Simplex presentation of the real world in the quantum mechanics

Isaeva E.A.
Institute of Physics of AzerbaijanNational Academy of Sciences, 33H.Javid av., Baku, Az-1143, e-mail: elmira @physics.ab.az

It is known that superposition of states of states taking place in the quantum world can occur in the macroworld too due to the mechanism of intensification. "Schrodinger cat" is a fact of such intensification. It is known that in an open system the "Schrodinger cat" paradox is explained by the decoherence phenomenon but in a close system it is explained by the many-world interpretation of quantum mechanics Everett-Wheeler. The quantum real world is presented as a certain complex multi-spatial geometric figure and what we call the "classical" world is one of face of this figure. In the paper it has shown that the this complex figure constitutes the simplex notion in the functional analysis. It has been shown that such an interpretation of mechanics enables one to obtain the non-uniform wave equation, and Schrodinger equation is the uniform equation of this equation.

1. **Simplex**.

From the functional analysis [1] it is known that the sequence of points $\{x_{n+1}\}$ are in general provisions when these points are not in (n-1)-dimensional space. If these points are connected with each other, they form n- dimensional simplex. For example, one point – zero-dimensional simplex, the piece - one-dimensional one, the triangle – two-dimensional one, the tetrahedral - three-dimensional simplex , etc. (Table 1).

Table 1

THE SIMPLEX					
0-dimensional	1-dimensional	2-dimensional	3-dimensional	4-dimensional	etc.
Point	Piece	Triangle	Tetrahedral	It isn't possible to imagine	
THE NUMBER OF FACES OF SIMPLEX $C_n^K = \frac{n!}{K!(n-K)!}$					
1	2	6	14	30	

It is known that if $x_1, x_2... x_n$ points are in the general provisions, any (κ+1) points of them, where k < n, are also in the general provisions and form k - dimensional simplex named k-dimensional face of the given simplex. For example, the three-dimensional simplex – the tetrahedral - has 4 two-dimensional faces (triangles), 6 one-dimensional faces (pieces) and 4 zero-dimensional faces (points). In total the sum of the faces equals 14 . Let's note that a cube are not the simplex because for creation of the simplex from the 8 points it is necessary all these points are in the 6- dimensional space. Let's consider the 4-dimensional simplex. Here the number of the points are 5. All of them should not be located in the three-dimensional space. It is impossible to imagine such a figure. This 4-dimensional simplex has 30 faces: 5 three-dimensional faces (tetrahedron), 10 two-dimensional faces (triangles), 10 one-dimensional faces (pieces) and 5 zero-dimensional faces (points). Thus the formed from more than four points simplex cannot be presented in our three-dimensional space. It is the complex volumetric figure.

The simplex in n-dimensional space is the minimal convex set, i.e. all points of a kind $\Sigma a_n x_n$, where $\Sigma a_n = 1$ belong to this simplex. From the theory of probabilities [2] it is known that the probability of event is closely connected to random and average value . The belonging to a simplex points are the set of all average value if we count that the tops of the simplex $\{x_n\}$ are the random value and a_n are probabilities of x_n . In this point of view the consideration of a simplex is expedient. Also from the point of view of

A.Einstein who wrote [4]: "… We should refuse from the description of the nuclear phenomena as phenomena in the space and the time . We should go away from old mechanical review. The quantum physics formulates the laws which manager sets but not individuals. It is described the probabilities and not the properties. It is formed the laws opening the future of systems but not the laws managing the changes of probabilities in the time and concerning to large sets of individuals."

2. **Superposition in quantum mechanics.**

It is known that by using imagination experiment "shrodinger cat" [5] it is proved that the superposition of two microstate (atom decayed and atom didn't decay) transform in the superposition of two macrostate (cat is alive and cat is dead). If in the open system this paradox is solved by the "decoherence" [12] than in the close system that is solved by the theory in which the consciousness is included. One of such theory is many- word interpretation of quantum mechanics of Evverett-Willer [9,10].

The author of paper Menskii M.B. [6] the quantum world symbolically represented as some complex volumetric figure and what we name "a classical reality " is only one of projections of this figure. He presents the following interesting scheme.

