EFFECT OF SPIN-VIBRATIONAL 1^+ STATES ON GROUND STATE CORRELATIONS OF NUCLEI

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We have considered a number of features of the ground state correlations (GSC) in deformed nuclei connected with the effective spin-spin forces in the framework FR-QRPA, R-QRPA and QRPA. The result of calculations has indicated a significant role of the high energy 1^+ states and the importance to use a complete set of the RPA solutions for reliable predictions of the theory in GSC. It is concluded that collective excitations, which form M1 resonance give dominant contribution to GSC. The total number of quasiparticles in the ground state N_{qp} and the nuclear transition matrix elements are significantly affected due to the collective states of the M1 resonance energy region. The results are practically insensitive to the QRPA, R-QRPA and FR-QRPA methods with isovector spin-spin interactions in particle-hole (p-h) channel.

1. Introduction

Quasiparticle Random Phase Approximation (QRPA) is based on the quasiboson approach (QBA), which treats the two quasi-particle states as bosons [1]. An improvement of this approach is the Renormalized QRPA (R-QRPA), which takes into account the Pauli principle for the fermion pairs in the correlated ground state [2-4]. A further development of this approach for realization of a self-consistent condition between ground state and RPA matrix elements was presented in [5-7]. Recently the pn R-QRPA for charge-exchange processes was formulated in [8]. An extended version of this approach with proton-neutron pairing (Full Renormalized QRPA) was proposed and applied to the double beta decay [9]. The R-QRPA has been intensively used in previous studies of the double beta decay [10,11].

But R-QRPA violates the Ikeda sum rule, which is fulfilled within QRPA and must be fulfilled for an exact solution [11-13]. In the last decade different ways were used to extend R-QRPA in order to cure this drawback of R-QRPA [12-14]. But the drawback has not been cured neither in the self-consistent QRPA [14] nor in the second order QRPA [12] and self-iterative BCS+RQRPA [13]. The main disadvantage of the R-QRPA is inconsistency between the model Hamiltonians and one phonon wave functions. Modification of the phonon operators by including scattering terms is unavoidable if one wants to restore the ISR [15] or EWSR within R-QRPA [16].

Recently the Fully Renormalized QRPA (FR-RQRPA) was formulated in [17]. In this approach the phonon operator is constructed from a given quasiparticle structure of the effective interactions by means of invariance principles. By requiring that the phonon operators must have a good angular momentum *J* and commute with the total particle number operator a consistent description with given Hamiltonian is achieved. Because of this full consistency between the phonon structure and Hamiltonian the FR-RQRPA fulfills ISR exactly.

The calculation beyond QRPA requires the solution of the complex nonlinear system of equations of motion. It follows to take all RPA solutions for all the R-QRPA calculations into account. Indeed in this approach the collective states are important in ground state correlations. In numerical calculations we must, in principle, take into account all the eigenstates of QRPA Hamiltonian in the GSC calculations. Many R-QRPA calculations [2,13-18] take into account only the first or some lowest QRPA solutions neglecting high energy QRPA solutions in the ground state expectation value of operator equations. Excepting [13], as a rule, the description of Double Beta Decay (DBD) [10,11] in all R-QRPA calculations have been performed without taking into account the effects of high energy state, since there are no physical grounds whatever for neglecting the effect of high energy solutions. It is impossible to guarantee the smallness of the effects associated with these approximations. Therefore, the results of numerical calculations are not fully correct because of the collective high energy admixtures. One can expect large effects of the collective high energy states of resonance energy region on the average number of quasiparticles in ground state ($N_q = 2\sum_{i,q'} Y^{i\ 2}_{qq'}$) [7] if one keep in mind that for collective states backward amplitudes are large Y^{i}_{ssr} than non collective states [1,19]. Therefore their influence on the results and contribution to GSC should be large.

Here we want to study the influence of high energy spin-vibrational 1^+ states and the FR-RQRPA approach in comparison with QRPA and R-QRPA on the ground state correlations in deformed nuclei. Up to now, the important aspect of high energy RPA solutions on GSC has not been examined. In connection with this, it is interesting to establish how important the role of high energy RPA solutions for the ground state correlations is and to estimate their contributions to the average quasiparticle number N_{qp}. In order to demonstrate the order of magnitude of the effects of high energy solutions in the GSC we have calculated N_{qp} and B(M1) value of the 1^+ excitations generated by the isovector spin-spin forces for the deformed ^{154}Sm, ^{156}Gd, ^{168}Er and ^{178}Hf of the rare-earth nuclei.

