THE EQUATIONS FOR THE MULTI-QUARK GREEN FUNCTION IN NAMBU-JONA-LAZINIO MODEL

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The equations for the multi-quark Green functions have been obtained in the decay of the average field with bilocal source in NJL model.

The equations for many-particle functions play very important role at the description of the connected states, the scattering of connected states on particles, the scattering of the connected states on connected states and e.t.c. The first representative of the family of the many-particle equations is the well-known Bethe-Salpeter (BS) equation [1] for the two-particle Green function. The generalization of BS equation in the case of three or more particles is given in the refs [2-4]. These generalizations are based on the analysis of the Feynman diagrams of the perturbation theory and all statements concerning nucleus structure have clearly perturbative character. The main disadvantage of these diagram methods is that practically all statements can be formulated only as the verbal recipes and they aren't subordinated to the formalization that makes the investigation more difficult.

However, there is natural language for the description of the many-particle equations in the limits of Legendre field theory. Legendre functional transformations allow to obtain the equations for the many-particle Green functions as the direct consequences of the field equations. In the ref [5] Legendre transformations are applied to the study of *n*-particle equations for fermions.

In the present ref in the limits of Nambu-Jona-Lazinio (NJL) [6,7] the equations for the many pointed functions are obtained. At the obtaining of these equations the iterative scheme (one of the variants of field degradation), supposed in refs [8,9], which is based on the approximation system of Shwinger-Dayson (SD) equations for the functional of Green functions with the exactly solvable equation.

Let's consider NJL model with Lagrangian [7,10]:

$$L = \overline{\psi}\left(i\hat{\partial} - m_0\right)\psi + \frac{g}{2}\left[\left(\overline{\psi}\psi\right)^2 + \left(\overline{\psi}i\gamma_5\tau^a\psi\right)^2\right], \quad (1)$$

which we will call SU(2) model of NJL. Here, $g>0$ is the constant of the connection with the dimension of square of reverse mass. Lagrangian is invariant with respect to the transformations of chiral group $SU(2)_V \times SU(2)_A$. $\psi \equiv \psi^\alpha(x)^c_j$ and $\alpha = 1,...,4$; $c = 1,...,n_c$; $j = 1,2$. τ^α_{jk} – are generators of SU(2) group (Pauli matrix):

$$\tau^a\tau^b = \delta^{ab} + i\varepsilon^{abc}\tau^c,\ a = 1,2,3;\ ;$$

which are normed by the following method:

$$tr\,\tau^a\tau^b = 2\delta^{ab}..$$

The SD equation is defined from the ratio [10]:

$$0 = \int D(\psi,\overline{\psi})\frac{\delta}{\delta\overline{\psi}^\alpha(x)^c_j}\overline{\psi}^\beta(y)^d_k \times$$

$$\times \exp i\left[\int dx' L(x') - \int dx'dy'\overline{\psi}(y')\eta(y',x')\psi(x')\right] \quad (2)$$

where η is the bilocal source of the quarks and

$$\overline{\psi}(y)\eta(y,x)\psi(x) \equiv \overline{\psi}^\beta(y)^d_k\eta^{\beta\alpha}(y,x)^{dc}_{kj}\psi^\alpha(x)^c_j,$$

$$\overline{\psi}^\beta(y)^d_k\psi^\alpha(x)^c_j \to i\frac{\delta}{\delta\eta^{\beta\alpha}(y,x)^{dc}_{kj}}..$$

