THE ENERGY SPECTRUM OF CARRIERS IN KANE TYPE SEMICONDUCTOR MICROCRYSTALS WITH SINQULAR OSCILLATOR POTENTIAL

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The energy spectrum of light carriers in narrow band gap semiconductor microcrystal are studied theoretically taking into account the nonparabolicity of the electrons, light holes and spin-orbit splitting holes dispersion laws. The confinement potential of microcrystal is approximated as $\lambda r^2+\lambda_1 r^{-2}$, and the dispersion laws are considered within the framework of three-band Kane model. Confinement potential introduced in the Kane equations by non-minimal substation.

In recent years there has been a great interest to the nanostructures, which was developed from narrow-gap semiconductors. In these nanostructures much smaller effective mass of electrons resulting has higher size quantization energy. In order to investigate the optical and kinetic properties of nanostructures it will be possible to observe size quantization energy states. On the other hand in narrow-gap semiconductors the spin-orbit interaction and the non-parabolicity energy spectrum of carriers must be taken into account. It is impossible solve the equation analytically when the confinement potential is added to the Kane equations as scalar potential. Because of this in this study we have added the potential to the Kane equations by non-minimal interaction applied in the works of [1,2]. We refereed to the obtained equation as the singular Kane oscillator by analogy with the Dirac oscillator [2].

Confinement potentials are of the form $V(r)=\lambda\cdot r^2+\lambda_1 r^{-2}$ describe a quantum dot, an antidot, or a quantum ring, depending on the values of λ and λ_1 [3].

In the three-band Kane's Hamiltonian the valence and conduction bands interaction is taken into account via the only matrix element P (so called Kane's parameter). The system of Kane equations including the nondispersional heavy hole bands as in the case of Dirac equation can be written in the following matrix form:

$$(\vec{б}\cdot\vec{k}\cdot P+в\cdot G-E)Ш=0 \qquad (1)$$

In equation (1) 8x8 matrix α,β and G have the following nonzero elements:

$$\alpha_{1,3}{}^{x}=\alpha_{3,1}{}^{x}=-\alpha_{2,6}{}^{x}=-\alpha_{6,2}{}^{x}=-\frac{1}{\sqrt{2}} \qquad (2)$$

$$\alpha_{1,5}{}^{x}=\alpha_{5,1}{}^{x}=-\alpha_{2,4}{}^{x}=-\alpha_{4,2}{}^{x}=\frac{1}{\sqrt{6}} \qquad (3)$$

$$\alpha_{1,8}{}^{x}=\alpha_{8,1}{}^{x}=\alpha_{2,7}{}^{x}=\alpha_{7,2}{}^{x}=\frac{1}{\sqrt{3}} \qquad (4)$$

$$\alpha_{1,3}{}^{y}=-\alpha_{3,1}{}^{y}=\alpha_{2,6}{}^{y}=-\alpha_{6,2}{}^{y}=\frac{i}{\sqrt{2}} \qquad (5)$$

$$\alpha_{1,5}{}^{y}=-\alpha_{5,1}{}^{y}=\alpha_{2,4}{}^{y}=-\alpha_{4,2}{}^{y}=\frac{i}{\sqrt{6}} \qquad (6)$$

$$\alpha_{1,8}{}^{y}=-\alpha_{8,1}{}^{y}=-\alpha_{2,7}{}^{y}=\alpha_{7,2}{}^{y}=\frac{i}{\sqrt{3}} \qquad (7)$$

$$\alpha_{1,4}{}^{z}=\alpha_{4,1}{}^{z}=\alpha_{2,5}{}^{z}=\alpha_{5,2}{}^{z}=\sqrt{\frac{2}{3}} \qquad (8)$$

$$\alpha_{1,7}{}^{z}=\alpha_{7,1}{}^{z}=-\alpha_{2,8}{}^{z}=-\alpha_{8,2}{}^{z}=\frac{1}{\sqrt{3}} \qquad (9)$$

$$G_{11}=G_{11}=E_g,\,G_{11}=G_{22}=Д \qquad (10)$$

$$Ш=\left(Ш_1,Ш_2,Ш_3,Ш_4,Ш_5,Ш_6,Ш_7,Ш_8\right)^T \qquad (11)$$

Here P is the Kane parameter, $E_{.g.}$ - the band gap energy, Δ - the value of spin-orbital splitting and $k_{\pm}=k_x\pm ik_y, \vec{k}=-i\vec{\nabla}$. The zero of energy is chosen at bottom of the conduction band.