2. Theory

Let us consider a system of nucleons in an axially symmetric mean field interacting via pairing forces. In this case the corresponding single-particle Hamiltonian of the system is given as

$$H_{sqp} = \sum_s E_s(\tau)(\alpha_s^+(\tau)\alpha_s(\tau) + \alpha_{\tilde{s}}^+(\tau)\alpha_{\tilde{s}}(\tau)) \quad (1)$$

here E_s are the single-quasiparticle energies of the nucleons and isospin index τ takes the value n(p) for neutrons (protons), $\alpha_s^+(\alpha_s)$ are the quasiparticle creation (annihilation) operator and single particle states $|\tilde{s}\rangle$ are the time-reversed of $|s\rangle$.Now supposing the isovector spin-spin interactions generate the 1^+ states in deformed nuclei, then the model Hamiltonian of system can be written as [20]

$$H = H_{sqp} + V_{\sigma\tau} \quad (2)$$

where

$$V_{\sigma\tau} = \frac{1}{2}\chi_{\sigma\tau}\sum_{i,j}\sigma_i\sigma_j\tau^Z{}_i\tau^Z{}_j\ . \quad (3)$$

Here $V_{\sigma\tau}$ is charge-exchange spin-spin interaction, $\chi_{\sigma\tau}$ denote spin-isospin coupling parameter. σ and τ are Pauli matrices representing the spin and isospin, respectively .In FR-RQRPA the modified phonon operator of the collective excitations in even-even deformed nuclei can be written as

$$|\Psi_i\rangle = Q_i^+|\Psi_0\rangle = \frac{1}{\sqrt{2}}\sum_{ssr,\tau}[X_{ssr}^i(\tau)\tilde{C}_{ssr}^+(\tau) - Y_{ssr}^i(\tau)\tilde{C}_{ssr}(\tau)]|\Psi_0\rangle \quad (4)$$

$$\sum_{ssr\,\tau}[X_{ssr}^{i\ 2}(\tau) - Y_{ssr}^{i\ 2}(\tau)] = 1 \quad (5)$$

Hereafter we use all definition of ref. [21], now, we present the expressions of the Hamiltonian (1) and σ_{+1} in FR-QRPA representation

$$V_{\sigma\tau} = \frac{1}{2}\chi_{\sigma\tau}\sum_\mu \sigma_\mu^+\sigma_\mu \quad (6)$$

where

$$\sigma_{+1} = \sqrt{2}\sum_{ssr}\sqrt{G_{ssr}}\,\sigma_{ssr}(u_s v_{sr}\tilde{C}_{ssr}^+ - u_{sr}v_s\tilde{C}_{ssr}) \quad (7)$$

Here, G_{ssr} takes into account the blocking effect due to the Pauli principle (scattering terms, exact commutator etc.), $\sigma_\mu^+ = (-1)^\mu\sigma_{-\mu}$ and $\sigma_{ssrr} = \langle s|\sigma_{+1}|sr\rangle$. Employing the conventional procedure [19,20] of QRPA with the equation of motion:

$$[H_{sqp} + V_{\sigma\tau}, Q_i^+] = \omega_i Q_i^+ \quad (8)$$

one can obtain the equation for the energy ω_i of one-phonon 1^+ excitations in the form (see, e.g., refs. [21])

$$D_\sigma = 1 + \chi_{\sigma\tau}F_\sigma = 0 \quad (9)$$

where

$$F_\sigma = 2\sum_{ssr}\frac{G_{ssr}E_{ssr}L_{ssr}^2\sigma_{ssr}^2}{E_{ssr}^*E_{ssr} - \omega_i^2} \quad (10)$$

