tag{1}$$

and for the following more complex equation

$$\square \vec{I}_e = -4\pi \vec{P} + \varepsilon^{-1} \vec{\nabla} \varepsilon (\vec{\nabla} \vec{I}_e) - \left[\vec{\nabla} \varepsilon, \partial \vec{I}_m / c \partial t \right], \tag{2}$$

for the electric vector \vec{I}_e , in the right part of which the two last members are caused by the medium inhomogeneous respect of the dielectric constant, the change of which on the layer thickness of the medium inhomogeneous is the reason of the creation of the radiation field and change of the eigen field; \vec{M} and \vec{P} are vectors of the magnetic and electric polarization. They are defined by the following expressions:

$$\vec{M} = \vec{m} \delta(\vec{r} - \vec{v}t), \quad \vec{P} = [\vec{\beta} \vec{m}] \delta(\vec{r} - \vec{v}t),$$

where $\square = \vec{\nabla}^2 - \frac{\varepsilon}{c^2} \cdot \frac{\partial^2}{\partial t^2}$ is D'Alembert's operator in the case of the nonhomogeneous medium, \vec{m} is the magnetic moment and $[\vec{\beta} \vec{m}] = \vec{p}$ is the electric dipole moment, combined with the magnetic moment, moving with the constant velocity.

In the general case the equations (1) and (2) impossible to solve. They are solved exactly or approximately only when the dielectric constant depends on the only one variable. In the present paper the dependence $\varepsilon = \varepsilon(z)$ of the dielectric constant of the medium, called the plane-layered is considered. The solutions of the equations (1) and (2) are obtained by the method of the consequent approximation; in addition, one takes into consideration, that:

$$\vec{I}_m(\vec{r}, t) = \vec{I}_m^0(\vec{r}, t) + \delta \vec{I}_m(\vec{r}, t), \tag{3}$$

$$\vec{I}_e(\vec{r}, t) = \vec{I}_e^0(\vec{r}, t) + \delta \vec{I}_e(\vec{r}, t), \tag{4}$$

$$\varepsilon(z) = \varepsilon^0 + \delta \varepsilon(z), \tag{5}$$

where $\delta \vec{I}_m$, $\delta \vec{I}_e$ and $\delta \varepsilon(z)$ are small values of the first order, ε^0 is the dielectric constant of the homogeneous medium. The summand $\delta \varepsilon(z)$ in the function (5), caused by the dielectric inhomogeneous, has to change gradually from $-\Delta \varepsilon / 2$ to the $+\Delta \varepsilon / 2$ on the all inhomogeneous, in addition, $\varepsilon_1 = \varepsilon^0 - \Delta \varepsilon / 2$ and $\varepsilon_2 = \varepsilon^0 + \Delta \varepsilon / 2$. From (1) and (2) with (3-5) we obtain the following equations:

$$\square_0 \vec{I}_m^0(\vec{r}, t) = -4\pi \vec{M}(\vec{r}, t), \tag{6}$$

$$\square_0 \vec{\Pi}_e^o(\vec{r}, t) = -4\pi \vec{P}(\vec{r}, t) , \quad (7)$$

$$\square_0 \delta \vec{\Pi}_m(\vec{r}, t) = \frac{\delta \varepsilon}{c^2} \cdot \frac{\partial^2 \vec{\Pi}_m^o(\vec{r}, t)}{\partial t^2} , \quad (8)$$

$$\square_0 \delta \vec{\Pi}_e(\vec{r}, t) = \frac{\delta \varepsilon}{c^2} \cdot \frac{\partial^2 \vec{\Pi}_e^o(\vec{r}, t)}{\partial t^2} + \vec{e}_3 \frac{1}{\varepsilon^o} \cdot \frac{\partial \delta \varepsilon}{\partial z} \left(\vec{\nabla} \vec{\Pi}_e^o(\vec{r}, t) \right) - \frac{\partial \delta \varepsilon}{\partial z} \left[\vec{e}_3, \frac{1}{c} \cdot \frac{\partial \vec{\Pi}_m^o(\vec{r}, t)}{\partial t} \right] \quad (9)$$

where $\square_0 = \nabla^2 - \frac{\varepsilon^o}{c^2} \cdot \frac{\partial^2}{\partial t^2}$ is the D'Alembert's operator for

the homogeneous nonmagnetic medium.