The transmission invariance of the measure of functional integration in (2) leads to the functionally-differential SD equation for the generating functional:

$$\delta^{\alpha\beta}\delta^{cd}\delta_{jk}\delta(x-y)G + \left(i\hat{\partial} - m^0\right)^{\alpha\alpha'}_{ji}\frac{\delta G}{\delta\eta^{\beta\alpha'}(y,x)^{dc}_{ki}} +$$

$$+ig\left\{\frac{\delta}{\delta\eta^{\beta\alpha}(y,x)^{dc}_{kj}}tr\left[\frac{\delta G}{\delta\eta(x,x)}\right] - \gamma_5^{\alpha\alpha'}\tau^a_{jj_1}\frac{\delta}{\delta\eta^{\beta\alpha'}(y,x)^{dc}_{kj_1}}tr\left[\gamma_5\cdot\tau^a\cdot\frac{\delta G}{\delta\eta(x,x)}\right]\right\} =$$

$$= \int dx_1\eta^{\alpha\beta_1}(x,x_1)^{cc_1}_{ji}\frac{\delta G}{\delta\eta^{\beta\beta'}(y,x_1)^{dc_1}_{ki}}.. \quad (3)$$

The solution of this equation we will find by the method, supposed in the refs [8,9]. Further, we will consider the chiral limit ($m_0=0$). The unique connected function of the main approximation is the free quark propagator. In the first step of iteration the equations for the two-particle Green function (four-tail) and for the addition to the quark propagator appear [10].

The equation for the functional of *n*-step $G^{(n)}$ has the form:

$$\delta^{\alpha\beta}\delta^{cd}\delta_{jk}\delta(x-y)G^{(n)}+(i\hat{\partial})^{\alpha\alpha'}_{ji}\frac{\delta G^{(n)}}{\delta\eta^{\beta\alpha'}(y,x)^{dc}_{ki}}+$$

$$+ig\left\{\frac{\delta}{\delta\eta^{\beta\alpha}(y,x)^{dc}_{kj}}tr\left[\frac{\delta G^{(n)}}{\delta\eta(x,x)}\right]-\gamma_5^{\alpha\alpha'}\tau^a_{jj_1}\frac{\delta}{\delta\eta^{\beta\alpha'}(y,x)^{dc}_{kj_1}}tr\left[\gamma_5\cdot\tau^a\cdot\frac{\delta G^{(n)}}{\delta\eta(x,x)}\right]\right\}=$$

$$=\eta*\frac{\delta G^{(n-1)}}{\delta\eta(x,x_1)}. \quad (4)$$

The solution for the equation (4) we will find in the form $G^{(n)}=P^{(n)}G^{(0)}$, where $P^{(0)}\equiv 1$. The equation for $P^{(n)}$ will have the form

$$-\frac{\delta P^{(n)}}{\delta\eta^{\beta\alpha}(y,x)^{dc}_{kj}}+ig\int dx_1 S^{(0)\alpha\alpha'}(x-x_1)^{cc_1}_{ji}\left\{\frac{\delta}{\delta\eta^{\beta\alpha'}(y,x_1)^{dc_1}_{ki}}tr\left[\frac{\delta P^{(n)}}{\delta\eta(x_1,x_1)}\right]-\right.$$

$$-\gamma_5^{\alpha'\alpha_1}\tau^a_{il}\frac{\delta}{\delta\eta^{\beta\alpha_1}(y,x_1)^{dc_1}_{kl}}tr\left[\gamma_5\cdot\tau^a\cdot\frac{\delta P^{(n)}}{\delta\eta(x_1,x_1)}\right]+$$

$$\left.+S^{(0)\alpha'\beta}(x_1-y)^{c_1d}_{ik}tr\left[\frac{\delta P^{(n1)}}{\delta\eta(x_1,x_1)}\right]-\gamma_5^{\alpha'\alpha_1}\tau^a_{il}S^{(0)\alpha_1\beta}(x_1-y)^{c_1d}_{ik}tr\left[\gamma_5\cdot\tau^a\frac{\delta P^{(n1)}}{\delta\eta(x_1,x_1)}\right]\right\}=$$

$$=\int dx_1dx_2S^{(0)\alpha\alpha'}(x-x_1)^{cc_1}_{ji}\eta^{\alpha'\beta'}(x_1,x_2)^{c_1c_2}_{il}\times$$

$$\times\left[P^{(n-1)}S^{(0)\beta'\beta}(x_2-y)^{c_2d}_{lk}+\frac{\delta P^{(n-1)}}{\delta\eta^{\beta\beta'}(y,x_2)^{dc_2}_{kl}}\right]. \quad (5)$$

