COLLECTİVE 1^+ STATES IN $^{176,178}Hf$ DEFORMED NUCLEI

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In this study, properties of the collective $I^\pi = 1^+$ (K=0 and K=1) states, generated by the residual paired and isovector spin-spin interactions, in the deformed nuclei ^{176,178}Hf are investigated in the Quasiparticle Random Phase Approximation (QRPA) method using the deformed Saxon-Woods Potential. Furthermore, contribution of *M1* transition matrix element to the energy weighted sum rules and energy distribution of *M1* excitation strength functions were investigated for these states. We have observed that the low-lying 1^+ states have weak correlations and small value of B(M1, $0 \rightarrow 1$), and the collective 1^+ states with larger values of B(M1) are in the energy region 8-10 MeV for K = 0 and 10-12 MeV for K = 1. We have also shown that the number of states for K = 0 is less than K = 1 states.

INTRODUCTION

Magnetic dipole excitations in heavy nuclei are of considerable interest in modern nuclear structure physics. Both low lying orbital and high lying magnetic resonance excitations were studied systematically to test nuclear models. An outstanding example is the so-called M1 scissors mode to which most attention was paid in the past. This low-lying orbital 1^+ states were first observed in ^{156}Gd in high resolution inelastic (e, e') reactions in 1984 [1]. Two of the important properties of these states are their low energy ($\omega \leq 4$ MeV) and having a value for their reduced magnetic dipole probability B($0^+ \rightarrow 1^+$) in the interval 0.6-1.8 μ_N^2. The different aspects of the magnetic dipole transition of 1^+ states have been investigation in inelastic proton scattering at small angles for the nuclei ^{154}Sm, ^{156}Gd and ^{164}Dy [2]. Before these experimental studies, existence of a state with the energy E=3.18 MeV and B(M1,$0^+ \rightarrow 1^+$)=1.8 μ_N^2 in the nucleus ^{168}Er was observed in theoretical calculations using quasiparticle model in the framework of RPA and taking into account the residual paired and spin-spin interactions [3]. Calculations have shown that collective states in the energy range 2-4 MeV and occur in the nuclei ^{154}Sm and ^{168}Er [4]. These states expected by the theory were experimentally studied using (γ, γ') resonance scattering reactions and a state with E=3.39 MeV and B(M1)=$(0.71 \pm 0.08)\,\mu_N^2$ was observed in the nucleus ^{168}Er [5].

Recently, magnetic dipole resonances ($I^\pi = 1^+$) have been experimentally found in wide region from light spherical nuclei up to actinides [6-11]. These experiments shown that a very broad M1 resonance at energies between 7 and 11 MeV exits in heavy spherical and deformed nuclei.

Spin-spin interactions in the spherical even-even nuclei happen in connection with the particle hole transitions between the spin-orbital elements of the neutron-neutron and proton-proton single particle states [12]. Unlike the potential of spherical nuclei, the picture is more complicated. In such case, the magnetic quantum number of every J-shell splits into excitations characterized by the angular momentum projection on the symmetry axis K=0 and K=1. Due to split mentioned, shell structure of the nucleus is destroyed and this causes an increase in the density of 1^+ states ($\rho \approx 10$ MeV^{-1}). Based on the information given, two independent branches of 1^+ states, namely K=0 and K=1, exist due to axial symmetry in deformed nuclei. K=0 corresponds to the spin vibrations along with the symmetry axis, and K=1 corresponds to the perpendicular ones. K=1 branch is easy to investigate in photon scattering experiments since electromagnetic waves are transverse. Results of such experiments have shown that in low energy spectroscopic region K=1 states are denser compare to K=0 levels.

The well-known Random Phase Approximation (RPA) is one the most popular method in theoretical microscopic study of nuclear structure and describes many-body systems. In nuclear physics RPA has been exploited to model properties of the excited states that allows to calculate intensities of various nuclear reactions, including decay probability of electromagnetic, beta and double beta decay. For the nuclei away from closed shells there appear static pairing correlations within the quasi-particle representation is usually referred to as quasi-particle version of RPA(QRPA), which consider the quasi-particle correlations and excitations.