In considered problem the all values it is need to expand in the Fourier integral on the time and transverse component of the radius vector because of the homogeneous in the time and on the directions, which are perpendicular to the field source velocity [2]:

$$\vec{\Pi}_m^o(\vec{r}, t) = \int \vec{\Pi}_{m\omega\vec{\chi}}^o(z) \exp(i\vec{\chi}\vec{r}_\perp - i\omega t) d\omega d\vec{\chi} \quad (10)$$

$$\mathcal{L}\delta \vec{\Pi}_{e\omega\vec{\chi}}(z) = -\frac{\omega^2}{c^2} \left(\delta \varepsilon - i \frac{c^2}{\omega \nu} \cdot \frac{\partial \delta \varepsilon}{\partial z} \right) \vec{\Pi}_{e\omega\vec{\chi}}^o(z) + i\vec{e}_3 \frac{1}{\varepsilon^o} \cdot \frac{\partial \delta \varepsilon}{\partial z} \left(\vec{\chi} \vec{\Pi}_{e\omega\vec{\chi}}^o(z) \right) , \quad (14)$$

where $\mathcal{L} = \partial^2 / \partial z^2 + k_{rz}^2$, $k_{rz} = \frac{\omega}{c} \sqrt{\varepsilon^o - \chi^2 c^2 / \omega^2}$ is

the longitudinal component of the vector of the radiation field and

$$\vec{M}_{\omega\vec{\chi}}(z) = \frac{\vec{m}}{(2\pi)^3 \nu} \exp(i\omega z / \nu) , \quad (15)$$

$$\vec{P}_{\omega\vec{\chi}}(z) = \frac{[\vec{\beta}\vec{m}]}{(2\pi)^3 \nu} \exp(i\omega z / \nu) \quad (16)$$

$$\vec{\Pi}_{m\omega\vec{\chi}}^o(z) = -\frac{4\pi \vec{m} c^2}{(2\pi)^3 \nu \omega^2} \left(\varepsilon^o - c^2 / \nu^2 - \chi^2 c^2 / \omega^2 \right)^{-1} \exp(i\omega z / \nu) , \quad (17)$$

$$\vec{\Pi}_{e\omega\vec{\chi}}^o(z) = -\frac{4\pi c^2 [\vec{\beta}\vec{m}]}{(2\pi)^3 \nu \omega^2} \left(\varepsilon^o - c^2 / \nu^2 - \chi^2 c^2 / \omega^2 \right)^{-1} \exp(i\omega z / \nu) . \quad (18)$$

the Fourier images of Hertzian vectors $\vec{\Pi}_{m\omega\vec{\chi}}^o$ and $\vec{\Pi}_{e\omega\vec{\chi}}^o$ define the eigen field of the source in the homogeneous medium with the dielectric constant ε^o (the radiation field in the homogeneous medium is supposed to be absent). The main problem is that solving equations (13) and (14) it is necessary to find the additions to the zero solutions (17) and (18), corresponding to the eigen field, and the general solutions of the homogeneous equation, defining the radiation field.

For the solutions of the equations (13) and (14) firstly it is need to expand $\partial \mathcal{E}(z)$ in the Fourier integral:

$$\delta \varepsilon(z) = \int \delta \varepsilon_\eta \cdot \exp(i\eta z) d\eta . \quad (19)$$

and e.t.c. In addition, we obtain the Fourier images of the equations (7-9):

$$\mathcal{L}\vec{\Pi}_{m\omega\vec{\chi}}^o(z) = -4\pi \vec{M}_{\omega\vec{\chi}}(z) , \quad (11)$$

$$\mathcal{L}\vec{\Pi}_{e\omega\vec{\chi}}^o(z) = -4\pi \vec{P}_{\omega\vec{\chi}}(z) , \quad (12)$$

$$\mathcal{L}\delta \vec{\Pi}_{m\omega\vec{\chi}}(z) = -\frac{\omega^2}{c^2} \delta \varepsilon \vec{\Pi}_{m\omega\vec{\chi}}^o(z) , \quad (13)$$

the Fourier images of the magnetic and electric polarizations.