According to the equations for $P^{(n)}$ (5) the equation for the second step of iteration will have the form:

$$-\frac{\delta P^{(2)}}{\delta\eta^{\beta\alpha}(y,x)^{dc}_{kj}}+ig\int dx_1 S^{(0)\alpha\alpha_1}(x-x_1)^{cc_1}_{ji}\left\{\frac{\delta}{\delta\eta^{\beta\alpha_1}(y,x_1)^{dc_1}_{ki}}\cdot tr\frac{\delta P^{(2)}}{\delta\eta(x_1,x_1)}-\right.$$

$$-\gamma_5^{\alpha_1\alpha_2}\tau^a_{il}\frac{\delta}{\delta\eta^{\beta\alpha_2}(y,x_1)}\cdot tr\gamma_5\tau^a\frac{\delta P^{(2)}}{\delta\eta(x_1,x_1)}+$$

$$\left.+S^{(0)\alpha_1\beta}(x_1-y)^{c_1d}_{ik}tr\frac{\delta P^{(2)}}{\delta\eta(x_1,x_1)}-\gamma_5^{\alpha_1\alpha_2}\tau^a_{il}S^{(0)\alpha_2\beta}(x_1-y)^{c_1d}_{ik}tr\gamma_5\tau^a\frac{\delta P^{(2)}}{\delta\eta(x_1,x_1)}\right\}=$$

$$=\int dx_1dx_2S^{(0)\alpha\alpha_1}(x-x_1)^{cc_1}_{ji}\eta^{\alpha_1\alpha_2}(x_1,x_2)^{c_1c_2}_{il}\times.$$

$$\times\left\{P^{(1)}S^{(0)\alpha_2\beta}(x_2-y)^{c_2d}_{lk}+\frac{\delta P^{(1)}}{\delta\eta^{\beta\alpha_2}(y,x_2)^{dc_2}_{kl}}\right\} \quad (6)$$

The solution of the equation (6) we will find in the form:

$$P^{(2)}=\frac{1}{4!}Tr\left(S_4^{(2)}*\eta^4\right)+\frac{1}{3!}Tr\left(S_3^{(2)}*\eta^3\right)+\frac{1}{2}Tr\left(S_2^{(2)}*\eta^2\right)+Tr\left(S^{(2)}*\eta\right) \tag{7}$$

(here Tr means the step in the operative meaning, and $*$ - means operator multiplication). The solution of the equation of the first step of iteration has been chosen in the form [10].

$$P^{(1)}=\frac{1}{2}Tr\left(S_2^{(1)}*\eta^2\right)+Tr\left(S^{(1)}*\eta\right) \tag{8}$$

Substituting expressions (7) and (8) in (6), making one differentiation and switching on searches η (i.e. $\eta=0$) we obtain the equation for propagator of second step of iteration

$$S^{(2)\alpha\beta}(x-y)_{jk}^{cd}=ig\int dx_1 S^{(0)\alpha\alpha_1}(x-x_1)_{jj_1}^{cc_1}\left\{S_2^{(2)}\begin{pmatrix}x_1 & y\\ x_1 & x_1\end{pmatrix}_{\alpha_2\alpha_2,c_2c_2,j_2j_2}^{\alpha_1\beta,c_1d,j_1k}-\right.$$
$$\left.-\gamma_5^{\alpha_1\alpha_2}\tau_{j_1k_1}^a S_2^{(2)}\begin{pmatrix}x_1 & y\\ x_1 & x_1\end{pmatrix}_{\alpha_4\alpha_3,c_2c_2,k_2j_2}^{\alpha_2\beta,c_1d,k_1k}\gamma_5^{\alpha_3\alpha_4}\tau_{k_2j_2}^a+S^{(0)\alpha_1\beta}(x_1-y)_{j_1k}^{c_1d}\,trS^{(2)}(0)\right\}. \tag{9}$$