In this study, properties of $I^\pi = 1^+$ (K=0 and K=1) spin-vibration states in the even-even ^{176,178}Hf isotopes were studied using QRPA method. Contribution of M1 transition matrix element to the energy weighted sum rules and energy distribution of M1 excitation strength functions were investigated for these states.

THEORY

For a system with large number of particles, like nucleus, it is difficult to solve the Schrödinger equation due to large number of the degrees of freedom. Therefore, in the microscopic model, it is assumed that neutron and proton in the nucleus move in a common field produced by them and assumed that they interact with each other. It is based on the Shell model. In this model the component of the effective force responsible for the excitations is taken into account in the microscopic calculations.

Assuming that 1^+ states are produced by the spin-spin forces in deformed nuclei, Hamiltonian of the system can be chosen as

$$H = H_{sqp} + V_{\sigma\tau} \qquad (1)$$

where H_{sqp} is the single quasi-particle Hamiltonian represented by

$$H_{sqp} = \sum_{s\tau} \varepsilon_s(\tau)(\alpha_s^+(\tau)\alpha_s(\tau) + \alpha_{\tilde{s}}^+(\tau)\alpha_{\tilde{s}}(\tau)) \quad (2)$$

In this expression, $\varepsilon_s = \sqrt{(E_s - \lambda)^2 + \Delta^2}$ is quasi-particle energy of the nucleons, and E_s is energy of the average field. Δ and λ are the gap and chemical potential parameters of the super fluid model, respectively. $\alpha^+(\alpha)$ are the quasi-particle creation(annihilation) operators.

The second term in Eq. 1

$$V_{\sigma\tau} = \frac{1}{2}\chi_{\sigma\tau}\sum_{i\neq j}\sigma_i\sigma_j\tau_i^z\tau_j^z \quad (3)$$

represents the isovector spin-spin interactions. Here, σ and τ are spin and isotropic Pauli matrices, respectively. All the unexplained expressions used here are given in Ref. [13].

The isovector spin-spin interactions can be written in terms of particle operators:

$$V_{\sigma\tau} = \chi_{\sigma\tau}\sum_{\mu=0,\pm1} D_\mu^+ D_\mu \quad (4)$$

In case of $\chi_{np} = \chi_{pn} = q\chi$ and $\chi_{nn} = \chi_{pp} = \chi$ q=-1. In the quasi-particle representation the D_μ operator splits up into quasi-boson and scattering terms in the form

$$D_\mu(\tau) = \sum_{ss'}\sigma_{ss'}\left\{\mu_{ss'}B_{ss'} + \frac{1}{\sqrt{2}}L_{ss'}(C_{ss'}^+ + C_{ss'})\right\} + \sum_{ss'}\sigma_{s\tilde{s}'}\left\{\mu_{ss'}\bar{B}_{ss'} - \frac{1}{\sqrt{2}}L_{ss'}(\bar{C}_{ss'}^+ + \bar{C}_{ss'})\right\} \quad (5)$$

Here, $M_{ss'} = u_s u_{s'} + v_s v_{s'}$ and $L_{ss'} = u_s v_{s'} + u_{s'} v_s$ are the Bogoliubov canonical transformation parameters, expressed through u_s and v_s, $D_\mu^{(\nu)} = \langle s|D_\mu|s'\rangle$ are single-particle matrix elements of the Pauli spin operator, and

$$C_{ss'} = \frac{1}{\sqrt{2}}\sum_{\rho=\pm1}\rho\alpha_{s'\rho}\alpha_{s-\rho}\ ,\ C_{ss'}^+ = \frac{1}{\sqrt{2}}\sum_{\rho=\pm1}\rho\alpha_{s-\rho}^+\alpha_{s'\rho}^+ \quad (6)$$

