GREEN FUNCTION METHOD IN A FERROMAGNETIC SUPERLATTICE

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The expression of Green function for different layers in a ferromagnetic superlattice is derived by the recurrence relations technique. The elementary unit cell of the superlattice under consideration consists of alternating layers of two simple-cubic Heisenberg ferromagnets. The results are illustrated numerically for a particular choice of parameters

In the past few years, there has been growing interest in the magnetic properties of artificially layered structures. With the advance of modern vacuum science, in particular the epitaxial growth technique, it is possible to grow very thin films of predetermined thickness, even of a few monolayers [1-3]. Superlattice structures composed of two different ferromagnetic layers (Fe/Co, Fe/Cr, Fe/Ni, Co/Cr, Dy/Gd etc.) have already been artificially fabricated. They have potential applications in magnetic information technology. Green's function method interface rescaling technique transfer matrix formalism as well as recurrence relations technique is used for their studies [4-6]. Green function method is the most useful among these methods. The physical characteristics, such as spectrum of magnons, the temperature dependence of magnetization, magnetic susceptibility and others of magnetic layered structures are obtained using Green function method [7,8]. The investigation of Green function in SLs is not new, but many earlier papers considered only the case the SLs composed of two different ferromagnetic or antiferromagnetic atomic layers [9,10]. J. Mathon derived the exact local spin-wave Green function in an arbitrary ferromagnetic interface, superlattice and disordered layer structure in ref. [11].

As indicated in fig1. we consider in this article a superlattice in which the elementary unit cell n_1 layers of material 1 alternate with n_2 layers of material 2. Both material are taken to be simple-cubic Heisenberg ferromagnets, having exchange constant J_1 and J_2 and lattice constant *a*. The exchange constant between constituents is *J*. The expression of Green function for different layers in the superlattice under consideration is derived by the recurrence relations technique.

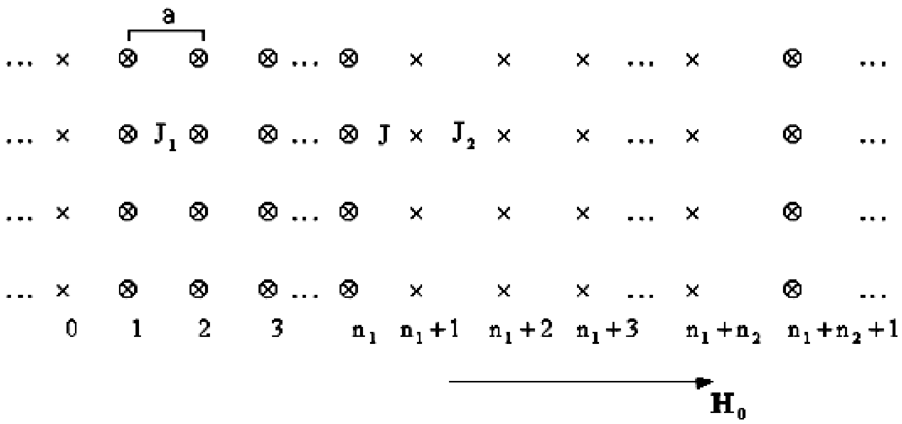

Fig.1. The elementary unit cell of *SL* consisting *N* different simple-cubic Heizenberg antiferromagnetic materials. The same lattice parameter *a* is assumed for all the materials. Antiferromagnetic layers consist of n_j (j=1,2,…,*N*) atomic layers. The layers are infinite in the direction perpendicular to the axes *z*.

The Hamiltonian of the system can be written in the form

$$H = -\frac{1}{2}\sum_{i,j} J_{i,j}\left(\vec{S}_i\vec{S}_j\right) - \sum_i g_i\mu_B\left(H_i^{(A)} + H_0\right), \qquad (1)$$

where the first term describes exchange interactions between the neighbouring spins and the last terms include the Zeeman's energy and magnetic anisotropy energy. The axis *z* of the coordinate system is normal to the film interfaces [001] and external field H_0 is assumed to be parallel to the axis z. $H_i^{(A)}(i=1,\ 2)$ anisotropy field for *a* ferromagnetic with simple uniaxial anisotropy along the *z* axis.

