UNIFICATION OF ELECTROMAGNETISM AND GRAVITATION IN THE FRAMEWORK OF GENERAL GEOMETRY[1]

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Abstract

General geometry including Riemannian geometry as a special case is constructed. It is proven that the most simplest special case of General Geometry is geometry underlying Electromagnetism. Action for electromagnetic field and Maxwell equations are derived from curvature function of geometry underlying Electromagnetism. And it is shown that equation of motion for a particle interacting with electromagnetic field coincides exactly with equation for geodesics of geometry underlying Electromagnetism. It is also shown that Electromagnetism can not be geometrized in the framework of Riemannian geometry. Using General Geometry we propose a unified model of electromagnetism and gravitation which reproduces Electromagnetism and Gravitation and predicts that electromagnetic field is a source for gravitational field. This theory is formulated in four dimensional space-time and does not contain additional fields.

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[1]This talk is based on papers [1] and [2].

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1 Introduction

As we know equation for geodesics of Riemannian geometry

$$\frac{d^2x^\lambda}{du^2} = -\Gamma'^{\sigma}_{\lambda\nu}(x)x^\nu_u x^\lambda,$$

$$2\Gamma'_{\lambda,\mu\nu} = \frac{\partial g_{\lambda\nu}}{\partial x^\mu} + \frac{\partial g_{\lambda\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda}.$$

coincides with the equation of motion for a particle interacting with gravitational field $g_{\mu\nu}$. And equation for gravitational field is related to curvature charchteristics of Riemannian geometry

$$^{\mathrm{g}}S = \int dx\sqrt{-g}R'$$

$$R' = g^{\mu\nu}R'_{\mu\nu}, \quad R'_{\lambda\nu} = \partial_\nu\Gamma^\mu{}_{\lambda\mu} - \partial_\mu\Gamma^\mu{}_{\lambda\nu} + \Gamma^\mu{}_{\rho\nu}\Gamma^\rho{}_{\lambda\mu} - \Gamma^\mu{}_{\rho\mu}\Gamma^\rho{}_{\lambda\nu}, \quad g = det g_{\mu\nu},$$

where $R'_{\lambda\nu}$ is the curvature tensor of Riemannian geometry.

This can be generalized to a definition of underlying geomety for any theory. So, we understand geometrization of a theory as follows:1. Equation of motion for particle interacting with a given field must coincide with equation for geodesics of the corresponding geometry. 2. Equation of motion for the given field must be related to curvature characteristics of the corresponding geometry.

Because, Riemannian geometry and theory of gravitation satisfy these two requirements, Riemannian geometry is considered as an underlying geometry for gravitation.

As it is known equation or action function for gravitation could not be found using tools of field theory because in order to get conserved energy momentum for gravitational field it was required to add to the action infinite number of terms. Only geometrization principle made it possible to find a proper action for gravitation [3].

After this was realized [4]–[5] at the beginning of XX century, many physicists and mathematicians tried to geometrize electromagnetism and unify it with gravitation using geometrization principle [6]–[16]. All these approaches considered this problem in the framework of Riemannian geometry and failed to satisfy the above mentioned requirements completely or to reproduce Electromagnetism and Gravitation exactly. I will mention drawbacks of two well known theories, only. They are Weyl and Kaluza-Klein theories.

A Drawback of Weyl theory is that some of its predictions contradict experiment [17].

Drawbacks of Kaluza-Klein theory are that it has charge/mass problem and additional dilaton field. As it is noted by its originator, T. Kaluza, this theory is not applicable even to electrons because of the charge/mass problem [7].

At the end of this talk I will show that Electromagnetism can not be geometrized in the framework of Riemannian geometry.

To solve these problems, instead of increasing dimensionality of spacetime or choosing different metrics in Riemannian geometry, we construct a new geometry, called General geometry. We show that it includes Riemannian geometry as a special case, its the most simplest special case is the geometry underlying Electromagnetism. Using its another special case we propose a unified model of electromagnetism and gravitation which reproduces electromagnetism and gravitation exactly and predicts that electromagnetic field is a source for gravitational field. It is formulated in four dimensional spacetime and does not contain any additional fields.

