THE NONLINEAR THEORY OF GUN'S EFFECT

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The nonlinear theory of Gun's effect has been constructed. The amplitude and the frequency of the current oscillation in the second approximation by the method of Bogolubov-Mitropolski have been found. The amplitude and frequency of the current oscillation in the GaAs have been calculated for the typical experiment. The amplitude of the current oscillation as the function of the electric field has been calculated in the first approximation.

The phenomena of the current instability, which firstly was observed in the compounds GaAs and InP [1] are called Gun's effect. The frequency of the current installation mainly lies in the ultrahigh frequency range (UHF) and differs from the low-frequency oscillation [2]. The mechanism of Gun's effect was supposed in the refs. [3,4,5]. The authors of these works show, that in the materials GaAs, InP, probably, the number of the carriers with the low energy decreases at the increase of the external electric field strength. Moreover, the electrons transit to the superincumbent energetical bands or in the case of the low-frequency ranges they transit on the ionized traps in the forbidden band.

The mobility of the electrons decreases strongly with the comparison of the mobility in the lower trough at the electron transfer in the upper energetical bands. When transfer of the current carriers is higher, than the definite value, the material conductivity decreases and the negative differential conductance appears:

$$\frac{dI}{dE} = \sigma_d < 0 \qquad (1)$$

At the values $\sigma_d < 0$ the electric field E in the crystal becomes nonhomogineous and the acute regions of the electric field, i.e. "domains" creat. The theoretical investigations are strong nonlinear because of the domains and the amplitude of the current oscillation in the crystal depends on the time.

Moreover, at $\sigma_d < 0$ the redistribution of the space charge begins and this process leads to the UHF radiation. The small-signal theory (i.e. near the threshold $\sigma_d \approx 0$) of this phenomenon have been constructed in the ref. [6]. However, the dependence of the amplitude on the time and on the frequency of the oscillations hadn't been investigated theoretical.

In our paper we will declare the nonlinear theory of the amplitude and frequency of the current oscillation, based on the model of Gun's effect at the existence of the external constant electric field. We will consider, that transport time in the low-mobile states is neglible small and the carrier concentration in the main vale decreases inversely of the some degree of the electric field. Besides it, we will suppose, that the number of the ionized donors doesn't almost change and the current oscillations are only because of the electron transition in the upper energetical states. As it is shown in the ref. [6], the mobility in the upper energetical state in GaAs is more less, than in the central vale, D is the electron diffusion coefficient in the lower trough for GaAs 130 cm/sec.. The total carrier concentration $N=n+n'$, the carrier mobility μ and μ' correspondingly, diffusion coefficients D and D' satisfy to the following relations:

$$D >> D', \quad \mu >> \mu', \quad n >> n' \quad . \qquad (2)$$

The number of the carriers in the lower (central) vale will be equal to the some part of f of the total number of the carriers:

$$n = f\,N = f(E)N \qquad (3)$$

parameter $f(E) = (m-1)\left[m - 1 + \left(\frac{E}{E_a}\right)^m\right]^{-1}$ [7].

Here E_a is the electric field, at which the current oscillation begins. The parameter m is calculated experimentally as the ratio of the ohmic current to the real current in the point $E_0 = E_a$ (at $\sigma_d = 0$).

The static current is

$$I_0 = \sigma_0 E_a f_0 x_0, \quad x_0 = \frac{E_0}{E_a} \ , \qquad (4)$$

where σ_0 is the sample conductivity in the weak field. We will follow on the experiment [7] in calculations. The ration of the dynamic conductivity σ to the conductivity in the weak field σ_0 has the form:

$$S_0 = \frac{\sigma}{\sigma_0} = \frac{1}{\sigma_0 E_a}\frac{dI_0}{dx_a} = (f_0 + x_0\frac{df_0}{dx_a}) \qquad (5)$$

$$S_0 = f_0\left[1 - m(1 - f_0)\right]$$

The value S_0 is the ratio of the tangent of inclination angle of the VAC curve at $E = E_0$ (work point) to the tangent of inclination angle at the weak fields. At $E_0 > E_a$ it is negative. In the point of the zero inclination x_0=1 and $f_0 = -\frac{df_0}{dx_0}$. The statistic current in this point is:

$$I_p = \frac{m-1}{m}\sigma_0 E_a \quad \text{and} \quad m = \frac{1}{1 - \dfrac{I_p}{\sigma_0 E_a}} \ .$$

The dynamics of the current transition through the sample is characterized by the following equations:

$$I = e\,f\,N\,\mu E + De\frac{\partial(f\,N)}{\partial z}$$
$$\frac{\partial I}{\partial z} = e\frac{\partial N}{\partial t} \qquad (6)$$
$$I_1 + \varepsilon\frac{\partial E_1}{\partial t} = 0; \qquad \nu_0 = -\mu E_0 .$$

The equation system (6) is enough for our goals, i.e. they connect between themselves the three unknown values *I*, *E*, *N*.