Employing the equation of motion for the Green function $G_{i,j}(t,t') = \langle\langle S_i^+(t); S_j^-(t')\rangle\rangle$ one obtains the following equation after two dimensional Fourier transform [9]

$$\left\{\omega - g_i\mu_B\left(H_0 + H_i^{(A)}\right) - 4J_{n,n}\gamma(k_{||})\langle S_n^z\rangle - J_{n,n+1}\langle S_{n+1}^z\rangle - J_{n,n-1}\langle S_{n-1}^z\rangle\right\}G_{n,n'}(\omega,k_{||}) + \\ + J_{n,n+1}G_{n+1,n'}(\omega,k_{||}) + J_{n,n-1}G_{n-1,n'}(\omega,k_{||}) = 2\langle S_n^z\rangle\delta_{n,n'}, \qquad (2)$$

here, *n* is the index of an atomic layer and $\gamma(k_{||})$ is defined as follows $\gamma(k_{||}) = 1 - 0.5(\cos k_x a + \cos k_y a)$. Equation (2) are valid in the low-temperature limit and random-phase-approximation (RPA) has already been done.

The equation (2) can be solved by recurrence relations technique [12] to relate the Green functions for interface layers of the elementary unit cell

$$\begin{pmatrix} G_{n_1+n_2+1,n'} \\ G_{n_1+n_2,n'} \end{pmatrix} = T\begin{pmatrix} G_{1,n'} \\ G_{0,n'} \end{pmatrix} - \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \qquad (3)$$

where $T = T_{2R}T_2^{n_2-2}T_{2L}T_{1R}T_1^{n_1-2}T_{1L}$ and the matrix T_{2R}, T_2, T_{21},`T_{IR}, T_1 and T_{11} have the form:

$$T_{1(2)}=\begin{pmatrix} 2\cos(\theta_{1(2)}) & -1 \\ 1 & 0 \end{pmatrix}$$

$$T_{1(2)R}=\frac{-1}{J\langle S_{2(1)}\rangle}\begin{pmatrix} \omega-\lambda_{12(21)} & J_{1(2)}\langle S_{1(2)}\rangle \\ -J\langle S_{2(1)}\rangle & 0 \end{pmatrix},$$

$$T_{1(2)L}=\frac{-1}{J_{1(2)}\langle S_{1(2)}\rangle}\begin{pmatrix} \omega-\lambda_{12(21)} & J\langle S_{2(1)}\rangle \\ -J_{1(2)}\langle S_{1(2)}\rangle & 0 \end{pmatrix} \qquad (4)$$

$$\begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \begin{cases} T_{2R}T_2^{n_2-2}T_{2L}T_{1R}T_1^{n_1-2}\begin{pmatrix} -2/J_1 \\ 0 \end{pmatrix} & n'=1 \\ T_{2R}T_2^{n_2-2}T_{2L}T_{1R}T_1^{n_1-n'-1}\begin{pmatrix} 0 \\ -2/J_1 \end{pmatrix} & 2\le n'\le n_1-1 \\ T_{2R}T_2^{n_2-2}\begin{pmatrix} 2\langle S_1\rangle(\omega-\lambda_{21})/JJ_2\langle S_2\rangle^2 \\ -2\langle S_1\rangle/J\langle S_2\rangle \end{pmatrix} & n'=n_1 \\ T_{2R}T_2^{n_2-2}\begin{pmatrix} -2/J_2 \\ 0 \end{pmatrix} & n'=n_1+1 \\ T_{2R}T_2^{n_2-n_2-n'-1}\begin{pmatrix} -2/J_2 \\ 0 \end{pmatrix} & n_1+2\le n'\le n_1+n_2-1 \\ \begin{pmatrix} -2\langle S_2\rangle/J\langle S_1\rangle \\ 0 \end{pmatrix} & n'=n_1+n_2 \end{cases} \qquad (5)$$

$$\lambda_{12(21)} = g_{1(2)}\mu_B(H_0 + H^{(A)}_{1(2)}) + J_{1(2)}\langle S^z_{1(2)}\rangle + J\langle S^z_{2(1)}\rangle + 4J_{1(2)}\langle S^z_{1(2)}\rangle\gamma(k_{||}),$$