Geometry underlying the proposed model is created by interacting particles and sources for electromagnetic and gravitational fields unlike geometry underlying gravitation,Riemannian geometry, which is created by sources for gravitational field only.

2 General Geometry

In this section we construct a new geometry. This geometry includes Riemannian geometry, geometry underlying Electromagnetism (see next section), geometry underlying a unified model of Electromagnetism and Gravitation, and infinite number of geometries, physical interpretation of which is not known at the present time, as special cases. Because of this we call it General Geometry. Besides mathematical applications, this new geometry has important physical applications. We demonstrate it in the next section.

Let M be a manifold with coordinates $x^\lambda, \lambda = 1, ..., n$. Consider a curve on this manifold $x^\lambda(u)$. Vector field

$$V = \xi^\lambda \frac{\partial}{\partial x^\lambda}$$

has coordinates ξ^λ. In Riemannian geometry it is accepted that

$$\frac{d\xi^\lambda}{du} = -\Gamma'^\sigma_{\lambda\nu}(x) x^\nu_u \xi^\lambda, \tag{1}$$

where $\Gamma'^\sigma_{\lambda\nu}(x)$ are functions of x only.

To construct General Geometry we assume that

$$\frac{d\xi^\sigma}{du} = -\Gamma^\sigma_\lambda(x, x_u)\xi^\lambda. \tag{2}$$

$\Gamma^\sigma_\lambda(x, x_u)$ are general functions of x and x_u. The next step is to consider x as a function of two parameters u, v and find $\lim\limits_{\substack{\Delta v \to 0 \\ \Delta u \to 0}} \Delta\xi^\sigma/\Delta u \Delta v$. In order to do that we need

$$\frac{d\xi^\sigma}{du} = -\Gamma^\sigma_\lambda \xi^\lambda, \quad \frac{d\xi^\sigma}{dv} = -\tilde{\Gamma}^\sigma_\lambda \xi^\lambda,$$

$$\Gamma^\sigma_\lambda = \Gamma^\sigma_\lambda(x, x_u, x_v), \quad \tilde{\Gamma}^\sigma_\lambda = \tilde{\Gamma}^\sigma_\lambda(x, x_u, x_v).$$

After simply calculations we arrive at

$$\lim_{\substack{\Delta v \to 0 \\ \Delta u \to 0}} \frac{\Delta\xi^\sigma}{\Delta u \Delta v} = R^\sigma_\lambda \xi^\lambda,$$

where

$$R^\sigma_\lambda = \frac{d}{dv}\Gamma^\sigma_\lambda - \frac{d}{du}\tilde{\Gamma}^\sigma_\lambda + \tilde{\Gamma}^\sigma_\rho \Gamma^\rho_\lambda - \Gamma^\sigma_\rho \tilde{\Gamma}^\rho_\lambda.$$

We call R^σ_λ curvature function.

Representing $\Gamma^\sigma_\lambda(x, x_u)$ as

$$\Gamma^\sigma_\lambda(x, x_u) = F^\sigma_\lambda(x) + \Gamma^\sigma_{\lambda\nu}(x)x^\nu_u + \Gamma^\sigma_{\lambda\nu\mu}(x)x^\nu_u x^\mu_u + ...$$

and considering each order in x_u or their combinations separately we define a set of new geometries. Only the first order in x_u is already known Riemannian geometry. Let us show how curvature function is related to curvature tensor in the case of Riemannian geometry. Let

$$\Gamma^\sigma_\lambda(x, x_u, x_v) = \Gamma^\sigma_{\lambda\nu}(x)x^\nu_u, \quad \tilde{\Gamma}^\sigma_\lambda(x, x_u, x_v) = \Gamma^\sigma_{\lambda\nu}(x)x^\nu_v.$$

Curvature function for this case is

$$R^\sigma_\lambda = R^\sigma_{\lambda\mu\nu}(x^\nu_u x^\mu_v - x^\nu_v x^\mu_u),$$

where

$$R^\sigma_{\lambda\mu\nu} = \partial_\mu \Gamma^\sigma_{\lambda\nu} - \partial_\nu \Gamma^\sigma_{\lambda\mu} + \Gamma^\sigma_{\rho\mu}\Gamma^\rho_{\lambda\nu} - \Gamma^\sigma_{\rho\nu}\Gamma^\rho_{\lambda\mu}$$

is the curvature tensor of Riemannian geometry.