$$B_{ss'} = \sum_{\rho=\pm1}\rho\alpha_{s\rho}^+\alpha_{s'\rho} \qquad B_{ss'}^+ = \sum_{\rho=\pm1}\rho\alpha_{s'\rho}^+\alpha_{s\rho} \quad (7)$$

are the quasi-particle operators. In RPA, collective 1^+ states are considered as one-phonon excitations given by

$$|\psi_i\rangle = Q_i^+|\psi_0\rangle = \frac{1}{\sqrt{2}}\left[\sum_{n\mu t.}\left\{\psi_{ss'}^i C_{ss'}^+ - \varphi_{ss'}^i C_{ss'}\right\} + \sum_{prot.}\left\{\psi_{vv'}^i C_{vv'}^+ - \varphi_{vv'}^i C_{vv'}\right\}\right]|\psi_0\rangle \quad (8)$$

where Q_i^+ is the phonon creation operator, $|\Psi_0\rangle$ is the phonon vacuum. The two quasi-particle amplitudes $\psi_{ss'}$ and $\varphi_{ss'}$ are normalized by

$$\sum_{\mu\tau}[\psi_\mu^{i\,2}(\tau) - \varphi_\mu^{i\,2}(\tau)] = 1 \quad (9)$$

The dispersion equation for the excitation frequency w is obtained in the form

$$(1+\chi F_n)(1+\chi F_p) - q^2\chi^2 F_n F_p = 0 \quad (10)$$

using a variational method and

$$\delta\left\{\langle\psi|Q_i H Q_i^+|\psi\rangle - \langle\psi|H|\psi\rangle - \omega_i\left(\sum_i(\psi_s^{i\,2} - \varphi_s^{i\,2}) - 1\right)\right\} = 0 \quad (11),$$

together with the RPA procedure. The roots of this equation gives the energy of 1^+ states.

MAGNETIC PROPERTIES OF THE COLLECTIVE 1^+ STATES

The characteristic quantity of the spin vibrational 1^+ states is the probability of the M1 transitions. The M1 transition operator is given by

$$\vec{M}_i = \langle\psi_i|\vec{\mu}|\psi_0\rangle \quad (12)$$

where $\vec{\mu}$ is the magnetic dipole operator expressed as

$$\vec{\mu} = \sum_{\tau,i}\left[(g_s^\tau - g_l^\tau)\vec{s}_i^\tau + g_l^\tau\vec{J}_i^\tau\right] \quad (13)$$

In this equation, J is the total angular momentum operator, g_s^τ ve g_l^τ are the orbital and spin g factors of the nucleons, respectively. Using Eqs. 8, 12 and 13, the M1 transition probability for the state 1^+ can be written as

$$B(M1,0\to 1^+)=\frac{3}{16\pi}\left[\sum_{n\mu t.}\mu^n_{ss'}L_{ss'}g_{ss'}+\sum_{prot.}\mu^p_{vv'}L_{vv'}g_{vv'}\right] \quad (14)$$

where $\mu^{(\tau)}_{ss'}$, $\hat{\mu}$ are the single-particle matrix elements of the magnetic dipole operator.

SUM RULE

In quantum mechanics, the probability of a transition from one state to another is bounded by some definite relations called sum rules. There are two kinds of sum rules; energy-weighted (EWSR) and non-energy-weighted sum rule (NEWSR). The sum rules are used in the microscopic nuclear theory in order to investigate the properties of collective excitations. These rules, in case of an arbitrary potential, allow one to calculate the vibration of the giant dipole and quadrupole resonant energy.

For the case of M1 transitions, the EWSR is written as

$$\frac{1}{4}\left\langle\psi_0\left|\left[\vec{\mu},\left[H,\vec{\mu}\right]\right]\right|\psi_0\right\rangle=\frac{8}{3}\pi\sum_i\omega_i B_i(M1,0^+\to 1^+) \quad (15)$$

and we are specifically interested in the energy region of saturation of the right-hand side of Eq. 15, namely the energy dependence of the function

$$\chi_n(\omega_i)=\frac{8}{3}\pi\sum_i\omega_i B_i(M1,0^+\to 1^+) \quad (16)$$

The left-hand side of Eq. 15 does not depend on the strength parameter χ, and represents the quasi-particle estimate of the sum rule. Thus, the sum rules help one make conclusions about the accuracy of RPA solutions, while the contribution of different 1^+ states to the sum rule is given by the function χ.