In the frameworks of "small-signal" theory, by method [7] for the solution of these equations it was supposed:

$$I_0 = I_0 + I_1 = I_0 + I_1(0)e^{iwt}, \qquad I_1 << I_0$$
$$E = E_0 + E_1 = E = E_0 + E_1(0)e^{iwt}, \quad E_1 << E_0$$
$$N = N_0 + N_1 = N_0 + N_1(0)e^{iwt}, \qquad N_1 << N_0 \qquad (7)$$

For the finding of the amplitude values *E*, *N*, *I* the condition (7) isn't enough and it is need to find the solutions (6) at the any values E_1, N_1 and I_1, i.e.

$$\begin{gathered} I = I_0 + I_1,\ I_1 < I_0,\ I_1 > I_0,\quad N = N_0 + N_1,\ N_1 < N_0,\ N_1 > N_0, \\ E = E_0 + E_1,\ E_1 < E_0,\ E_1 > E_0 \end{gathered} . \qquad (8)$$

We will find the solutions of the equation systems (6) at the conditions (8) by the Bogolubov- Mitropolski method [7]. Substituting (8) into (6) and denoting $x = \frac{I_1}{\sigma_0 E_0}$ we obtain:

$$\begin{gathered} x = \left(f_0\frac{E_1}{E_0} + f_0\frac{N_1}{N_0} + f_0\frac{N_1}{N_0}\cdot\frac{E_1}{E_0} + f_1 + f_1\frac{E_1}{E_0} + f_1\frac{N_1}{N_0} + f_1\frac{N_1E_1}{N_0E_0}\right) + \\ + \frac{D}{\nu_0}\frac{\nu}{\partial z}\left(f_0\frac{N_1}{N_0} + f_1 + f_1\frac{N_1}{N_0}\right) \end{gathered} . \qquad (9)$$

$$I_1 = -\varepsilon\frac{\partial E_1}{\partial t}\ ; \quad \frac{\partial I_1}{\partial z} = e\frac{\partial N_1}{\partial t}\ ; \quad f_1 = (S_0 - f_0)\frac{E_1}{E_0}$$

We will express the change of the electric field through the change of the current carriers for the reducing the equation (9) to one nonlinear equation by the following way:

$$E_1 = \frac{E_0}{N_0 k\nu_0}\frac{\partial N_1}{\partial t} \quad \text{and then} \quad I_1 = -\frac{\varepsilon}{N_0 k\mu}\frac{\partial^2 N_1}{\partial t^2} \qquad (10),$$

where *k* is the wave vector, propagating on *Z* axis. If we denote $y = \frac{N_1}{N_0}$, so taking into consideration (10) from (9), we will obtain the following nonlinear equation for *y* for finding:

$$\begin{gathered} \frac{\partial^2 y}{\partial t^2} + \omega_0^2 y = \frac{\sigma_0 f_0}{\varepsilon}\left[-\frac{\partial y}{\partial t} - y\frac{\partial y}{\partial t} + m(1-f_0)\frac{1}{k\nu_0}\left(\frac{\partial y}{\partial n}\right)^2 + m(1-f_0)y\frac{\partial y}{\partial n} + \right. \\ \left. + m(1-f_0)y\frac{1}{k\nu_0}\left(\frac{\partial y}{\partial n}\right)^2\right] + f_0\frac{m(1-f_0)\sigma_0 Dk}{\nu_0}\left(\frac{\partial y}{\partial t} + y\frac{\partial y}{\partial t}\right) \end{gathered} . \qquad (11)$$

Here $\omega_0^2 = \frac{\sigma_0 f_0}{\varepsilon}(k\,\nu_0 + Dk^2)$.

