EFFECT DEFORMATİON İN M1 TRANSİTİONS BETWEEN STATES WİTH DİFFERENT SHAPES

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In Random Phase Approximation (RPA), using the analytic properties of the nucleus transition matrix elements and by means of contour integrals and residue theorem, we obtain an analytic formula containing the dependence of deformation of the energy-weighted sum rule (EWSR) for the magnetic dipole transformations. Numeric calculations show that the transition probability between levels, which have different forms decreases sharply compatible with experimental data.

Keywords: sum rules, transitions M1, Residue theorem and contour integrals.

Microscopic nuclear models are successfully used to investigate the properties of nuclear collective excitations [1]. Approximate calculation methods are used to investigate the structure of nucleus within the framework of assumed models. In order to evaluate the success of used methods and nucleus models, the obtained results are compared with suitable experimental data.

In quantum mechanics, the transition probability of the system from one state to the other is restricted to the sum rules which are independent from the model and subject to transitions matrix elements. The sum rules in nucleus physics are very important to finding parameters and understanding the reliability of used models [2]. Experimental reviews in heavy nuclei show that the theoretical values of sum rules of matrix elements of the electromagnetic transition parameters are greater by a factor of almost 1,5-2 than the experimental values corresponding to them. The reason for this disagreement between the theory and the experiments is not exactly explained. According to our hypothesis, the main reason of these disagreements is the change of nucleus shape caused by the transitions between different energy levels. It is experimentally known that in some nuclei, the velocity of transition between levels having different structure decrease [4,5].

While investigating the structure of nucleus, electromagnetic and β-transition probabilities play a very important role in determination of spin forces among nucleons in nucleus. For this reason, analytically calculating the sum rules for the transition matrix elements in levels, which have deformation parameters different from the ground state is very important.

There are two types of sum rules: Energy-weighted sum rule and none energy-weighted sum rule and the energy-weighted sum rule (EWSR) is widely used in the nucleus physics:

$$\sum_{k>0}(E_k-E_0)|\langle k|M|0\rangle|^2=\frac{1}{2}\langle 0|[M^+,[H,M]]|0\rangle. \quad (1)$$

Here energy E_k and wave function $\langle k|$ are eigenvalues and eigenfunctions of Hamiltonian operator of nucleus, respectively. M is transition operator, E_0 and $|0\rangle$ also denote the energy and wave function of the ground state, respectively. Since the right-hand side of the sum rule (1) does not contain the internal movement parameters of nucleus, this is independent from models and has constant values. On the other hand, since the left-hand side of the sum rule (1) contains wave functions and energy levels of nucleus, its values depend on models and used methods. Thus, the sum rule (1) is very important in investigating the structure of nucleus. In Ref. [6], assuming that the spin forces creates 1^+ excitations in deformed nuclei, the Hamiltonian describing these levels can be expressed in the following form [6]:

$$H = H_{sqp} + V_{\sigma\tau} \quad (2)$$

where

$$V_{\sigma\tau} = \frac{1}{2}\chi_{\sigma\tau}\sum_{i\neq j}\sigma_i\sigma_j\tau_i^z\tau_z^j. \quad (3)$$

Here $V_{\sigma\tau}$ and H_{sqp} denote isovector spin forces and quasi-particle Hamiltonian in the model of superfluid, respectively. Further, σ and τ are Pauli matrices which represent respectively spin and isotopic spin. All relations which are used and not explained all relations in this paper are similar to those in Ref. [6]. In RPA, the wave functions of 1^+ states can be viewed as single-phonon function:

$$|k\rangle = Q_k^+|0\rangle = \frac{1}{\sqrt{2}}\sum_\mu(\psi_\mu^k C_\mu^+ - \varphi_\mu^k C_\mu)|0\rangle \quad (4)$$

Where Q_k^+ and $|0\rangle$ describe, respectively, phonon-creation operator and phonon vacuum corresponding to the ground state in even-even nuclei. Our system has a discrete spectrum and the wave functions $|k\rangle$ form complete set satisfying $\sum_k |k\rangle\langle k| = 1$. Thus, two quasi-particle amplitudes ψ_μ ve φ_μ corresponding to the operator C_μ and C_μ^+ are normalized as follows:

$$\sum_\mu\left(\psi_\mu^i\psi_\mu^k-\varphi_\mu^i\varphi_\mu^k\right)=\delta_{i,k}. \quad (5)$$

In order to find the eigenvalues and the eigenfunctions of Hamiltonian H, by solving the motion equation

$$\left[H_{sqp} + V_{\sigma\tau}, Q_k^+\right] = \omega_k Q_k^+ \quad (6)$$

and using the well-known RPA procedure, dispersion equation for ω_k roots which are energy of 1^+ states is taken in the form:

$$D(\omega_k) = 1 + \chi\left(F_k^n(\omega_k) + F_k^p(\omega_k)\right) = 0 \quad (7)$$

where

$$F_k^\tau(\omega_k) = 2\sum_\mu{}^{\tau} \frac{\varepsilon_\mu \sigma_\mu^2 L_\mu^2}{\varepsilon_\mu^2 - \omega_k^2}, \quad \tau = \text{n,p.} \quad (8)$$