Here, the function F_σ is defined as usually for the spin-vibration, $E_{ss'} = E_s + E_{s'}$ are two-quasiparticle energies, $L_\mu \equiv u_s v_{s'} - u_{s'} v_s$, u_s and v_s the Bogolyubov transformation parameters and

$$E_{ssr}^* = E_{ssr} + \tilde{E}_{ssr} \quad (11)$$

$$\tilde{E}_{ss^r} = (E_s - E_{sr})\frac{u_s v_s u_{sr} v_{sr}}{G_{ssr}(v_{sr}{}^2 - v_s{}^2)^2}(N_s - N_{sr}) \quad (12)$$

and

$$G_{ssr} = 1 - \frac{1}{2}\frac{u_s{}^2 - v_s{}^2}{v_{sr}{}^2 - v_s{}^2}N_s + \frac{1}{2}\frac{u_{sr}{}^2 - v_{sr}{}^2}{v_{sr}{}^2 - v_s{}^2}N_{sr} \quad (13)$$

Here N_q is average quasiparticle occupation number in the ground state modified by interactions between nucleons. The calculation of the N_{qp} can be performed with help of the fermion-boson mapping [7]. In the phonon representation one gets $N_q = 2\sum_{i,q'} Y_{qq'}^{i\ 2}$. We show that in FR-QRPA the two-quasiparticle energies E_{ssr}^* are naturally modified by the given interactions between nucleons. As a result the interactions between quasiparticles and the transition matrix elements are changed since it involves particles with modified properties. $\tilde{E}_{ssr}$ is responsible for the modification of the quasiparticle energies. Note that in the R-QRPA and QRPA the expression (12) is zero. The two-quasiparticle amplitudes of the one phonon wave function (4) can be written as

$$X_\mu^n = -\sqrt{\frac{G_\mu}{4\omega_i Z_\sigma}}\frac{\sigma_\mu L_\mu (E_\mu + \omega_i)}{E_\mu^* E_\mu - \omega_i^2}, \quad X_\mu^p = \sqrt{\frac{G_\mu}{4\omega_i Z_\sigma}}\frac{\sigma_\mu L_\mu (E_\mu + \omega_i)}{E_\mu^* E_\mu - \omega_i^2} \qquad (14)$$

$$Y_\mu^n = -\sqrt{\frac{G_\mu}{4\omega_i Z_\sigma}}\frac{\sigma_\mu L_\mu (E_\mu - \omega_i)}{E_\mu^* E_\mu - \omega_i^2}, \quad Y_\mu^p = \sqrt{\frac{G_\mu}{4\omega_i Z_\sigma}}\frac{\sigma_\mu L_\mu (E_\mu - \omega_i)}{E_\mu^* E_\mu - \omega_i^2} \qquad (15)$$

where

$$Z_\sigma = 2\sum_{ssr}\frac{G_{ssr} E_{ssr} L_{ssr}^2 \sigma_{ssr}^2}{(E_{ssr}^* E_{ssr} - \omega_i^2)^2}, \qquad (16)$$

3. Results and Discussion

Numerical calculations have been carried out for the deformed nuclei in the rare-earth region 150<A<172. The experimental values of deformation parameters of these nuclei were taken from [22]. The single particle energies are obtained from the Warsaw deformed Woods-Saxon potential [23]. The basis contains all discrete and quasi-discrete levels in the energy region up to 3 MeV. The pairing interaction constants chosen according to Soloviev [1] are based on single particle levels corresponding to the nucleus in question. The isovector spin-spin interaction strength was chosen to be $\chi_{\sigma\tau}$ =40/A MeV [20].

As the R-QRPA and FR-QRPA calculations are very time consuming processes in the deformed case, it is interesting to determine which spin matrix elements are important to be taken into account while calculating the RPA solutions. We calculate the energy weighted and non-energy weighted sum rules of M1 transition matrix elements. The calculation shows that the matrix elements $\langle s | s_{+1} | sr \rangle^2 \leq 10^{-4}$ do contribute only little to the sum rules (the effect does not exceed 2%). We perform all the numerical calculations with this restriction on the matrix elements.

The aim of the present calculations is to demonstrate the role of collective high energy RPA solutions in GSC calculations. This can be obtained by the comparison with the results calculated in different energy regions. The results of calculated N_{qp} values for ^{154}Sm and ^{178}Hf taking into account RPA solutions in different energy region are given in Table I for the various FR-QRPA, R-QRPA and QRPA methods.