3 Geometry of Electromagnetism

In the case of Electromagnetism we do know equation for the field and particles interacting with it but we do not know geometry underlying it. This is the reversed case for gravitation. We need to know geometry underlying electromagnetism because in that case we can construct geometry underlying unified model of Electromagnetism and gravitation and as in the case of gravitation, derive equation for the unified model.

For geometry of Electromagnetism, we consider the most simplest case of General Geometry

$$\Gamma^\sigma_\lambda(x, x_u, x_v) = F^\sigma_\lambda(x(u, v)), \quad \tilde{\Gamma}^\sigma_\lambda(x, x_u, x_v) = F^\sigma_\lambda(x(u, v)),$$

when $\Gamma^\sigma_\lambda(x, x_u)$ does not depend on x_u and show that it is an underlying geometry for electromagnetism. In order to prove, that geometry defined by

$$\frac{d\xi^\sigma}{du} = -F^\sigma_\lambda(x)\xi^\lambda \tag{3}$$

with the length of a curve

$$ds = \sqrt{\eta_{\mu\nu}dx^\mu dx^\nu} + \frac{q}{cm}A_\mu dx^\mu$$

is an underlying geometry for electromagnetism we must show that equation of motion for a particle interacting with electromagnetic field coincides with equation of geodesics in this geometry, and Maxwell equations and Lagrangian for electromagnetic field are related to its curvature characteristics.

Geometry defined by (3) has different properties than Riemannian geometry defined by (1). We do not get into details here. We simply mention that in this geometry the notion of parallel transport is not defined. As we show in the sequel this makes it be underlying geometry for Electromagnetism.

To obtain equations for geodesics we substitute ξ^λ in (3) by x^λ_u and arrive at

$$\frac{d^2 x_\sigma}{du^2} = -F_{\sigma\lambda}(x) x^\lambda_u.$$

This is exactly equation of motion for a charged particle moving in electromagnetic field A_μ, if we choose

$$F_{\sigma\lambda} = \frac{q}{cm}(\partial_\sigma A_\lambda - \partial_\lambda A_\sigma).$$

So, the first requirement is satisfied with this choice of function $F_{\sigma\lambda}$. In [18], we have proved this relation between $F_{\sigma\lambda}$ and A_μ.

It remains to show that Maxwell equations and Lagrangian for electromagnetic field is related to curvature characteristics of geometry (3). To this end let us find curvature function for (3)

$$R^\sigma_\lambda = R^\sigma_{\mu\lambda}(x^\mu_v - x^\mu_u),$$

where

$$R^\sigma_{\mu\lambda} = \partial_\mu F^\sigma_\lambda.$$

This tensor is an analog of curvature tensor of Riemannian geometry. After summing by two of the three indices we obtain

$$R_\lambda = R^\mu_{\mu\lambda} = \partial_\mu F^\mu_\lambda.$$

Vector R_λ is an analog of Ricci tensor. Equations $R_\lambda = 0$ coincide with the Maxwell equations. In order to construct a Lagrangian we need a scalar function. In our case we have two quantities R_λ and A^λ. A^λ originates from the length of a curve (metric) as $g_{\mu\nu}$ originates from the length of a curve in Riemannian geometry. We can construct from R_λ and A^λ a Lagrangian

$$R = A^\lambda R_\lambda = \partial_\mu(A^\lambda F^\mu_\lambda) - \frac{1}{2} F_{\mu\lambda} F^{\mu\lambda}.$$

This coincides with the Lagrangian of electromagnetic field up to total derivative.

We see that as in the case of Riemannian geometry and gravitation we can find equations and action functional for electromagnetic field from geometric characteristics of geometry underlying Electromagnetism. And equation for geodesics coincides with the equation of motion for a particle interacting with electromagnetic field.