Here ε_μ and σ_μ are quasi-particle energy of nucleons and single-particle matrix elements of the spin operator, respectively. Two-quasi particle amplitudes ψ_μ and φ_μ have the form:

$$\psi_\mu^i = \frac{1}{\sqrt{Y(\omega_i)}} \cdot \frac{\sigma_\mu L_\mu}{\varepsilon_\mu - \omega_i} \quad (9)$$

$$\varphi_\mu^i = \frac{1}{\sqrt{Y(\omega_i)}} \cdot \frac{\sigma_\mu L_\mu}{\varepsilon_\mu + \omega_i} \quad (10)$$

where

$$Y(\omega_i) = 4\omega_i \sum_i \frac{\varepsilon_\mu \sigma_\mu^2 L_\mu^2}{(\varepsilon_\mu^2 - \omega_i^2)^2}. \quad (11)$$

On the other hand, since energies of the magnetic dipole 1^+ states are the solutions of the function $D(\omega_i)$, we can write

$$Y(\omega_i) = \frac{1}{\chi} D'(\omega_i) \quad (12)$$

where

$$D' = \frac{dD(z)}{dz}.$$

Due to the symmetry between the used spin-spin forces and magnetic dipole operator, the most characteristic quantity of the 1^+ states is transition matrix elements M1 from ground state to all excited states in nucleus:

$$M_k = \langle k | M | 0 \rangle \quad (13)$$

where the magnetic-dipol operator is

$$\vec{M} = \sqrt{\frac{3}{4\pi}} \sum_{m,\tau} \left[(g_s^\tau - g_l^\tau) \overrightarrow{s_m^\tau} - g_e^\tau \overrightarrow{j_m^\tau} \right]. \quad (14)$$

Here g_s^τ and g_l^τ are the spin and the orbital gyromagnetic ratios of nucleons, respectively. By using the wave function (4) and by means of (9) and (10), transition matrix elements of 1^+ states from ground state to excited states can be expressed as

$$M_i = \sqrt{\frac{3}{4\pi}} \frac{\sum_{,\tau} \left[\frac{1}{2}(g_s^\tau - g_l^\tau) \overrightarrow{F_i^\tau} - g_e^\tau \overrightarrow{J_i^\tau} \right]}{\sqrt{Y(\omega_i)}} \quad (15)$$

where

$$\vec{J}_i^\tau = 2\sum_i{}^{\tau} \frac{\varepsilon_\mu j_\mu^2}{\varepsilon_\mu^2 - \omega_i^2}. \quad (16)$$

Here j_μ denote the single-particle matrix elements of the angular momentum operator.

Let us now generalize the sum rule in Eq. (1) for transitions between the ground and excited states which have different form. Let us suppose that shape of the excited state levels $|k\rangle$ have different deformation from the ground state. After this, the quantities corresponding to excited states which have different form from the ground state are denoted by ~(*tilda*) over themselves. Also, by taking the fact that the excited level wave function $|i\rangle = Q_i^+|0\rangle$ in the ground state bases form a complete set into consideration, we obtain the generalized sum rule of Eq. (1) for different forms as follows:

$$S(\delta_i, \delta_k) = \sum_{k>0} \omega_k \left| \sum_{i>0} M_i \langle i | k \rangle \right|^2 \quad (17)$$

$$\langle i | k \rangle = \left(\psi_\mu^i \psi_\mu^k - \varphi_\mu^i \varphi_\mu^k \right). \quad (18)$$

Here δ_i and δ_k are quadrupole deformation parameters characterizing the forms of the ground and excited states. The sum rules are successfully calculated in spherical nuclei. However, in the deformed nuclei, since the levels of the nucleus have high density, computing eigenvalues ω_i from Eq. (7) is numerically quite difficult. Hence, when computing transition matrix elements M_i and the sum rules corresponding to them may occur considerable errors. In [7], the solution methods of this problem are given and by exploiting mathematical properties of β-transition matrix elements, even β-decomposition sum rule is analitically calculated. Later, the method developed in [7] is successfully applied to electromagnetic transitions in [8]. In this study, we calculate this method which is put forward in [7] by applying to the sum rule given in [15] that we have generalized for transitions having different form.