NUMERICAL CALCULATIONS AND DİSCUSSİON

In calculations, the single-particle model is used taking the deformed Saxon-Woods potential as the average field potential. The Schrodinger Eq. is solved by means of the method mentioned in Ref. [14]. Calculations are performed for ^{176,178}Hf isotopes, the deformation parameters and interaction constants are taken from Ref. [15] and Ref. [16], respectively. For this reason, throughout this study, the RPA method with harmonic approach is used. The isovector spin-spin interaction constant is chosen as $\chi_{\sigma\tau}=40/A$ [17].

Table 1.

Ton-pair correlation parameters and the deformation parameters for ^{176,178}Hf isotopes

A	N	δ^2	Δ_n	Δ_p	λ_n	λ_p
176	104	0.2731	0.655	0.75	-4.139	-6.098
178	106	0.2563	0.72	0.75	-3.664	-6.412

Calculations have shown that, small probability of the M1 transition from the ground state to the 1^+ excitation levels appear in the energy region up to 5 MeV. Information on the low energy 1^+ (K=0 and K=1) levels and the state structure of them for ^{176,178}Hf in the spectroscopic region is given in Table 2 and 3.

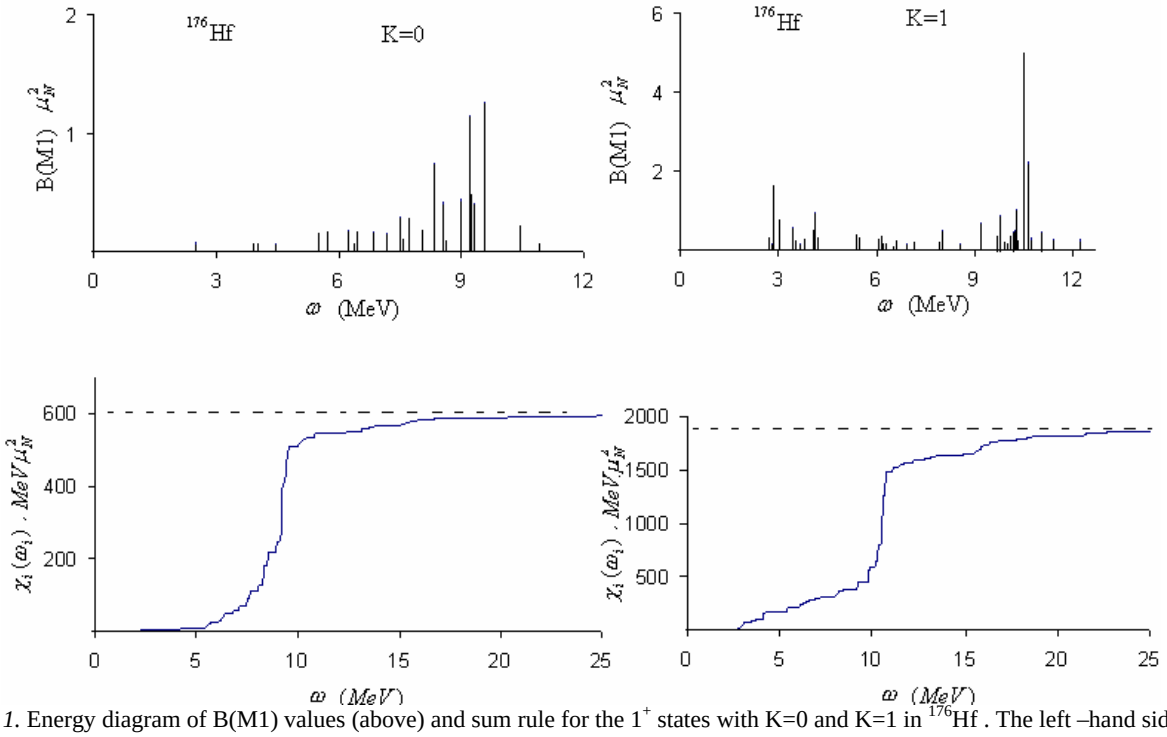

Fig. 1. Energy diagram of B(M1) values (above) and sum rule for the 1^+ states with K=0 and K=1 in ^{176}Hf . The left –hand side of the sum rule is shown by dotted line. The solid line corresponds to the function χ_i [Eq. 16].

Table 2.

The characteristic 1^+ (K=0 and K=1) states and the transition probability B(M1) for ^{176}Hf in the spectroscopic energy region.

^{176}Hf K=0			
Energy ω (MeV)	**BM1 (μ_N^2)**	**Amplitudes ψ_μ**	**Structure of states**
5.717	0.159	-0.187 0.280 0.215 0.135 0.553	nn512-503 nn512-752 pp530-550 pp541-532 pp411-402