The objective quantum world
↓
The illusion of the classical world

In this scheme the quantum world is objective because it does not depend on consciousness of the observer. In this case the system is close, i.e. our consciousness inside volumetric figure. The objective real world exists in the form of the parallel worlds, each of which is not realer than the rest. In scheme the classical world is illusion because it depend on consciousness of the observer. In this case the system is open, i.e. our consciousness outside volumetric figure Being outside the volumetric figure our consciousness is interacting with the surrounding world and in consequence the decoherence take place. Picture of the world seen by us is the result of the coupling of wave functions of our consciousness and surrounding world [13]. We always see only one of the parallel worlds, but other worlds do not cease to exist. Therefore the classical world is only one of many variants and it arises in our consciousness. From the beginning of existence of the quantum mechanics the famous scientists Pauli [11], Wigner[12], Shredinger [5] said about the necessity of inclusion of the observer's consciousness in the quantum theory of measurements. Wigner paper contains more stronger statement: the consciousness not only should be included in the theory of measurements but the consciousness may influence on the reality. As M. Plank wrote:
" … We are compelled to recognise that behind the sensual world there is a second, real world, which exists independently from the man , the world which we cannot directly study but which we comprehend through the sensual world, known signs which inform us just as if we could consider the subject interesting for us through glasses, optical properties of which are completely unknown for us."

3. Construction of the simplex

Apparently the above mentioned complex volumetric figure is the simplex.
Let's imagine that on two-co-ordinate plane xoy, on an axis x the number of dead "Schrodinger cats" and on an axis y the number of alive cats are marked. Let's suppose in experiment with 100 "Schrodinger cats" 80 cats are alive and 20 cats are dead. It is may be different numbers alive and dead cats from the total number all cats. But we consider numbers 100, 80, 20. In this case the probabilities are approximately equal 0,8

(alive cat) and 0,2(dead cat). The points 20 and 80 are two tops of the simplex. In the other case or at other moment of time with 100 "Schrodinger cats " there are 60 alive cats and 40 dead cats. These points we put in other system of coordinates x^1oy^1 in 3 dimensional space. If we connect given 4 points we receive three-dimensional simplex - tetrahedral. (Fig.1)

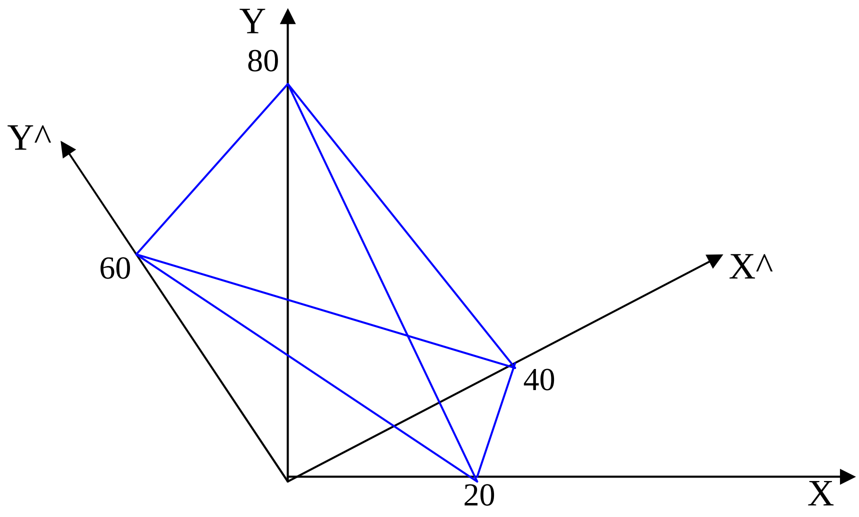

Fig.1

If we would consider a lot of points we shall receive the complex volumetric figure – n-dimensional simplex. Tetrahedral is the final simplex that we can represent in our three-dimensional space. The simplex of higher order have faces taken from these tetrahedron. The ribs of the tetrahedral indicate to various probabilities. For example, the rib linking the points 80 alive cats and 40 dead cats point out at 80/120=2/3 of probability of case in that cat is alive. In the case 60 alive and 20 dead cats the rib of the simplex shows the probability that is equal 60/80=3/4 and etc. The rib linking the points 20 dead and 40 dead cat and the rib linking the points 80 alive and 60 alive cats point out to probability of 1. Let's consider the faces of the simplex. In the case of alive cat on one of them the probability changes from 2/3 to 0,8 ; on another face – from 3/4 to 0,6; on third - from 2/3 up to 0,6; on fourth – from 3/4 to 0,8 etc.

4.The directing cosines and the probabilities.

From the quantum mechanics [7] it is known the following: Let's assume that e is the energy eigenvalue operator $\hat{E}$.

$$e = \Sigma C_k e_k$$

where e_k are eigenvectors (the basis),

C_k are coefficients and $\Sigma C_k e_k = 1$.

$$(e_k , e) = | e_k | * | e | * \cos \alpha$$

where $\cos^2\alpha$ is the corresponding to e_k part of e.