For the sum rule (17), by exploiting the equations (7)-(12), we obtain:

$$S = \frac{3}{16\pi} \sum_{\mu\nu} d_\mu d_\nu . \Omega_{\mu\nu} \quad (19)$$

where

$$d_\mu = \sum_i M_i w_\mu^i = 2\chi \sigma_\mu L_\mu \sum_i \frac{\omega_i}{\left(\varepsilon_\mu^2 - \omega_i^2\right)} \frac{F(\omega_i)}{D'(\omega_i)} \quad (20)$$

$$\Omega_{\mu\nu} = \sum_k \omega_k g_\mu^k g_\nu^k \qquad (21)$$

$$g_\mu = \psi_\mu + \varphi_\mu \ , \ w_\mu = \psi_\mu - \varphi_\mu \ .$$

Here, the sums with respect to *i* and *k* contain all positive and negative forces of the Eq. (7). Since ω_i are the zeros of the function $D(\omega_i)$, the basic theorem of the theory of residues [8] now allow us to write the expression for d_μ in the form of the contour integral:

$$d_\mu = 2\chi\sigma_\mu L_\mu \sum_i \oint_{L_i} \frac{zF(z)}{\left(\varepsilon_\mu^2 - z^2\right)D(z)} dz \qquad (22)$$

The contour L_i in the complex plane are shown in Fig. 1 and contain first-order singularities of the integrands at $z = \pm\omega_i$ which are the zeros of the corresponding function $D(z)$.

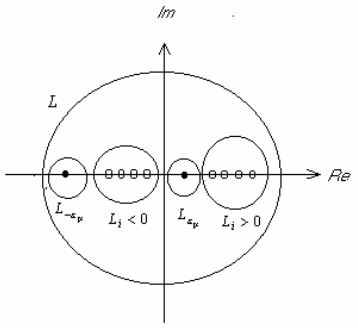

Fig. 1. The contour of integration in the complex plane for (20)

However, it is well-known from analysis that, outside L_i, the integrand in Eq. (22) have simple poles at $z = \pm\varepsilon_\mu$ and the corresponding residues can be evaluated easily. Since the sum of all residues of the an analytic function is equal to zero, we have

$$\oint_L = \sum_i \frac{\omega_i F\left(\omega_i^2\right)}{\left(\varepsilon_\mu^2 - \omega_i^2\right)D'(\omega_i)} + \mathrm{Re}\, z\left(f, -\varepsilon_\mu\right) + \mathrm{Re}\, z\left(f, +\varepsilon_\mu\right) = 0 \qquad (23)$$

Using Cauchy theorem, we find

$$d_\mu = -\oint_{L_\mu} \frac{zF(z)}{\left(\varepsilon_\mu^2 - z^2\right)D(z)} dz = 2\sigma_\mu L_\mu \ . \qquad (24)$$

After laborious calculations, we show that $\Omega_{\mu\neq\nu} = 0$, hence the Eq. (21) becomes

$$\Omega_{\mu\nu} = 2\widetilde{\varepsilon}_\mu \delta_{\mu\nu} \ . \qquad (25)$$

Finally, by exploiting (19), (22) and (23) formulas, we derived very simple formula for the generalized expression of the sum rule (17):

$$S(\delta_i, \delta_k) = \frac{3}{2\pi} \sum_{\mu,\tau} \widetilde{\varepsilon}_\mu^\tau [(g_s^\tau - g_l^\tau) s_\mu^\tau - g_e^\tau j_\mu^\tau]^2 \qquad (26)$$

where s_μ^τ and $\widetilde{\varepsilon}_\mu$ are the single particle matrix elements of the spin operator in the ground state base and two-quasi particle energies corresponding to the form of excited states which are calculated in different base. According to this, when $\delta_i = \delta_k$, the formula (24) is transformed into the known sum rule expression [6] for magnetic dipole transitions. For the spin transitions, the sum rule becomes

$$S_\sigma(\delta_i, \delta_k) = 2\sum_{\mu,\tau} \widetilde{\varepsilon}_\mu^\tau \sigma_\mu^{\tau\,2} L_\mu^{\tau\,2} \ . \qquad (27)$$

Numerical calculations are performed in a wide interval of the deformation parameter for the nucleus ^{140}Ce in the deformed Woods-Saxon potential [10]. The calculations are performed by using the sum rule (26) for the spin taransitions. When calculating the spin transitions using the sum rule (26), the quadrupole deformation parameter δ_0 =0.09 given in [11] from an experimental data of the ground state is taken and the deformation parameter of excited levels is changed in the 0.05 and 0.3 interval. As shown in Fig. 2., while the difference between the forms of the transition levels increases, the transition probability or the numerical values of the sum rule S declines sharply.

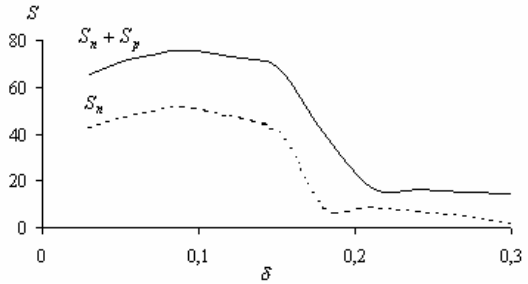

Fig. 2. The change in the sum rule (25) with respect to δ for the spin transitions (in MeV)

Consequently, the known energy weighted sum rule for the magnetic dipole transitions is generalized for transitions to levels, which have different forms from the ground state. Later, by using the theory of residue and contour integrals, analytical expressions is obtained for these sum rules. In numerical calculations, in the case of the spin transition operator, while the form of the nucleus changes, the numerical values of the energy-weighted sum rule decline sharply.

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