8.332	0.726	0.364 -0.236 -0.165 0.146 0.238 0.122 -0.186 -0.329	nn640-631 nn642-633 pp301-530 pp530-521 pp541-521 pp532-523 pp523-514 pp413-404
8.544	0.399	0.278 -0.507 -0.132 -0.130 -0.263 -0.118	nn530-750 nn640-631 nn532-501 nn642-633 pp523-514 pp413-404
8.974	0.432	-0.529 -0.428	nn514-505 pp523-514
9.228	1.144	-0.415 -0.102 -0.346 0.162 -0.190 0.287	nn530-501 nn642-633 nn514-505 pp431-640 pp532-512 pp523-514
9.264	0.488	0.554 -0.191 0.279 -0.148 0.174	nn530-501 nn514-505 pp431-640 pp532-512 pp523-514
9.540	1.252	-0.131 0.638 0.165	Nn514-505 pp532-512 pp523-514

^{176}Hf K=1			
Energy ω (MeV)	**BM1 (μ_N^2)**	**Amplitudes ψ_μ**	**Structure of states**
2.509	2.519	0.705	pp523-514
3.199	0.208	-0.704	pp550-532
3.355	1.110	0.687	pp411-402
3.623	0.689	0.112 -0.149 0.658	nn660-631 nn631-642 pp413-404
4.417	0.722	-0.704	pp411-400
4.496	0.509	-0.704	pp411-402
5.299	0.470	0.128 0.184	nn622-633 pp530-550
6.530	0.435	0.183 -0.209 0.162 -0.240 0.221 0.132	nn651-651 nn301-523 nn613-624 nn505-505 pp530-532 pp532-523
9.775	1.602	0.319 0.109 0.369 0.173	pp521-541 pp532-514 pp422-404 pp503-514
9.981	0.527	0.224 -0.266 0.101 0.572	nn550-750 pp521-541 pp532-514 pp503-514
10.350	1.529	0.462 0.212 -0.102 0.300 -0.192 -0.120	pp530-521 pp550-512 pp521-541 pp541-512 pp532-514 pp503-514
10.600	4.028	0.178 -0.291 -0.107 0.113 -0.177 0.193 0.234 -0.144 -0.114	nn523-505 pp530-521 pp521-541 pp640-660 pp541-512 pp532-303 pp521-512 pp532-514 pp503-514
10.640	0.666	-0.113 -0.645 -0.131	nn550-501 pp532-303 pp521-512
10.784	0.854	-0.248 -0.613	nn523-505 pp640-660

Table 3.
The characteristic 1^+ (K=0 and K=1) states and the transition probability B(M1) for ^{178}Hf in the spectroscopic energy region.