Let us carry out the non-minimal substitution

$$\vec{k}\rightarrow\vec{k}-i\beta\left(\lambda\vec{r}-i\lambda_1\frac{\vec{r}}{r^2}\right) \qquad (12)$$

in Kane system of equations. Expressing all components of the wave function by the first two we obtain two coupled equations for the spin-up and the spin-down conduction band

$$\left(A+BL_z\right)Ш_1+BL_+Ш_2=0 \qquad (13)$$

$$\left(A-BL_z\right)\Psi_2+BL_-\Psi_1=0 \qquad (14)$$

where

$$A=E_g-E+\frac{P^2(3E+2ДД)}{3(Д+E)E}\left(-\nabla^2+л^2r^2+3л+2лл_1+\frac{л_1\left(л_1+1\right)}{r^2}\right) \qquad (15)$$

$$B = \frac{2}{3}\frac{P^2\Delta}{E(\Delta+E)}\left(\lambda+\frac{\lambda_1}{r^2}\right) \quad (16)$$

where L_x, L_y, L_z are angular momentum operator compenents. Sense the problem has a spherical symmetry, we seek a solution to the differential equation in the form *F(r)Ylm(θφ)*..

By acting upon the equation (11) through the operator L_+ and using commutation relationships for the operators, we obtain $L_+\psi_2$. After substituting this quantity into expression (10), we derive two equations for *F(r)*.

$$\left(A+\frac{B}{2}\mp B(l+\frac{1}{2})\right)F(r)=0 \quad (17)$$

After substitution of the values of *A* and *B* from (15), (16) the equation (17) can be rewritten in the form:

$$\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+\frac{2m_n}{\hbar^2}(E'-\frac{l(l+1)}{r^2}\cdot\frac{\hbar^2}{2m_n}-\frac{a}{r^2}-br^2)\right)F(r)=0 \quad (18)$$

where

$$E'=\frac{E(E-E_g)(E+\Delta)}{3E+2\Delta}\cdot\frac{3}{P^2}\frac{\hbar^2}{2m_n}-\frac{\hbar^2}{2m_n}\left(2\lambda\lambda_1+3\lambda+\frac{2\Delta\lambda}{3E+2\Delta}\left(\frac{1}{2}\mp(l+\frac{1}{2})\right)\right) \quad (19)$$

$$a=\frac{\hbar^2}{2m_n}\left(\lambda_1(\lambda_1+1)+\frac{2\Delta\lambda_1}{3E+2\Delta}\left(\frac{1}{2}\mp(l+\frac{1}{2})\right)\right) \quad (20)$$

$$b=\frac{\hbar^2\lambda^2}{2m_n} \quad (21)$$

The eigenvalues of equations (18) take the following form [5]

$$E'=\frac{\hbar^2\lambda}{2m_n}\left(4n+2+\sqrt{(2l+1)^2+\frac{8m_n}{\hbar^2}a}\right) \quad (22)$$

The eigenfunctions corresponding to the eigenvalues of equation (18) are

$$F(\xi)=\exp\left(-\frac{\xi}{2}\right)\cdot\xi^2\cdot\Phi\left(-n,2s+\frac{3}{2},\xi\right) \quad (23)$$

where is the confluent hypergeometric function, *n* must be non-negative integer, $\lambda=\frac{m_n\omega}{\hbar},\xi=\sqrt{2m_nb}\cdot\frac{r^2}{\hbar}$.

Equation (18) determines the energies of electrons, light holes, and the spin-orbit split-of band of holes. Equation (18) can be useful for analyzing the influence of nonparabolicity on the energy spectrum of electrons in a quantum dot.The singular oscillator equation is obtained from a system of the equations for multiband Hamiltonian describing spectrum of electrons, light and heavy holes in Kane's semiconductors by the method of a non-minimal interaction.

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SİNGULYAR OSSİLYATOR POTENSİALİNA MALİK KEYN TİPLİ MİKROKRİSTALLARDA YÜKDAŞIYICILARIN ENERJİ SPEKTRLƏRİ

Qadağan olunmuş zonası dar olan yarımkeçirici microkristallarda yüngül yükdaşıyıcıların enerjı spektrləri nəzəri olaraq öyrənilmişdir. Mikrokristallarda saxlayıcı potensial olaraq $\lambda r^2+\lambda_1 r^{-2}$ şəklində götürülmüşdür. Potensial Keyn tənliklərinə qeyri-minimal qarşılıqlı təsir yolu ilə daxil edilmişdir. Yüngül yükdaşıyıcıların enerji spektrlərinin qeyri parabolikliyi nəzərə alınmışdır.

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ЭНЕРГЕТИЧЕСКИЙ СПЕКТР НОСИТЕЛЕЙ ЗАРЯДА В КЕЙНОВСКИХ МИКРОКРИСТАЛЛАХ С ПОТЕНЦИАЛОМ СИНГУЛЯРНОГО ОСЦИЛЛЯТОРА

Найден энергетический спектр и волновые функции кейновского сингулярного осциллятора, описывающего спектр энергии электронов, легких дырок и спин-орбитально отщепленной зоны дырок в квантовой точке с удерживающим потенциалом типа $\lambda r^2+\lambda_1 r^{-2}$.

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