From the geometrical point of view a charged particle interacting with electromagnetic field can be considered as a free particle in the spacetime with the length of a curve $ds = \sqrt{\eta_{\mu\nu} dx^\mu dx^\nu} + \frac{q}{cm} A_\mu dx^\mu$ and equation for geodesic

$$\frac{d^2 x_\sigma}{du^2} = \frac{q}{cm}(\partial_\lambda A_\sigma - \partial_\sigma A_\lambda) x^\lambda_u,$$

where A_μ is a solution to equation $R_\lambda = 0$.

This theory does not has any drawbacks like in the theories constructed before. It reproduces electromagnetism exactly is free from additional fields and extra dimensions.

4 Unification of electromagnetism and gravitation

Now, we consider a different special case of General Geometry. For geometry underlying our unified model we choose functions Γ and $\tilde{\Gamma}$ as

$$\Gamma^\sigma{}_\lambda(x, x_u, x_\upsilon) = F^\sigma{}_\lambda(x(u,\upsilon)) + \Gamma^\sigma{}_{\lambda\nu}(x)x^\nu_u, \quad \tilde{\Gamma}^\sigma{}_\lambda(x, x_u, x_\upsilon) = F^\sigma{}_\lambda(x(u,\upsilon)) + \Gamma^\sigma{}_{\lambda\nu}(x)x^\nu_\upsilon.$$

And the length of a curve as

$$ds = \sqrt{g_{\mu\nu}(x)dx^\mu dx^\nu} + \frac{q}{cm}A_\mu(x)dx^\mu.$$

For our choice of $\Gamma^\sigma{}_\lambda$, (2) becomes

$$\frac{d\xi^\sigma}{du} = -(F^\sigma{}_\lambda(x) + \Gamma^\sigma{}_{\lambda\nu}(x)x^\nu_u)\xi^\lambda, \tag{4}$$

We substitute ξ^σ in (4) by x^σ_u and obtain equation for geodesics

$$\frac{d^2x^\sigma}{du^2} = -F^\sigma{}_\lambda(x)x^\lambda_u - \Gamma^\sigma{}_{\mu\nu}(x)x^\mu_u x^\nu_u.$$

It coincides exactly with equation of motion for a particle with charge q and mass m interacting with electromagnetic and gravitational fields if we choose

$$F_{\mu\nu} = \frac{q}{cm}(\partial_\mu A_\nu - \partial_\nu A_\mu), \quad 2\Gamma_{\lambda,\mu\nu} = \frac{\partial g_{\lambda\nu}}{\partial x^\mu} + \frac{\partial g_{\lambda\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda}. \tag{5}$$

In this paper we assume these relations and declare A_μ as electromagnetic field and $g_{\mu\nu}$ as gravitational field. These relations between F and A, and Γ and $g_{\mu\nu}$ are proven in [18] and it is shown that A_μ can be identified with electromagnetic field, q with charge, m with mass of a particle interacting with A_μ, c with the velocity of the light, and $g_{\mu\nu}$ with gravitational field.

The corresponding curvature function is

$$R^\sigma{}_\lambda = (\partial_\mu F^\sigma{}_\lambda - \Gamma^\rho{}_{\lambda\mu}F^\sigma{}_\rho + \Gamma^\sigma{}_{\rho\mu}F^\rho{}_\lambda)(x^\mu_\upsilon - x^\mu_u) +$$

$$\frac{1}{2}(\partial_\nu\Gamma^\sigma{}_{\lambda\mu} - \partial_\mu\Gamma^\sigma{}_{\lambda\nu} + \Gamma^\sigma{}_{\rho\nu}\Gamma^\rho{}_{\lambda\mu} - \Gamma^\sigma{}_{\rho\mu}\Gamma^\rho{}_{\lambda\nu})(x^\nu_\upsilon x^\mu_u - x^\nu_u x^\mu_\upsilon). \tag{6}$$

From (6), we see that gravitational field is coupled to $F^\sigma{}_\lambda$ through covariant derivative

$$\Delta_\mu F^\sigma{}_\lambda = \partial_\mu F^\sigma{}_\lambda - \Gamma^\rho{}_{\lambda\mu}F^\sigma{}_\rho + \Gamma^\sigma{}_{\rho\mu}F^\rho{}_\lambda.$$