From the physical point of view this means that if we measure the energy of the identical systems that are in the *e* state then the part of the number of E_k energy measurements equals cosα (more exactly it equals |cosα |). Therefore |cosα | is called directing cosine and it is the probability that the system have E_k energy in the state *e*.

It is known that for many operators cosα is the complex number

$$\cos\alpha = x+iy = Ae^{i\beta}$$

where *A*=const is the length of the vector cosα.

Then $\cos^2\alpha = x^2 + y^2$.

Using the mentioned simplex, the above can be imagined and presented as the following.

Let's assume that e_1 and e_2 are eigenvectors.

E_1 and E_2 are the eigenvalue energy E (E_1 is energy of decayed atom, E_2 is energy of normal atom).

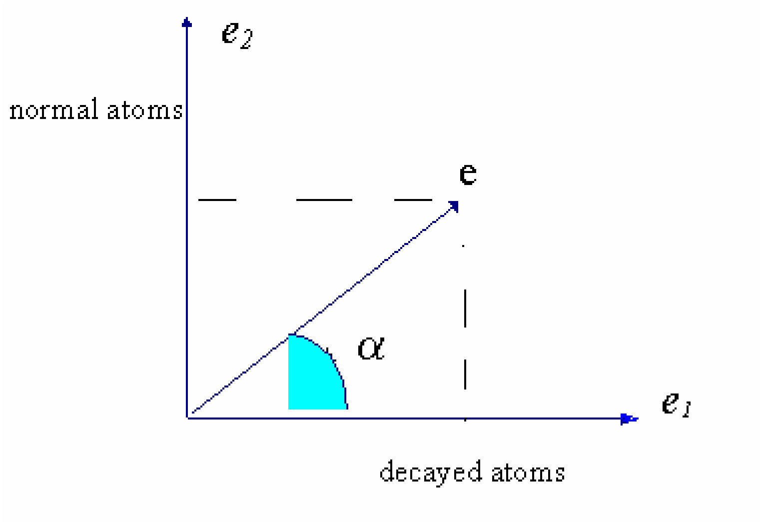

Fig.2

The information about an amounts of atoms with energy E_1 and E_2 are given us on the axis e_1, e_2, respectively.

The exact information about the amounts of normal or decayed atoms are presented on the e_1 and e_2 axes.

The statement of physical system and uncertainty of information are presented by vector *e* and angle α that is formed by rotation of *e* on the plane e_1oe_2, respectively (fig.2). Note that in [14] paper the statements of physical system are pictured by unit rays. The rotation of unit vector was considered by Orlov [8] and called by him "the intention" of the quantum system. Thus angle α is the uncertainty measure.

The vector *e* is non-eigenvector and one may be written in the form superposition of e_1 and e_2 .

$$e = C_1e_1 + C_2e_2$$

The projections of *e* on e_1 and e_2 axes are the number of decayed and normal atoms, respectively and cosα is the part of decayed atoms and sinα (sinα=cos(90-α)) is part of normal of total number of all atoms. Let's assume that e_1 is a real axis and e_2 is a imaginary one of a complex number. When constricting the simplex let's consider the case which we used at constricting the simplex: from 100 "schrodinger cats" (c=a+b=100) 80 cats are alive (a=80) and 20 cats are dead (b=20) (fig.3). It is clear that probabilities are equal to:

$$p_1 = \frac{a}{a+b} \quad \text{(cat is alive)}, \qquad p_2 = \frac{b}{a+b} \quad \text{(cat is dead).}$$

It is the main point that $p_1 + p_2 = 1$.
But it is possible take also other measure p^* for which the equality $\Sigma\, p^* = 1$ will be held true.
It is known that $\sin^2\alpha + \cos^2\alpha = 1$.
Assume $\cos^2\alpha = p^*_1$ and $\sin^2\alpha = p^*_2$.
Here p^*_1 and p^*_2 are other measures which we will call “the new probabilities”. From fig.3 it is clear that

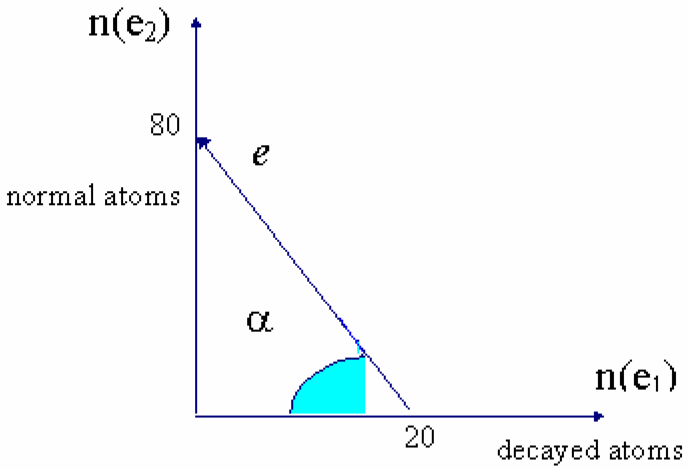

Fig.2

$$p_1^* = \frac{a^2}{a^2+b^2} = \frac{a}{a+ib} \times \frac{a}{a-ib} \quad \text{(cat is alive)}$$
$$p_2^* = \frac{b^2}{a^2+b^2} = \frac{ib}{a+ib} \times \frac{-ib}{a-ib} \quad \text{(cat is dead).}$$