After analogical procedure we will obtain the equations for the rest functions of the second step ($S_2^{(2)}$ – two-particle, $S_3^{(2)}$ – three-particle and $S_4^{(2)}$ – four-particle functions) of SU (2) of NJL model:

$$S_2^{(2)}\begin{pmatrix}x & y\\ x' & y'\end{pmatrix}_{\alpha'\beta',c'd',j'k'}^{\alpha\beta,cd,jk}=-S^{(0)\alpha\beta'}(x-y')_{jk'}^{cd'}S_2^{(1)\alpha'\beta}(x'-y)_{j'k}^{c'd}+$$
$$+ig\int dx_1 S^{(0)\alpha\alpha_1}(x-x_1)_{jj_1}^{cc_1}\cdot\left\{S_3^{(2)}\begin{pmatrix}x_1 & y\\ x' & y'\\ x_1 & x_1\end{pmatrix}_{\alpha_2\alpha_2,c_2c_2,j_2j_2}^{\alpha_1\beta,c_1d,j_1k;\ \alpha'\beta',c'd',j'k'}-\right.$$
$$-\gamma_5^{\alpha_1\alpha_2}\cdot\tau_{j_1k_1}^a S_3^{(2)}\begin{pmatrix}x_1 & y\\ x' & y'\\ x_1 & x_1\end{pmatrix}_{\alpha_4\alpha_3,c_2c_2,k_2j_2}^{\alpha_2\beta,c_1d,j_1k;\ \alpha'\beta',c'd',j'k'}\gamma_5^{\alpha_3\alpha_4}\cdot\tau_{j_2k_2}^a+$$
$$+S^{(0)\alpha_1\beta}(x_1-y)_{j_1k}^{c_1d}S_2^{(2)}\begin{pmatrix}x_1 & x_1\\ x' & y'\end{pmatrix}_{\alpha'\beta',c'd',j'k'}^{\alpha_2\alpha_2,c_2c_2,j_2j_2}-$$
$$\left.-\gamma_5^{\alpha_1\alpha_2}\cdot\tau_{j_1k_1}^a S^{(0)\alpha_2\beta}(x_1-y)_{k_1k}^{c_1d}S_2^{(2)}\begin{pmatrix}x_1 & x_1\\ x' & y'\end{pmatrix}_{\alpha'\beta',c'd',j'k'}^{\alpha_4\alpha_3,c_2c_2,j_2k_2}\gamma_5^{\alpha_3\alpha_4}\cdot\tau_{k_2j_2}^a\right\}, \tag{10}$$

$$S_3^{(2)}\begin{pmatrix}x & y\\ x' & y'\\ x'' & y''\end{pmatrix}_{\alpha''\beta'',c''d'',j''k''}^{\alpha\beta,cd,jk;\ \alpha'\beta',c'd',j'k'}=-S^{(0)\alpha\beta'}(x-y')_{jk'}^{cd'}S^{(0)\alpha'\beta}(x'-y)_{j'k}^{c'd}S^{(1)\alpha''\beta''}(x''-y'')_{j''k''}^{c''d''}-$$
$$-S^{(0)\alpha\beta'}(x-y')_{jk'}^{cd'}S_2^{(1)}\begin{pmatrix}x' & y\\ x'' & y''\end{pmatrix}_{\alpha''\beta'',c''d'',j''k''}^{\alpha'\beta,c'd,j'k}-$$
$$-S^{(0)\alpha\beta''}(x-y'')_{jk''}^{cd''}S^{(0)\alpha''\beta}(x''-y)_{j''k}^{c''d}S^{(1)\alpha'\beta'}(x'-y')_{j'k'}^{c'd'}-$$
$$-S^{(0)\alpha\beta''}(x-y'')_{jk''}^{cd''}S_2^{(1)}\begin{pmatrix}x'' & y\\ x' & y'\end{pmatrix}_{\alpha'\beta',c'd',j'k'}^{\alpha''\beta,c''d,j''k}-$$