We have $g_{\mu\nu}$, A_μ and curvature function to use to find an action for the unified model. First, we construct a tensor from (6)[2]

$$R^\sigma{}_{\lambda\mu\nu} = \frac{cm}{4q}(\Delta_\nu F^\sigma{}_\lambda A_\mu - \Delta_\mu F^\sigma{}_\lambda A_\nu) + \frac{1}{16\pi G}(\partial_\nu\Gamma^\sigma{}_{\lambda\mu} - \partial_\mu\Gamma^\sigma{}_{\lambda\nu} + \Gamma^\sigma{}_{\rho\nu}\Gamma^\rho{}_{\lambda\mu} - \Gamma^\sigma{}_{\rho\mu}\Gamma^\rho{}_{\lambda\nu}),$$

where G is gravitational constant. Finally we have a scalar

$$R = g^{\lambda\nu}R^\mu{}_{\lambda\mu\nu}$$

[2] One may construct different tensors from (6).

and action

$$
{}^{\mathbf{eg}}S = \int dx\sqrt{-g}R = \frac{c^2m^2}{4q^2}\int dx\sqrt{-g}F^{\mu\nu}F_{\mu\nu} + \frac{1}{16\pi G}\int dx\sqrt{-g}\,{}^{\mathbf{g}}R,
$$

$$
{}^{\mathbf{g}}R = g^{\mu\nu}\,{}^{\mathbf{g}}R_{\mu\nu}, \quad {}^{\mathbf{g}}R_{\lambda\nu} = \partial_\nu\Gamma^\mu{}_{\lambda\mu} - \partial_\mu\Gamma^\mu{}_{\lambda\nu} + \Gamma^\mu{}_{\rho\nu}\Gamma^\rho{}_{\lambda\mu} - \Gamma^\mu{}_{\rho\mu}\Gamma^\rho{}_{\lambda\nu}, \quad g = detg_{\mu\nu},
$$

where use has been made of

$$
\Delta_\nu F^{\mu\nu} = \frac{1}{\sqrt{-g}}\partial_\nu(\sqrt{-g}F^{\mu\nu}), \quad \Delta_\nu g_{\mu\lambda} = 0.
$$

Note, that the action is invariant under gauge transformations of fields and general transformations of coordinates. Covariant derivative appears naturally in this formalism. Hence, geometrization principle leads to an action which is invariant under gauge transformations of fields and general transformations of coordinates. We conclude that geometrization principle is more general than gauge principle.

Equation of motion for gravitational field is

$$
\frac{c^2m^2}{4q^2}(\frac{1}{2}F^{\rho\sigma}F_{\rho\sigma}g^{\mu\nu} - 2F^{\nu\sigma}F^\mu{}_\sigma) + \frac{1}{16\pi G}(-{}^{\mathbf{g}}R^{\mu\nu} + \frac{1}{2}{}^{\mathbf{g}}Rg^{\mu\nu}) = 0. \tag{7}
$$

From (7) it follows that ${}^{\mathbf{g}}R = 0$ for $n = 4$ (in the rest of the paper we restrict ourselves to four dimensional spacetime) and (7) becomes

$$
\frac{c^2m^2}{4q^2}(\frac{1}{2}g^{\rho n}g^{\sigma m}F_{nm}F_{\rho\sigma}g_{\mu\nu} - 2g^{\sigma m}F_{\nu m}F_{\mu\sigma}) - \frac{1}{16\pi G}{}^{\mathbf{g}}R_{\mu\nu} = 0. \tag{8}
$$

We see that electromagnetic field is a source for gravitational field. In the weak gravitational and strong electromagnetic field approximation $g^{\mu\nu} \sim \eta^{\mu\nu} = diag(1,-1,-1,-1)$ and

$$
{}^{\mathbf{g}}R_{00} \sim -\partial_\mu\Gamma^\mu_{00} = -\frac{1}{2}\mathbf{\Delta}g_{00}, \quad \mathbf{\Delta} = \frac{\partial}{\partial x^i}\frac{\partial}{\partial x^i}, \quad i = 1,2,3.
$$

The 00 component of equation (8) gives

$$
\mathbf{\Delta}\Phi = 4\pi c^2G(E_iE_i + H_iH_i) + O(hF), \quad g_{00} = 1 - 2\frac{\Phi}{c^2}, \tag{9}
$$

where Φ is the Newtonian potential, $E_i = \partial_0A_i - \partial_iA_0$ and $H_i = \frac{1}{2}\epsilon_{ijk}(\partial_jA_k - \partial_kA_j)$ are electric and magnetic fields respectively and ϵ_{ijk} is antisymmetric tensor. Accordingly, total energy of electromagnetic field produces gravitational field.