The nonlinear equation (11) respect of *y* can be reduced to the equation of Van-der-Pole one. For this we denote $\tau = \omega_0 t$ and $r = \frac{\sigma_0 f_0}{\varepsilon\,\omega_0}$ and reduce the equation (11) to the dimensionless equation. Then the Van-der-Pole equation assumes the following form:

$$\frac{\partial^2 y}{\partial\tau^2} + y = rF\left(\frac{dy}{d\tau}, y\right), \qquad (12)$$

where $r = \frac{\sigma_0 f_0}{\varepsilon\omega_0}$ is the small parameter for the typical crystal GaAs, $D = 130\frac{cm^2}{sec}$, $\nu_0 \approx 10^7\frac{cm}{sec}$, $\omega_c \approx \frac{\sigma_0}{\varepsilon} \approx 10^{12}\ sec^{-1}$, $r << 1$.

The solution of the equation (12) is as follows

$y = a\cos(\tau+\theta) = a\cos(\omega t+\theta) = a\cos\psi$ at r=0. (13)

We will use Bogolubov-Mitropolski method [7] for the finding of the solution (12) at $r\neq 0$ by the following way:

$$\frac{da}{d\tau} = rA_1(a) + r^2A_2(a) + r^3A_3(a) + ...$$
$$\frac{d\psi}{d\tau} = \omega_0 + rB_1(a) + r^2B_2(a) + r^3B_3(a) + ... \quad (14)$$

wher $A_1(a) = -\frac{\omega_0}{2\pi}\int_0^{2\pi} F\left(y, \frac{dy}{d\tau}\right)\sin\psi\, d\psi$; $B_1(a) = -\frac{\omega_0}{2\pi a}\int_0^{2\pi} F\left(y, \frac{dy}{d\tau}\right)\cos\psi\, d\psi$

$$A_2(a) = -\frac{\omega_0}{2}\left(2A_1B_1 + A_2\frac{dB_1}{da}a\right) - \frac{\omega_0}{2\pi}\int_0^{2\pi}\left[u_1\frac{dF}{dY} + \left(A_1\cos\psi - aB_1\sin\psi + \omega_0\frac{du_1}{d\psi}\right)\frac{dF}{dy'}\right]\sin\psi\, d\psi$$

$$B_2(a) = -\frac{\omega_0}{2}\left(B_1^2 - \frac{A_1}{a}\frac{dA_1}{da}\right) - \frac{\omega_0}{2\pi a}\int_0^{2\pi}\left[u_1\frac{dF}{dy} + \left(A_1\cos\psi - aB_1\sin\psi + \omega_0\frac{du_1}{d\psi}\right)\frac{dF}{dy'}\right]\cos\psi\, d\psi$$

$$u_1 = g_0 - \sum_{n=2}^{\infty}\frac{g_n\cos n\psi + h_n\sin n\psi}{n^2-1} \ ; \quad g_n = \frac{1}{\pi}\int_0^{2\pi} F\left(y, \frac{dy}{d\tau}\right)\cos n\psi d\psi \quad . \quad (15)$$

$$h_n = \frac{1}{\pi}\int_0^{2\pi} F\left(y, \frac{dy}{d\tau}\right)\sin n\psi d\psi$$

We confine ourselves only by the second approximation. Firstly, we will find the amplitude in the first approximation from equation (14):

$$\frac{da_1}{d\tau} = rA_1(a)\,. \quad (16)$$

Substituting the equation (13) in the equation (15) after the integration, we will obtain

$$\frac{da_1}{dt} = \frac{r\omega_0}{2}\left[\frac{m(1-f_0)Dk\varepsilon}{\nu_0} - 1\right]a :$$

Solving above mentioned equation we find:

$$a_1 = a_0 e^{\frac{r}{2}\left[\frac{m(1-f_0)Dk_0\varepsilon}{\nu_0} - 1\right]t\omega_0} ; \quad y = a_1\cos\psi\,. \quad (17)$$

From the equation (17) it is seen, that the amplitude of the current oscillation increases exponentially for the crystal GaAs:

$$\frac{mDk\varepsilon}{\nu_0}(1-f_0) > 1 \quad (18)$$

As $f_0 < 1$, so we obtain the values of the change interval of the constant electric field:

$$E_0 > E_a\left(\frac{2\nu_0}{3Dk\varepsilon}\right)^{1/3} \quad (19),$$

at which the current oscillates with the frequency ω_0. Indeed, it is clearly seen from the equation (15), that $B_1(a) = 0$ and that's why the frequency is ω_0. As it is seen from the equation (17) the amplitude a_1 tends to the initial value a_0, $(a_1 \to a_0)$ at $t \to 0$.