Table 1

Comparison of the value of N_{qp} calculated with Hamiltonian (1) using FRQRPA approach, RQRPA and QRPA

	^{154}Sm			^{178}Hf		
ω (MeV)	FR-QRPA	R-QRPA	QRPA	FR-QRPA	R-QRPA	QRPA
2-5	0.0100	0.0100	0.0100	0.0097	0.0097	0.0097
5-9	0.0108	0.0108	0.0108	0.0160	0.0160	0.0160
9-13	0.1361	0.1364	0.1364	0.1485	0.1482	0.1489
13-25	0.0077	0.0077	0.0078	0.0048	0.0048	0.0048
2-25	0.1646	0.1649	0.1650	0.1790	0.1787	0.1794

There is a fragmentation of the 1^+ states at low energy. For these 1^+ states the summed N_{qp} is order of 0.07 and contains about 8 % of the total N_{qp}. Contributions of the 1^+ states with large value N_{qp} are obtained in the energy region 9-13 MeV. The absolute value N_{qp} in this region is in the order of 0.15. The relative contribution of the high energy RPA solutions which form M1 resonance gives more than 80% of the total number of quasiparticles in the ground state of the nuclei considered. Thus we see that the models, which take into account only low-lying states overestimate ground state correlations strongly.

Besides the N_{qp} another important quantity of the 1^+ states is $N_{qp}(\omega)$ distribution over RPA solutions. The distribution of the average number of quasiparticles in the ground state $N_{qp}(\omega)$ for the different RPA solutions ω_i gives important information about the role of the 1^+ states and their relative contributions on ground state correlations. In Fig.1 we show examples of these calculations for the nucleus ^{168}Er.

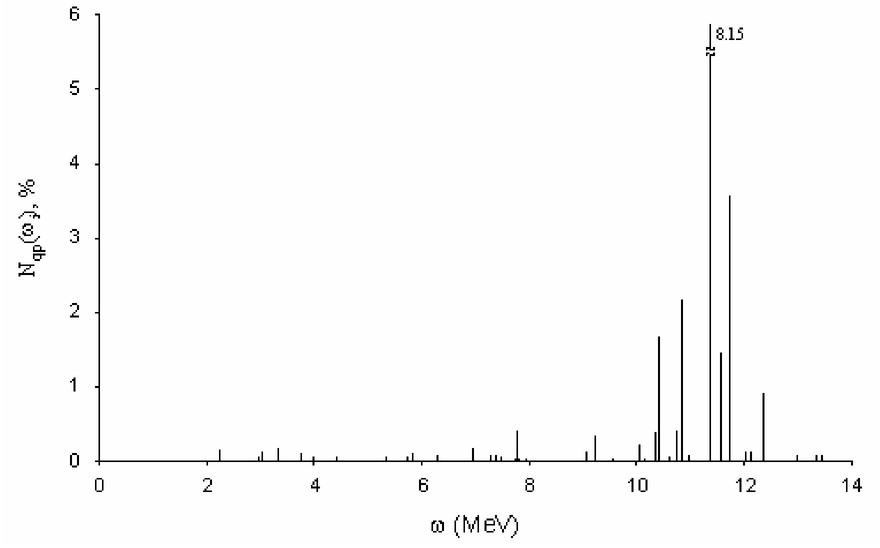

Fig.1. Distribution of the relative values of $N_{qp}(\omega)$ in relation to the total N_{qp} value for the 1$^+$ states in ^{154}Sm.

As seen from the figure, most contributions come from the high energy collective states which form M1 resonance. We note that the 1$^+$ states in the M1 resonance region take up to about 80% of the total N_{qp}. Unlike previous calculations [2,8,18] we conclude that the collective states of the resonance region exhaust the main part of the total N_{qp} and their role is very important for GSC.

The comparison of the calculated value of N_{qp} for ^{154}Sm, ^{156}Gd, ^{168}Er and ^{178}Hf in framework QRPA, R-QRPA and FR-QRPA approaches are given in Fig.2.