θ_1 and θ_2 are defined by the expression

$$\cos(\theta_{1(2)}) \equiv b_{1(2)} = \left(g_{1(2)}\mu_B(H_0+H^{(A)}_{1(2)}) + 2J_{1(2)}\langle S^z_{1(2)}\rangle + 4J_{1(2)}\langle S^z_{1(2)}\rangle\gamma(k_{||}) - \omega\right)/2J_{1(2)}\langle S_{1(2)}\rangle \qquad (6)$$

For $|b_{1(2)}|>1$, $b_{1(2)} = \cosh(\theta_{1(2)})$, and one replaces $\sin(n\theta_{1(2)})$ by $\sinh(n\theta_{1(2)})$ for $b_{1(2)}>1$ and $(-1)^n\sinh(n\theta_{1(2)})$ for $b_{1(2)}<-1$. The matrix elements of T is given by the following expression

$$T_{11} = (x_{11}y_{11} - x_{12}y_{12})/(J^2J_1J_2\langle S_1\rangle^2\langle S_2\rangle^2\sin(\theta_1)\sin(\theta_2))$$

$$T_{12} = (x_{12}y_{11} + x_{22}y_{12})/(J^2J_1J_2\langle S_1\rangle^2\langle S_2\rangle^2\sin(\theta_1)\sin(\theta_2))$$

$$x_{11} = (\omega-\lambda_{12})^2\sin((n_1-1)\theta_1) + 2J_1\langle S_1\rangle(\omega-\lambda_{12})\sin((n_1-2)\theta_1) + J_1^2\langle S_1\rangle^2\sin((n_1\ \ 3)\theta_1) \qquad (7)$$

$$x_{12} = J\langle S_2\rangle[(\omega-\lambda_{12})\sin((n_1-1)\theta_1) + J_1\langle S_1\rangle\sin((n_1-2)\theta_1)]$$

$$x_{22} = -J^2\langle S_2\rangle^2\sin((n_1-1)\theta_1)$$

the matrix elements T_{21}, T_{22} and Y_{11}, Y_{12}, Y_{22} by replacing all subscript 1 by 2, 2 by 1 in - T_{21}, T_{22} and Y_{11}, Y_{12}, Y_{22}, respectively.

The system is also periodic in the z direction, which lattice constant is $L = (n_1 + n_2)a$. According to Bloch's theorem we can write [13,14]

$$\begin{pmatrix} G_{n_1+n_2+1,n'} \\ G_{n_1+n_2,n'} \end{pmatrix} = exp[iK_zL] \begin{pmatrix} G_{1,n'} \\ G_{0,n'} \end{pmatrix} \quad (8)$$

The expression of Green function $G_{1,n'}(\omega, k_{||})$ and $G_{0,n'}(\omega, k_{||})$ are obtained by using equation (3) and (8).

$$G_{1,n'}(\omega,k_{||}) = \frac{(\delta_1T_{22} - \delta_2T_{12})exp[-iK_zL] - \delta_1}{2cos(K_zL) - T_{11} - T_{22}}, \quad G_{0,n'}(\omega,k_{||}) = \frac{(\delta_2T_{11} - \delta_1T_{21})exp[-iK_zL] - \delta_2}{2cos(K_zL) - T_{11} - T_{22}} \quad (9)$$

The Green's function for all layers of elementary unit of superlattice are related with $G_{1,n'}(\omega, k_{||})$ and $G_{0,n'}(\omega, k_{||})$ by reccurence relation technique. Using (4), (5) and (8) one can calculate the Green's function for different layers in elementary unit cell of the superlattice under the consideration.

Green function for the left-hand $(n' = 1)$ and right-hand $(n' = n_1)$ layers of components 1 in elementary unit cell of the superlattice have the form

$$G_{1,1}(\omega, k_{||}) = G_{n_1,n_1}(\omega, k_{||}) = (2S_1T_{12}/(JS_2))/(2\cos(K_zL) - T_{11} - T_{22}) \quad (10)$$