Because geometrization principle gave true equation for gravitational field, we can be sure that this equation is also true. The proposed theory gives exactly Gravitation when electromagnetic field is equal to zero and Electromagnetism when gravitational field is equal to zero. It predicts that electromagnetic field is a source for gravitational field. This theory is formulated in four dimensional spacetime and does not contain any additional fields.

5 Discussion

For Riemannian geometry

$$\frac{d\xi^\lambda}{du} = -\Gamma'^{\sigma}_{\lambda\nu}(x)x^\nu_u\xi^\lambda,$$

it is possible to make a change of coordinates so that its right hand side will be equal to zero, because of its right hand side structure. In new coordinates x', equation for geodesics becomes

$$\frac{d^2x'^\sigma}{du^2} = 0.$$

From physical point of view this corresponds to finding a reference frame where trajectory of particles is strait line, because this equation must coincide with the equation of motion. For gravitational interaction we can find a reference frame where gravitational interaction is absent. Accordingly, Riemannian geometry is suitable for gravitational interaction only. For electromagnetic interactions it is not possible to find a reference frame where it is absent. Therefore, all attempts to geometrize electromagnetism or unify it with gravitation in the framework of Riemannian geometry must fail.

On the other hand for geometry

$$\frac{d\xi^\sigma}{du} = -(F^\sigma{}_\lambda(x) + \Gamma^\sigma{}_{\lambda\nu}(x)x^\nu_u)\xi^\lambda$$

it is not possible to eliminate its right hand side by changing coordinates because of $F^\sigma{}_\lambda$ term. And this property makes it to be underlying geometry for the proposed unified model.

In general relativity, geometry underlying gravitation and metric are independent of properties of interacting particles. This is a consequence of equivalence principle. Geometry and metric depends on the characteristics of sources for gravitational field $g_{\mu\nu}$ only. For electromagnetic interactions there is no equivalence principle, so geometry and metric underling electromagnetism and unified model of electromagnetism and gravitation must depend on characteristics of interacting particles, because particles of different charges move in electromagnetic field differently.

For our model we have

$$\frac{d\xi^\sigma}{du} = -(\frac{q}{cm}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \Gamma^\sigma{}_{\lambda\nu}(x)x^\nu_u)\xi^\lambda.$$

Accordingly, geometry underlying unified model of electromagnetism and gravitation depends not only on the characteristics of sources for A_μ and $g_{\mu\nu}$ but also on the characteristics of interacting particles q and m. This means that geometry and the length of a curve (metric) $ds = \sqrt{g_{\mu\nu}(x)dx^\mu dx^\nu} + \frac{q}{cm}A_\mu(x)dx^\mu$ are created by interacting particles too, together with sources unlike gravitational interaction. This gives us a new understanding of problem of geometry and matter.

Next, we note that in General Relativity, if we consider sources for gravitational field we must add the so called source term $g_{\mu\nu}T^{\mu\nu}$ to the action

$$^{\mathrm{g}}S = \int dx(\sqrt{-g}R' + g_{\mu\nu}T^{\mu\nu}).$$

$T^{\mu\nu}$ is the energy– momentum tensor of sources for gravitational field. We can not simply replace it with energy- momentum tensor of any field, if it is not a source for $g_{\mu\nu}$. Some people say that General Relativity also predicts that electromagnetic field is a source for gravitational field trough inclusion of it to $T^{\mu\nu}$. I would like to stress that in General Relativity there is no natural place for electromagnetic field because inclusion of electromagnetic field in $T^{\mu\nu}$ declares it as a source for gravitational field which is the assumption but not prediction.

Resuming, we can say that we have eliminated the need for extra dimensions and additional fields for formulating unified model of electromagnetism and gravitation by formulating a new geometry. This approach can be useful for formulating a unified electroweak model without Higgs fields and for unifying strong interactions with the other ones.

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