$$+ig\int dx_1 S^{(0)\alpha\alpha_1}\left(x-x_1\right)^{cc_1}_{jj_1}\left\{S_4^{(2)}\begin{pmatrix} x_1 & y \\ x_1 & x_1 \\ x' & y' \\ x'' & y'' \end{pmatrix}^{\begin{matrix}\alpha_1\beta,c_1d,j_1k \\ \alpha_2\alpha_2,c_2c_2,j_2j_2 \\ \alpha'\beta',c'd',j'k' \\ \alpha''\beta'',c''d'',j''k''\end{matrix}}-\right.$$

$$-\gamma_5^{\alpha_1\alpha_2}\cdot\tau^a_{j_1k_1}S_4^{(2)}\begin{pmatrix} x_1 & y \\ x_1 & x_1 \\ x' & y' \\ x'' & y'' \end{pmatrix}^{\begin{matrix}\alpha_2\beta,c_1d,k_1k \\ \alpha_4\alpha_3,c_2c_2,j_2k_2 \\ \alpha'\beta',c'd',j'k' \\ \alpha''\beta'',c''d'',j''k''\end{matrix}}\gamma_5^{\alpha_3\alpha_4}\cdot\tau^a_{k_2j_2}+$$

$$+S^{(0)\alpha_1\beta}\left(x_1-y\right)^{c_1d}_{j_1k}S_3^{(2)}\begin{pmatrix} x_1 & x_1 \\ x' & y' \\ x'' & y'' \end{pmatrix}^{\begin{matrix}\alpha_2\alpha_2,c_2c_2,j_2j_2 \\ \alpha'\beta',c'd',j'k' \\ \alpha''\beta'',c''d'',j''k''\end{matrix}}-$$

$$\left.-\gamma_5^{\alpha_1\alpha_2}\cdot\tau^a_{j_1k_1}S^{(0)\alpha_2\beta}\left(x_1-y\right)^{c_1d}_{k_1k}S_3^{(2)}\begin{pmatrix} x_1 & x_1 \\ x' & y' \\ x'' & y'' \end{pmatrix}^{\begin{matrix}\alpha_4\alpha_3,c_2c_2\,j_2k_2 \\ \alpha'\beta',c'd',j'k' \\ \alpha''\beta'',c''d'',j''k''\end{matrix}}\gamma_5^{\alpha_3\alpha_4}\cdot\tau^a_{k_2j_2}\right\}, \tag{11}$$

$$S_4^{(2)}\begin{pmatrix} x & y \\ x' & y' \\ x'' & y'' \\ x''' & y''' \end{pmatrix}^{\begin{matrix}\alpha\beta,cd,jk \\ \alpha'\beta',c'd',j'k' \\ \alpha''\beta'',c''d'',j''k'' \\ \alpha'''\beta''',c'''d''',j'''k'''\end{matrix}}=$$

$$=-S^{(0)\alpha\beta'}\left(x-y'\right)^{cd'}_{jk'}S^{(0)\alpha'\beta}\left(x'-y\right)^{c'd}_{j'k}S_2^{(1)}\begin{pmatrix} x'' & y'' \\ x''' & y''' \end{pmatrix}^{\begin{matrix}\alpha''\beta'',c''d'',j''k'' \\ \alpha'''\beta''',c'''d''',j'''k'''\end{matrix}}-$$