Green function for the bulk layers $(2 \le n' \le n_1 - 1)$ of components 1 in elementary unit cell of the superlattice has the form

$$G_{n',n'}(\omega, k_{||}) = [x'_{22}x'_{12}(T_{22}-T_{11}) + (x'_{22})^2T_{21} - (x'_{12})^2T_{12}] / [0.5J_1^2J^3\langle S_1\rangle \sin^2(\theta_1)(2\cos(K_zL) - T_{11} - T_{22})] \quad (11)$$

$$x'_{11} = (\omega - \lambda_{12})^2 \sin((n'-1)\theta_1) + 2J_1\langle S_1\rangle(\omega - \lambda_{12})\sin((n'-2)\theta_1) + J_1^2\langle S_1\rangle^2 \sin((n'-3)\theta_1)$$

$$x'_{12} = J\langle S_2\rangle[(\omega-\lambda_{12})\sin((n'-1)\theta_1) + J_1\langle S_1\rangle \sin((n'-2)\theta_1)], \quad x'_{22} = -J^2\langle S_2\rangle^2 \sin((n'-1)\theta_1)$$

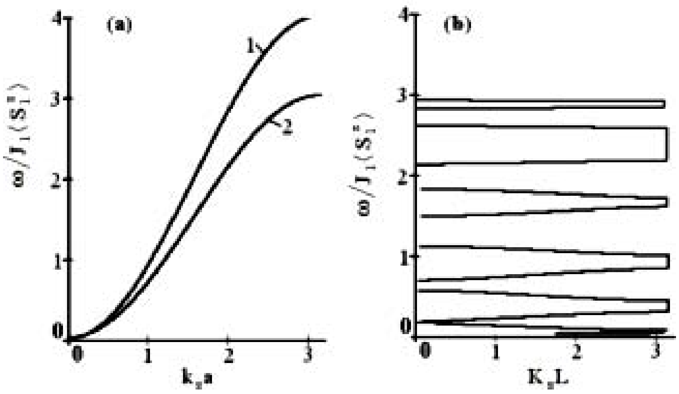

Fig.2. Bulk spin – wave dispersion graphs for [001] propagation with parameters $J_2/J_1=2$; $g_1=g_2$; $g_1\mu_BH_1^{(A)}/J_1\langle S_1^z\rangle = 0.01$; $g_2\mu_BH_2^{(A)}/J_1\langle S_1^Z\rangle = 0.03$.

Green function for the left-hand layer $(n'= n_1+1)$, the bulk layers $(n_1+2 \le n' \le n_1+n_2-1)$ and right-hand layer $(n'=n_1+n_2)$ of components 2 in elementary unit cell of the superlattice are obtained by replacing all subscript 1 by 2, 2 by 1 and $(n' \to n'+n_1)$ in the Green's function for the same layers of components 1 in elementary unit cell of the superlattice, respectively.

For numerical illustration of our result we consider the spin wave dispersion-curve of the superlattice under consideration. As known the spin-wave spectrum is obtained from the poles of Green function. fig 2(a) shows the bulk spin-wave dispersion curves of the component 1 and 2 for a particular choice of parameters, while fig.2(b) shows the spin-wave dispersion curves of the superlattice. In the frequence range, where k_{1z} and k_{2z} are real, the superlattice dispersion curve exhibits broud pass bands and narrow stop bands. The pass bands are narrow and the stop bands are broud where at least one of the wave vectors is complex.

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FERROMAGNİT İFRAT QƏFƏSDƏ QRİN FUNKSİYASI METODU

Rekurens əlaqələr metodu ilə ferromagnit ifrat qəfəsin müxtəlif layları üçün Qrin funksiyasının ifadəsi tapılıb. Baxılan ifrat qəfəsin elementar özəyi iki müxtlif sadə kubik Heyzenberq tip ferromaqnit layların növbələşməsindən təşkil olunub. Alınan nəticələr parametrlərin seçilmiş qiymətləri üçün kəmiyyətcə təsvir olunub.

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МЕТОД ФУНКЦИЙ ГРИНА В ФЕРРОМАГНИТНОЙ СВЕРХРЕШЕТКЕ

С помощью техники рекуррентных соогношений найдены выражения функции Грина для различных слоев ферромагнитной сверхрешетки. Элементарная ячейка расматриваемой сверхрешетки состоит из чередующихся слоёв двух различных простых Гейзенберговских ферромагнетиков. Результаты проиллюстрированы количественно для выбранных значений параметров.

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