^{178}Hf K=0			
Energy ω (MeV)	**BM1 (μ_N^2)**	**Amplitudes ψ_μ**	**Structure of states**
5.716	0.194	-0.141 -0.354 0.556 -0.135 0.122	nn512-503 pp530-550 pp420-411 pp541-532 pp411-402
6.341	0.415	-0.103 -0.148 -0.107 0.642	nn651-642 nn642-622 pp530-550 pp541-532
7.581	0.379	0.111 -0.164 0.124 0.367 -0.405 0.131 0.299	nn640-651 nn651-642 nn523-752 nn642-633 nn633-613 pp420-400 pp532-523
8.126	0.289	-0.137 -0.233 -0.182 -0.325 -0.114 0.480	nn640-631 nn642-633 pp530-521 pp541-521 pp523-514 pp413-404
8.277	0.435	-0.411 -0.180 -0.155 -0.177 -0.451	nn640-631 nn642-633 pp541-521 pp523-514 pp413-404
9.026	2.293	0.237 0.132 -0.132 -0.109 -0.163 0.124 -0.239 0.476 -0.104	nn530-501 nn640-631 nn532-761 nn651-642 nn642-633 pp532-523 pp532-512 pp523-514 pp413-404
9.363	1.018	0.651 0.157	pp532-512 pp523-514
9.746	0.116	0.551 0.433	nn514-505 pp550-510
9.756	0.160	0.420 -0.560	nn514-505 pp550-510

^{178}Hf K=1			
Energy ω (MeV)	**BM1 (μ_N^2)**	**Amplitudes ψ_μ**	**Structure of states**
8.032	0.469	0.111 -0.458 -0.131 -0.131 0.294 0.107 0.237	nn660-651 nn631-651 pp550-301 pp420-400 pp420-402 pp541-523 pp512-523
8.140	0.435	0.381 0.124 0.529	nn631-651 pp541-523 pp512-523
9.727	0.650	0.168 0.141 0.111 0.626	nn530-512 pp 521-541 pp431-651 pp422-404
9.820	1.780	0.516 0.137 -0.274	pp521-541 pp532-514 pp422-404
10.029	0.707	-0.152 0.583 -0.241 0.200	nn530-510 nn510-532 pp521-541 pp532-514
10.042	0.923	-0.421 -0.379 -0.247 0.256	nn530-510 nn510-532 pp521-541 pp532-514
10.058	0.909	0.543 -0.127 -0.217 0.298	nn530-510 nn510-532 pp521-541 pp532-514
10.254	1.856	-0.332 0.200 -0.139 -0.199 -0.403 0.135	pp301-521 pp530-521 pp521-541 pp411-422 pp532-514 pp503-514
10.461	0.404	-0.357 -0.293 -0.212 0.450	pp530-521 pp411-431 pp541-512 pp503-514
10.593	1.508	0.437 0.208 -0.192 -0.117 -0.307	nn642-402 nn503-514 pp530-521 pp541-512 pp503-514
10.604	0.933	-0.552 -0.142 0.200 -0.219	nn642-402 pp530-521 pp532-303 pp503-514