In quantum mechanics the wave function ψ is interpreted as the following. The square of wave function amplitude $|\psi|^2$ is the probability p that the particle is in state E. We suppose that in our consideration $|\psi|^2$ isn’t the probability p, but that is ”the new probability p^* .” Thus:

$|\psi|_1^2 = \frac{a}{a+ib} \times \frac{a}{a-ib}$. Then $\psi_1 = \frac{a}{a+ib} = \cos\alpha\, e^{-i\alpha}$ and $\psi_1^* = \frac{a}{a-ib} = \cos\alpha\, e^{i\alpha}$

$|\psi|_2^2 = \frac{bi}{a+bi} \times \frac{-bi}{a-bi}$. Then $\psi_2 = \frac{bi}{a+bi} = \sin\alpha\, e^{-i\alpha}$ and $\psi_2^* = \frac{-bi}{a-bi} = -\sin\alpha\, e^{i\alpha}$

So we suppose that in quantum mechanics the directional cosines can be presented not in the form $\cos\alpha = a + ib$, but as $\cos\alpha = \frac{a}{a+ib}$. Taking $\cos\alpha = a + ib$ and $\cos^2\alpha = a^2 + b^2$, we can’t see that $\cos^2\alpha$ is the part of the total amount, on the contrary taking $\cos\alpha = \frac{a}{a+ib}$ it can be seen that $\cos^2\alpha$ ($\cos^2\alpha$ =

$\frac{a^2}{a^2+b^2}$) is the part of the total amount .So the rubs of our simplex accordingly point on the different statement of physical system and probabilities.

5. The Schrodinger's equation and new wave equation.

It is known that Schrodinger equation

$$\frac{h^2}{2m}\Delta\psi - E\psi = 0 \quad (1)$$

(the solution of which is $\psi = Ae^{-\frac{i}{h}(Et-px)}$) cannot be derived and it was obtained intuitively in order to explain strange properties of the microscopic world. In this paper it has been shown that Schrodinger equation has been derived by using the above mentioned geometrical interpretation of quantum mechanics.

As it was shown in section 3,

$$\psi = \cos\alpha\ e^{-i\alpha} \quad (2)$$

Then it is easy to derive the following equation: $\Delta'\psi + 4\psi = 2$. Here $\Delta' = \frac{\partial^2}{\partial^2\alpha}$. However comparing $\psi = Ae^{-\frac{i}{h}(Et-px)}$ with $\psi = \cos\alpha\ e^{-i\alpha}$ we can write

$$\alpha = \frac{Et-px}{h} \quad (3).$$

In the stationary case $\alpha=\alpha(x)$. Then $\Delta = \frac{\partial^2}{\partial\alpha^2} = \frac{h^2}{p^2}\frac{\partial^2}{\partial x^2}$. But $p^2=m^2v^2=2m\frac{mv^2}{2}=2mE$. Therefore $\Delta = \frac{\partial^2}{\partial\alpha^2} = \frac{h^2}{2mE}\frac{\partial^2}{\partial x^2}$. Thus $\frac{h^2}{2mE}\frac{\partial^2}{\partial x^2}\psi + 4\psi = 2$ or

$$\frac{h^2}{2m}\Delta\psi + 4\ E\psi = 2 \quad (4)$$

In order to solve this non-uniform differential equation we should solve the corresponding uniform equation

$$\frac{h^2}{2m}\Delta\psi + 4\ E\psi = 0 \quad (5)$$

which is Schredinger equation (Eq.1) , if 4 factor substitutes for -1. From the theory of differential equations it is known that the general solution of Eq.4 is equal to the solution of Eq.5 plus one partial solution of Eq.4 which, taking into account the expression (3), is in given case $\cos\alpha\ e^{-i\alpha} = \cos\frac{Et-px}{h}\ \exp(-i\frac{Et-px}{h})$. Thus the general solution of Eq.4 has the form of $\psi = A\exp(-i\frac{Et-px}{h}) + \cos\frac{Et-px}{h}\ \exp(-i\frac{Et-px}{h})$.

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