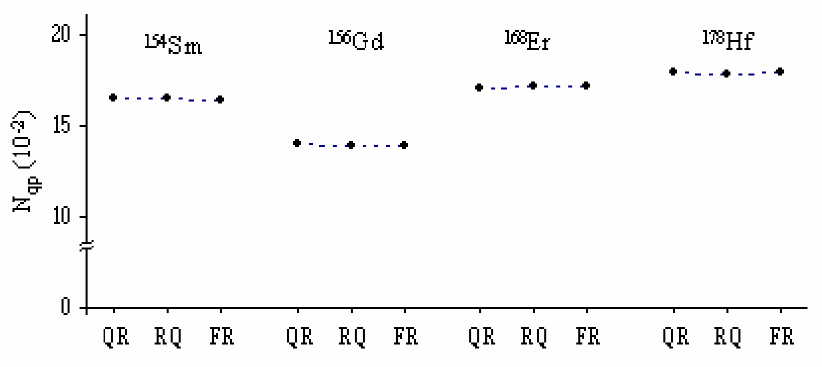

Fig.2. Comparison of the average number of quasiparticles in the ground state N_{qp} due to the vibrational 1$^+$ states calculated in the QRPA, R-QRPA and the FR-RQPA approaches. The three different approaches QRPA, R-QRPA and FR-QRPA are denoted below the axes as QR, RQ and FR, respectively.

It follows from Fig.2 that the total values of N_{qp} are almost the same in QRPA, R-QRPA and FR-QRPA approach (a difference is about 1-2% of the total N_{qp}). The results that we have obtained indicate a significant role of the high energy 1$^+$ states and the importance to use a complete set of the RPA solutions for reliable predictions of the theory in ground state correlations calculations. Besides the calculations show that the absolute values of N_{qp} are almost the same in QRPA, R-QRPA and FR-QRPA for the p-h isovector spin-spin interaction.

4. Conclusions and outlook

Thus, we have considered a number of features of the GSC in deformed nuclei connected with the effective spin-spin forces in p-h cannel. It is concluded that high energy collective excitations which form M1 resonance give dominant contribution to GSC. The total N_{qp} and renormalizied nuclear transition matrix elements are significantly affected by the collective states of the M1 resonance energy region. The study of ground state correlations seems to give some evidence for the importance of taking into account a full spectrum of RPA solutions for reliable predictions of the theory.

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SPİN–VİBRASİYA 1^+ SƏVİYYƏLƏRİNİN NÜVƏLƏRİN ƏSAS HALINDAKI KORELASİONLARINA TƏSİRİ

Məqalədə deformasiyalı nüvələrin əsas hal korelasionlarının effektiv spin-spin qüvvətləri ilə əlaqədar olan bəzi xassələri FR-QRPA, R-QRPA və QRPA metodları çərçivəsində tədqiq edilmişdir. Hesablamalar əsas hal korelasionlarında etibarlı nəticələr əldə edilməsi üçün yüksək enerjili kollektiv 1^+ səviyyələrin və RPA metodunun bütün köklərinin (tam set) nəzərə alınmasının mühüm rol oynadığını göstərmişdir.

M1 rezonansı meydana gətirən kollektiv səviyyəllərin əsas hal korelasionlarına əlavəsinin qalan səviyyələrdən daha böyük olduğu göstərilmişdir. Bu kollektiv səviyyələr, əsas haldakı kvazizərrəciklərin tam sayına və nüvə keçidlərinin matris elementlərinə daha çox təsir etməkdədir. Nəzəri hesablamalar spin-spin qüvvətlərini zərrəcik-deşik kanalında nəzərə alan QRPA, R -QRPA və FR-QRPA metodlarında əldə edilən nəticələrin bir-birinə yaxın olduğunu göstərmişdir.

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ЭФФЕКТ СПИН-ВИБРАЦИОННЫХ 1^+ УРОВНЕЙ НА КОРРЕЛЯЦИИ В ОСНОВНОМ СОСТОЯНИИ В ЯДРАХ

В статье рассмотрено несколько свойств корреляции в основных состояниях деформированных ядер, связнных с эффективными силами в рамках FR-QRPA, R-QRPA и QRPA. Результаты вычислений указывают на значительную роль высоких энергетических 1^+ состояний, важность использования полного набора RPA для надежного предсказания теории корреляции в основном состоянии. Показано, что коллективные состояния, формирующие M1 резонанс дают доминирующий вклад корреляции в основное состояние. Выше упомянутые состояния имеют значительное влияние на полное число квазичастиц в основном состоянии и матричные элементы ядерных переходов. Полученные результаты практически не чувствительны к использованию QRPA, R-QRPA и FR-QRPA методов, которые используют изовекторное спин-спиновое взаимодействие в канале частица-дырка.

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