It is need to calculate the $u_1, \frac{\partial\psi_1}{\partial\psi}, g_0$ in the second approximation for the finding of the amplitude a_2 and the frequency of the current oscillation ω_2.

After the simple calculation we find:

$$u_1 = 3R\left(\frac{\cos 2\psi}{6} - \frac{Dk^2\varepsilon}{6\omega_0}\sin 2\psi - 1\right); \quad \frac{du_1}{d\psi} = -R\left(\frac{Dk^2\varepsilon}{\omega_0}\cos 2\psi + \sin 2\psi\right)$$
$$R = \frac{m(1-f_0)}{3k\nu_0\omega_0}a^2 \ ; \quad g_0 = -\frac{m(1-f_0)\omega_0}{k\nu_0}a^2 \quad (20)$$

Substituting the equation (20) in the equation (15), we obtain the following values for $A_2(a)$ and $B_2(a)$ after the integration:

$$A_2(a) = -\frac{\omega_0 a^3}{8}\left[\frac{Dk^2\varepsilon}{\nu_0}\left(\frac{mDk\varepsilon}{\nu_0} + m\right)\frac{m\omega_0}{6k\nu_0} + \frac{2m\omega_0}{6k\nu_0}\right] = -La^3 .$$

$$B_2(a) = \frac{\omega_0}{8}\left[\left(\frac{mDk\varepsilon}{\nu_0}\right)^2 - 26\left(\frac{mDk\varepsilon}{\nu_0} + m\right)\frac{\omega_0 ma^2}{6k\nu_0} - \right. \tag{21}$$

$$\left. -4Dk^2\varepsilon\frac{ma^2}{6k\nu_0}\left(\frac{2m\omega_0}{k\nu_0} + m + \frac{mDk\varepsilon}{\nu_0}\right)\right] = B_2(0) + B_2' a^2$$

Substituting the equation (21) in the equation (14), we obtain the equations for the finding of the oscillation frequency in the second approximation:

$$\frac{da}{dt} = rA_1(0)a + r^2 La^3\omega_0 \tag{22}$$

$$\frac{d\psi}{dt} = \omega_{//} = \omega_0 + r^2 B_2(0) + B_2' r^2 a^2 \tag{23}$$

$$\frac{da}{dt} = \omega_0\left[rA_1(0)a + r^2 La^3\right] \tag{24}$$

$$\frac{d\psi}{dt} = \omega_2 = \omega_0\left[1 + r^2 B_2(0) + B_2' r^2 a^2\right] \tag{25}$$

where $A_1(0) = \frac{m(1-f_0)Dk\varepsilon}{\nu_0} - 1$; $B_2(0) = \frac{1}{8}\left(\frac{mDk\varepsilon}{\nu_0}\right)^2$

$$B_2' = \frac{13m\omega_0}{24k\nu_0}\left(1 - m - \frac{mDk\varepsilon}{\nu_0}\right) - \frac{4Dk^1}{12\nu_0}m^2\left(\frac{2\omega_0}{k\nu_0} + \frac{Dk\varepsilon}{\nu_0}\right)$$

$$L = \frac{m\omega_0}{48k\nu_0}\frac{Dk\varepsilon}{\nu_0}\left(1 - m - \frac{mDk\varepsilon}{\nu_0}\right) - \frac{m\omega_0}{24k\nu_0}$$

After the integration we obtain from the equation (22):

$$a_2 = \frac{a_0}{\left[a_0^2\frac{L}{A_1(0)}\left(e^{-\frac{rA_1(0)}{2}\omega_0 t} - 1\right) + e^{-\frac{rA_1(0)}{2}\omega_0 t}\right]^{1/2}} = \frac{a_0}{\Phi^{1/2}}; \quad y = a_2\cos\psi \tag{26}$$