3. THE RETARDED SOLUTIONS OF THE EQUATIONS

We know about the solutions of the equations (11) and (12) [3]:

By way of the concrete expressions for $\partial \mathcal{E}(z)$ we can choose the following functions:

$$\delta \varepsilon(z) = \frac{\Delta \varepsilon}{2} \operatorname{th} \frac{z}{\Delta z} , \quad (20)$$

$$\delta \varepsilon(z) = \frac{\Delta \varepsilon}{\pi} \operatorname{arctg} \frac{z}{\Delta z} , \quad (21)$$

$$\delta \varepsilon(z) = \frac{\Delta \varepsilon}{\sqrt{\pi}} \int_0^z \exp[-(x/\Delta z)^2] dx , \quad (22)$$

the Fourier images of which are defined by the appropriate expressions:

$$\delta \varepsilon_\eta = \frac{\Delta \varepsilon}{4i} \cdot \frac{\Delta z}{sh(\pi \eta \cdot \Delta z / 2)} , \quad (23)$$

$$\delta \varepsilon_\eta = \frac{\Delta \varepsilon}{2\pi i \eta} \cdot \exp(-|\eta| \Delta z) , \quad (24)$$

$$\delta \varepsilon_\eta = \frac{\Delta \varepsilon}{2\pi i \eta} \cdot \exp[-(\eta \cdot \Delta z / 2)^2] . \quad (25)$$

$$\bar{G}_{\omega \bar{z}}^{m,e}(\eta) = 4\pi \delta \varepsilon_\eta \frac{(\varepsilon^\circ - c^2 / v^2 - \chi^2 c^2 / \omega^2)^{-1}}{(2\pi)^3 \cdot v} \left\{ \begin{array}{l} \bar{m} \\ [\bar{\beta} \bar{m}] \left(1 + \eta \frac{c^2}{\omega v} \right) - \bar{e}_3 [\bar{\chi} \bar{m}]_z \frac{c v \eta}{\varepsilon^\circ \omega^2} \end{array} \right\} . \quad (28)$$

The inhomogeneous magnetic and electric polarizations appear in the layer-inhomogeneous medium at the magnetic moment motion. Taking into consideration the expressions (15) and (16), the right parts of the equations (26) and (27) can be expressed through the magnetic and electric polarizations correspondingly, in addition, the last three play role of the source functions. That's why at the solving it is need to take into consideration, that Green functions in the left part of the equalities have to be retarded, i.e. to describe the retarded fields:

$$\delta \bar{\Pi}_{m\omega \bar{z}}(z) = - \int \frac{\bar{G}_{\omega \bar{z}}^m(\eta) \exp[i(\eta + \omega / v)z]}{(\eta - \eta_1) \cdot (\eta - \eta_2)} d\eta , \quad (29)$$

$$\delta \bar{\Pi}_{e\omega \bar{z}}(z) = - \int \frac{\bar{G}_{\omega \bar{z}}^e(\eta) \exp[i(\eta + \omega / v)z]}{(\eta - \eta_1) \cdot (\eta - \eta_2)} d\eta , \quad (30)$$

where

$$\eta_{1,2} = -\frac{\omega}{v} \pm \frac{\omega}{c} \sqrt{\varepsilon^\circ - \chi^2 c^2 / \omega^2} . \quad (31)$$

In the considered case the Cerenkov radiation is absent as in the homogeneous so in the inhomogeneous parts of all medium, when the condition $\varepsilon^\circ < c^2 / v^2$ is carried out, the values η_1 and η_2 became the indeed in the high frequencies region $\varepsilon^\circ \gg \chi^2 c^2 / \omega^2$ and the expression $(\varepsilon^\circ c^2 / v^2 - \chi^2 c^2 / \omega^2) < 0$. It follows from the expression $\eta_1 \eta_2 = -\frac{\omega^2}{c^2} (\varepsilon^\circ - c^2 / v^2 - \chi^2 c^2 / \omega^2) > 0$ that both values are equal to each other on the sign, i.e. if $\eta_2 < 0$, then $\eta_1 < 0$. Introducing the designation $\xi = \eta + \omega / v$, then we obtain

$$\xi_1 = \omega \sqrt{\varepsilon^\circ - \chi^2 c^2 / \omega^2} / c = \xi_o > 0 ,$$

$$\xi_2 = -\omega \sqrt{\varepsilon^\circ - \chi^2 c^2 / \omega^2} / c = -\xi_o < 0 .$$