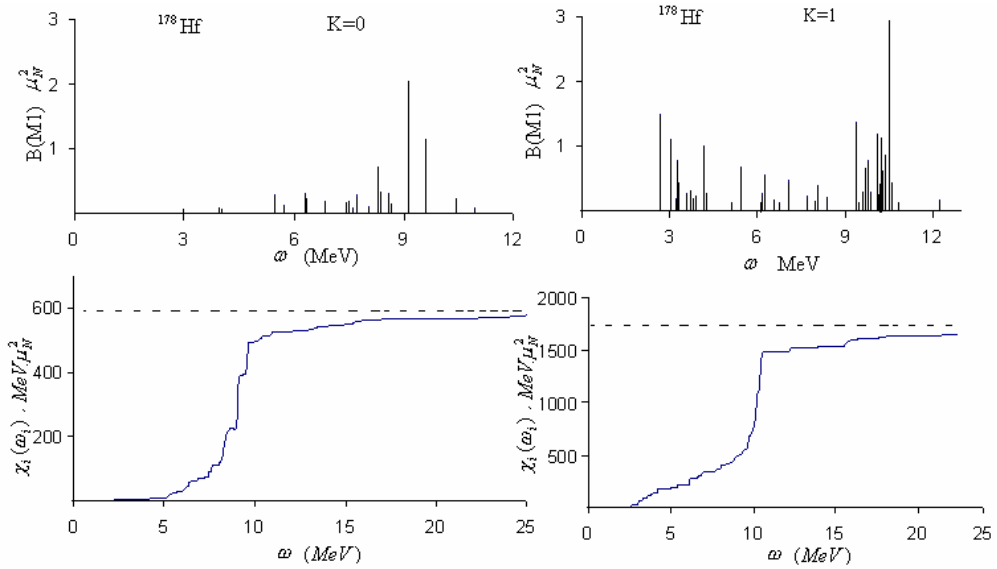

Fig. 2. Energy diagram of B(M1) values (above) and sum rule for the 1^+ states with K=0 and K=1 in ^{178}Hf . The left –hand side of the sum rule is shown by dotted line. The solid line corresponds to the function χ_i [Eq. 16].

Results of the calculations for ^{176,178}Hf nuclei are given in Figure 1 and 2. Probability of B(M1) transition as a function of energy for $I^\pi = 1^+$ (K=0 and K=1) excitations are shown in Fig. 1. As seen in the figure, there are several collective states with large transition probabilities $(B(M1) = (1.2-3.4)\mu_N^2$ exist for $\omega \le 4$ MeV for these isotopes. There are many states between 4 and 12 MeV and the most collective energy interval where these states are pached is 8-10 MeV for K=0 and 10-12 MeV for K=1.

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DEFORMASIYA OLMUŞ ^{176,178}Hf NÜVƏLƏRININ COLLEKTIV 1^+ SƏVIYYƏLƏRI

Məqalədə qalıq cütlənmə və izovektor spin-spin qarşılıqlı təsirləri tərəfindən yaradıldığı fərz edilən $I^\pi=1^+$ (K=0 və K=1) səviyyələrinin xüsusiyyətləri QRPA yanaşmasında deformasiya olmuş Saxon-Woods potensialı istifadə edilərək öyrənilmişdir. Ayrıca bu səviyyələrin M1 matris elemntlərinin enerji ağırlıqlı cəmləmə qanunlarına əlavəsi və M1 səviyyələrinin güc funksiyalarının enerji yayılması da öyrənilmişdir. Hesablamalar aşagı enerjilərdəki 1^+ səviyyələrinin zəif korellasiona və kiçik ehtimala sahib olduğunu göstərdi. K=0 budağının ən böyük ehtimallı səviyyələri 8-10MeV bölgəsində, K=1 budağı ücün isə 10-12 MeV bölgəsində olduğu təyin edildi. Ayrıca K=1 budağına aid olan səviyyələrin sayının K=0 budağından daha çox olduğu göstərildi.

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КОЛЛЕКТИВНЫЕ 1^+ СОСТОЯНИЯ ДЕФОРМИРОВАННЫХ ЯДЕР ^{176,178}Hf

В статье было изучено свойство $I^\pi=1^+$ (K=0 и K=1) состояний, взаимодействующих посредством остаточных парных и изовекторных спин-спиновых взаимодействий для ядер ^{176,178}Hf в приближении случайных фаз (СФ) с использованием деформированного потенциала Саксона- Вуда. Кроме того, для этих состояний были изучены вклад M1 матричных

элементов в закон энерговесомого суммирования и энергетическое распределение силовых функций для этих состояний. Вычисления показали, что низколежащие 1^+ состояния слабо коррелированы и маловероятны, так как для К=0 ветви возбуждении более вероятные состояния лежат в интервале 8-10 МеВ, а для К=1 ветви более вероятные состояния лежат в интервале 10-12 МеВ. Кроме того, было показано, что число состояний из ветви К=1 больше числа состояний из ветви К=0.

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