As $f_0 < 1$, so for the typical GaAs, $A_1(0) = \frac{mDk\varepsilon}{\nu_0}$ and

$$L = -\frac{m^2\omega_0}{48k\nu_0}\left(\frac{Dk\varepsilon}{\nu_0}\right)^2 - \frac{m\omega_0}{2k\nu_0} \approx -\frac{m^2\omega_0}{48k\nu_0}\left(\frac{Dk\varepsilon}{\nu_0}\right)^2$$

$$\frac{L}{A_1(0)} = -\frac{m\omega_0}{48k\nu_0}\frac{Dk\varepsilon}{\nu_0} << 1 \tag{27}$$

$$\omega_0\frac{rA_1(0)}{2} \approx \frac{mDk}{2\nu_0}\frac{\sigma_0 f_0}{\varepsilon\omega_0}\omega_0 \approx \frac{m}{2}\frac{Dk}{\nu_0}\sigma_0 f_0 t >> 1$$

.

Taking into consideration the experiment conditions [7] and (27), which has been proved in the experiment for the typical GaAs from the equation (26), we obtain:

$$a_2^2 = \frac{a_0^2}{e^{-\frac{rA_1(0)}{2}\omega_0 t} - \frac{a_0^2 L}{A_1(0)}} \quad \text{or}$$

$$a_2 = \frac{a_0}{\left[e^{-\frac{mDk\sigma_0 f_0}{2\nu_0}t} + \frac{m\omega_0}{48k\nu_0}\frac{Dk\varepsilon}{\nu_0}a_0^2\right]^{1/2}} \tag{28}$$

It is easy seen from the equation (28), that amplitude in the second approximation decreases and tends to the constant limit (i.e. to the value, not depending on time).

$$a_2 \to \left(\frac{48k\nu_0}{m\omega_0}\frac{\nu_0}{Dk\varepsilon}\right)^{1/2} \tag{29}$$

The frequency of the current oscillation is defined by the expression (23), i.e. $\frac{d\psi}{dt} = \omega_2$ and the expression phase low is $\psi = \omega_2 t$. It is possible to find the amplitudes of the current oscillation on the formula (10), substituting *a* from the equations (17) and (26) in the equation (10) serially in the first and second approximations. We will calculate the current amplitudes only in the first approximation because of the striving of the amplitude of the current oscillation to the constant limit in the second approximation (29). After the simple calculation we obtain:

$$J_1 = \sigma_0 E_0 f_0 (1 + v_0) e^{\beta\omega_0 t}\left[2\beta \sin\psi - (\beta^2 - 1)\cos\psi\right]$$
$$x = \frac{J_1}{\sigma_0 E_0} = f_0\left(1 + \frac{Dk}{v_0}\right)\left[2\beta \sin\psi - (\beta^2 - 1)\cos\psi\right] e^{\beta\omega_0 t} \qquad (30)$$

From the equation (30) it is seen, that oscillations of the current amplitude x have the oscillating form as the functions $\sin\psi$ and $\cos\psi$ in dependence on time. Using the equation (30) it is possible to find the experimental values x (i.e. $\frac{\partial x}{\partial t} = 0$) with the time and the maximum conditions for x i.e. $\frac{\partial^2 x}{\partial t^2} < 0$. However, for the finding $\bar{J}_1$ it is need to find $\bar{J}_1^2$, since the harmonic functions $\sin\psi$ and $\cos\psi$ are in the equation (30), and further it is need to find the values $\bar{J}_1 = \bar{J}_{1\min}$ and $\bar{J}_1 = \bar{J}_{1\max}$, which define the minimal and maximal values of the amplitude of the current oscillation.

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QANN EFFEKTİNİN QEYRİ-XƏTTİ NƏZƏRİYYƏSİ

Boqolyubov-Mitropolski riyazi metodu ilə GaAs kristalında cərəyan rəqslərinin amplitudu və tezliyi nəzəri olaraq hesablanmışdır. Birinci yaxınlaşmada cərəyan rəqslərinin amplitudu elektrik sahəsindən asılı olaraq hesablanmışdır.

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НЕЛИНЕЙНАЯ ТЕОРИЯ ЭФФЕКТА ГАННА

Построена нелинейная теория эффекта Ганна. Найдены амплитуда и частота колебания тока во втором приближении методом Боголюбова-Митропольского. Для типичного эксперимента в кристалле GaAs вычислены частота и амплитуда колебания тока. Вычислена в первом приближении амплитуда колебания тока как функция электрического поля.

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