Substituting the solutions (17) and (18) and the equality (19) in the right parts of (13) and (14), we obtain:

$$\mathcal{E} \delta \bar{\Pi}_{m\omega \bar{z}}(z) = \int \bar{G}_{\omega \bar{z}}^m(\eta) \exp[i(\eta + \omega / v)z] d\eta , \quad (26)$$

$$\mathcal{E} \delta \bar{\Pi}_{e\omega \bar{z}}(z) = \int \bar{G}_{\omega \bar{z}}^e(\eta) \exp[i(\eta + \omega / v)z] d\eta , \quad (27)$$

where

Taking into consideration the introduced designation, the integrals (29) and (30) are written in the form of:

$$\delta \bar{\Pi}_{m\omega \bar{z}}(z) = - \int \bar{G}_{\omega \bar{z}}^m(\xi) (\xi^2 - \xi_o^2)^{-1} \exp(i \xi z) d\xi , \quad (32)$$

$$\delta \bar{\Pi}_{e\omega \bar{z}}(z) = - \int \bar{G}_{\omega \bar{z}}^e(\xi) (\xi^2 - \xi_o^2)^{-1} \exp(i \xi z) d\xi . \quad (33)$$

For the diverging waves the ratio of the exponent in the exponential function, being in the integrand expression, must be positive $z > 0$. That's why at the forward radiation ($z > 0$) $\xi_1 > 0$, and at the back radiation ($z < 0$) $\xi_2 < 0$. It means that if $z > 0$, then the forward radiation field is proportional to $\exp(i \omega z \sqrt{\varepsilon^\circ - \chi^2 c^2 / \omega^2} / c)$, and if $z < 0$, then the back radiation field is proportional to $\exp(-i \omega z \sqrt{\varepsilon^\circ - \chi^2 c^2 / \omega^2} / c)$.

To obtain the retarded solutions of the equations (32) and (33), satisfying the principle of the causality, it is necessary to make the analytic continuation of the integrand function at $z > 0$ on the upper complex half-plane, and $z < 0$ on the low complex half-plane and instead of the detour of singular points to shift them from the indeed axis. It can be done, if we consider that ε° has the infinitesimal addition. In addition the pole $\xi_1 = \xi_o$ changing on $\xi_o + i \omega \delta / c$ passes to the upper half-plane, and the pole $\xi_2 = -\xi_o$ changing on $-\xi_o - i \omega \delta / c$, passes to the low half-plane (here δ is the infinitesimal positive number), and the pole $\xi_3 = \omega / v$, not having ε° , isn't shift staying on the indeed axis; the corresponding contours for $z > 0$ and $z < 0$ are given in the picture.

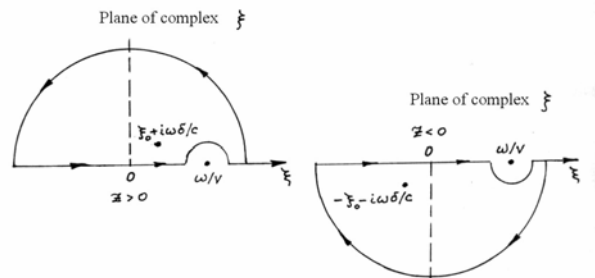


Fig. Circuit of integration in complex plane

The integrand functions (32) and (33) are analytic in the all points of the indeed axis, besides the points ξ_1, ξ_2, ξ_3 ,

being the simple poles, and satisfy $\bar{G}(\xi)(\xi^2 - \xi_0^2)^{-1} \rightarrow 0$ and $\xi \rightarrow \infty$. As the integrand functions have the finish number of the simple poles on the indeed axis, so integrals are understood by their main values [8,9]. That's why at $z > 0$ we have:

$$V.p. \int_{-\infty}^{\infty} \bar{\Phi}(\xi) d\xi = \pi i \operatorname{Re} s \left[\bar{\Phi}(\xi) \right]_{\xi=\omega/\nu} + 2\pi i \lim_{\delta \rightarrow 0} \operatorname{Re} s \left[\bar{\Phi}(\xi) \right]_{\xi=\xi_0+i\omega\delta/c} , \quad (34)$$

$$V.p. \int_{-\infty}^{\infty} \bar{\Phi}(\xi) d\xi = -\pi i \operatorname{Re} s \left[\bar{\Phi}(\xi) \right]_{\xi=\omega/\nu} + 2\pi i \lim_{\delta \rightarrow 0} \operatorname{Re} s \left[\bar{\Phi}(\xi) \right]_{\xi=-\xi_0-i\omega\delta/c} , \quad (35)$$

where

$$\bar{\Phi}(\xi) = \bar{G}_{\omega\bar{\chi}}^{m,e}(\xi)(\xi^2 - \xi_0^2)^{-1} \exp(i\xi z) . \quad (36)$$

In the formulas (34) and (35) the first summands define the change of the eigen field, accordingly, on the and against the direction of the source movement correspondingly and second summands define the forward and back radiation

field. The such solution corresponds with the diversing wave, distributing in two sides from the boundary of the blurred band.