$$-S^{(0)\alpha\beta''}\left(x-y''\right)^{cd''}_{jk''}S^{(0)\alpha''\beta}\left(x''-y\right)^{c''d}_{j''k}S_2^{(1)}\begin{pmatrix} x' & y' \\ x''' & y''' \end{pmatrix}^{\begin{matrix}\alpha'\beta',c'd',j'k' \\ \alpha'''\beta''',c'''d''',j'''k'''\end{matrix}}-$$

$$-S^{(0)\alpha\beta'''}\left(x-y'''\right)^{cd'''}_{jk'''}S^{(0)\alpha'''\beta}\left(x'''-y\right)^{c'''d}_{j'''k}S_2^{(1)}\begin{pmatrix} x' & y' \\ x'' & y'' \end{pmatrix}^{\begin{matrix}\alpha'\beta',c'd',j'k' \\ \alpha''\beta'',c''d'',j''k''\end{matrix}}+$$

$$+ig\int dx_1 S^{(0)\alpha\alpha_1}\left(x-x_1\right)^{cc_1}_{jj_1}\left\{S^{(0)\alpha_1\beta}\left(x_1-y\right)^{c_1d}_{j_1k}S_4^{(2)}\begin{pmatrix} x_1 & x_1 \\ x' & y' \\ x'' & y'' \\ x''' & y''' \end{pmatrix}^{\begin{matrix}\alpha_2\alpha_2,c_2c_2,j_2j_2 \\ \alpha'\beta',c'd',j'k' \\ \alpha''\beta'',c''d'',j''k'' \\ \alpha'''\beta''',c'''d''',j'''k'''\end{matrix}}-\right.$$

$$\left.-\gamma_5^{\alpha_1\alpha_2}\tau^a_{j_1k_1}S^{(0)\alpha_2\beta}\left(x_1-y\right)^{c_1d}_{k_1k}S_4^{(2)}\begin{pmatrix} x_1 & x_1 \\ x' & y' \\ x'' & y'' \\ x''' & y''' \end{pmatrix}^{\begin{matrix}\alpha_4\alpha_3,c_2c_2,j_2k_2 \\ \alpha'\beta',c'd',j'k' \\ \alpha''\beta',c''d'',j''k'' \\ \alpha'''\beta''',c'''d''',j'''k'''\end{matrix}}\gamma_5^{\alpha_3\alpha_4}\tau^a_{k_2j_{21}}\right\}, \tag{12}$$

The equations (11) and (12) are new in the given iteration scheme, and the equations (9) and (10) for the two-particle function $S_2^{(2)}$ and for propagator $S^{(2)}$ have the same forms (see [10]), that the equations of the first step have, except of the nonhomogeneous members, in which the four-particle function $S_4^{(2)}$ and three-particle function $S_3^{(2)}$ of the second step are included.

Here it is need to note, that the third step of iterations leads to the appearance of the equations for the six-particle (twelve-tail), fifth-particle (ten-tail), four-particle (eight-tail), three-particle (six-tail), two-particle (four-tail) one-particle (propagator) functions. The above obtained equations allow to make the consequent theoretico-field calculation of the characteristics of hadronic decays, and also to investigate the possibilities of the dynamic description in the limits of the supposed approach of the interaction of the nucleons and new exotic baryons of pent quark type.

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NAMBU-YONA-LAZİNİO MODELİNDƏ ÇOXKVARKLI QRİN FUNKSİYALARI ÜÇÜN TƏNLİKLƏR

Bilokal mənbəli orta sahə paylanmasında Nambu-Yona-Lazinio modelində çoxkvarklı Qrin funksiyaları üçün tənliklər alınmışdır.

Р.Г. Джафаров

УРАВНЕНИЯ ДЛЯ МНОГОКВАРКОВЫХ ФУНКЦИЙ ГРИНА В МОДЕЛИ НАМБУ-ИОНА-ЛАЗИНИО

В разложении среднего поля с билокальным источником в модели Намбу-Иона-Лазинио получены уравнения для многокварковых функций Грина.

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