In the result of the calculation of the residues of the first terms in the formulas (34) and (35) in the pole $\xi=\omega/\nu$, not depending on the concrete expressions for $\delta\varepsilon_{\xi}$, we obtain the similar additions to Hertizian vectors, defining the change of the eigen field:

$$\delta\bar{\Pi}_{m,e}^s(\omega, \bar{\chi}, z) = \frac{z}{|z|} \cdot \frac{\Delta\varepsilon \cdot c^2}{(2\pi)^2 \nu \omega^2} (\varepsilon^\circ - c^2/\nu^2 - \chi^2 c^2/\omega^2)^{-2} \left\{ \begin{array}{l} \bar{m} \\ [\bar{\beta}\bar{m}] \end{array} \right\} \exp(i\omega z/\nu) . \quad (37)$$

The Hertizian vectors describing the radiation field, are found by the calculation of the residues of the last terms in

the formulas (34) and (35) in the poles $\xi=\xi_1$ (the radiation forward) and $\xi=\xi_2$ (the radiation back) correspondingly:

$$\delta\bar{\Pi}_{m,e}^{r,2}(\omega, \bar{\chi}, z) = \mp \frac{ic}{2\pi\nu\omega} \delta\varepsilon_{\xi_{1,2}} \cdot \frac{(\varepsilon^\circ - c^2/\nu^2 - \chi^2 c^2/\omega^2)^{-1}}{\sqrt{\varepsilon^\circ - \chi^2 c^2/\omega^2}} \exp(\pm i\omega z \sqrt{\varepsilon^\circ - \chi^2 c^2/\omega^2}/c) \times \left\{ \begin{array}{l} \bar{m} \\ [\bar{\beta}\bar{m}] \left(1 - \frac{c^2}{\nu^2} \pm \frac{c}{\nu} \sqrt{\varepsilon^\circ - \chi^2 c^2/\omega^2} \right) + \bar{e}_3 [\bar{\chi}\bar{m}] \frac{c}{\varepsilon^\circ \omega} \left(1 \mp \frac{\nu}{c} \sqrt{\varepsilon^\circ - \chi^2 c^2/\omega^2} \right) \end{array} \right\} . \quad (38)$$

In the formulas (37) and (38) and the following ones the index s corresponds with the eigen field, and the index r corresponds with the radiation field; the upper sign and index 1 corresponds with the forward radiation, and the low sign

and index 2 corresponds with the back radiation; $\delta\varepsilon_{\xi_{1,2}}$ are values of the changing of the dielectric constant in the poles $\xi=\xi_1$ and $\xi=\xi_2$.

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MÜSTƏVİ TƏBƏQƏLİ MÜHİTDƏ QEYRİ-INVARIANT MƏNBƏNİN KEÇİD ŞÜALANMASI

Müstəvi təbəqəli qeyri-mağnit mühitdə qeyri-invariant relyativistik elektromağnit sahə mənbənin, xüsusi halda maqnit dipol momentinin, şüalanmasına baxılıb. Şüalanma sahəsi və məxsusi sahənin dəyişməsinə təsvir edən ümumi ifadələr alınmışdır. Maqnit momentinin ultrarelyativistik sürəti üçün alınan düsturların təhlili aparılır.

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ПЕРЕХОДНОЕ ИЗЛУЧЕНИЕ НЕИНВАРИАНТНОГО ИСТОЧНИКА В ПЛОСКОСЛОИСТОЙ СРЕДЕ I

Рассматривается процесс переходного излучения неинвариантного релятивистского источника электромагнитного поля, в частности, магнитного дипольного момента в плоскослоистой немагнитной среде. Получены общие выражения, описывающие поле излучения и изменение собственного поля. Проводится анализ полученных формул для ультрарелятивистской скорости магнитного момента.

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