EDGE STATES IN A KANE TYPE SEMICONDUCTOR QUANTUM WIRE

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The energy spectra of carriers confined to a cylindrical semiconductor quantum wire in an inhomogeneous magnetic field which is $B=0$ $r<r_0$ and $B\neq 0$ elsewhere are studied by, taking into account the real band structure of InSb type semiconductors; narrow energy gap and strong spin-orbit interaction. It's found that the eigen energy spectra for the magnetic quantum wire critically depend on the number of missing flux quanta. Since the spin effect is taken into account, each energy curve splits into two curves. The crossover point of energy curves for *m=0* state with opposite spins is obtained around the value of *S=8*. The magnetic field and the radius dependence of the edge g-factor is also studied.

1. Introduction

Recently there has been a great interest in the behavior of carriers with an inhomogeneous perpendicular magnetic field about low-dimensional systems both theoretically and experimentally [1,2]. On the other hand edge states play an important role in understanding the transport properties of quantum nanostructures [3]. Magnetic edge state in a magnetic quantum wire becomes quite popular, in particular conjunction with a possible candidate for a high density memory device or spintronic materials, so various magnetic nano-quantum structures are reviewed in detail [4]. Advances with respect to growth as well as high-resolution electron-beam lithography techniques allow the novel confined structures called quantum wires or quantum rings. Transport properties of edge states in quantum nanostructures have been discussed by many groups. Peeters, Matulis and İbrahim [5] presented energy levels in the magnetic antidote, while Reijineries, Peeters and Matulis [6] further performed a more detailed and complete study of the bound states of such a system. Solimay and Kroner [7] solved the classical and quantum mechanical equations for a magnetically confined quantum dot and discussed the eigen energies. Badalyan and Peeters have developed a theory for the non-homogeneous magnetic field induced magnetic edge states and their transport in a quantum wire formed by a parabolic confining potential [8]. Ihm et al [9,10] investigated the two dimensional electrons further confined in an inhomogeneous magnetic field. They have found that the eigenstates deviated from Landau levels, due o the non-uniform magnetic field distributions, forming the magnetic edge states which critically depend on the number of missing flux quanta within the dot or the ring. Recently Young Guo et al [11] investigated the electron spin effect on quantum states and magneto-conductance in a magnetic quantum antidote with inhomogeneous magnetic field results in further splitting of energy levels. In the work of [12] the electron states and circulating probability currents due to the inhomogeneous field distribution formed in a magnetic quantum ring is studied. However, the experimental advantages of using narrow-gap semiconductors for the reduced dimensionality systems make it necessary to account for the real band structure of these materials. The purpose of our work is taking into account the coupling of the conduction and valence bands and the non-parabolicity of the electron dispersion while studying the narrow and medium gap semiconductors. It's also aimed to study the magnetic field and the radius change of edge g-factor. In the present study, using eight band Kane's model including the conduction band, light and spin orbital hole bands, the energy spectrum and edge g-factor of electrons confined to a cylindrical semiconductor quantum wire (InSb) in an inhomogeneous magnetic field which is $B=0$ $r<r_0$ and $B\neq 0$ elsewhere are investigated. In the eight-band Kane's Hamiltonian the valence and conduction bands interaction is taken into account via the only matrix element *P* (so called Kane's parameter). We also neglect the free-electron term in the diagonal part and the Pauli spin term as they give small contributions to the effective mass and the spin g-value of electrons in InSb. The system of Kane equations including the non-dispersional heavy hole bands have the form [13,14,15,16]:

$$-EC_1-\frac{Pk_-}{\sqrt{2}}C_3+\sqrt{\frac{2}{3}}Pk_zC_4+\frac{Pk_+}{\sqrt{6}}C_5+\frac{Pk_z}{\sqrt{3}}C_7+\frac{Pk_+}{\sqrt{3}}C_8=0 \quad (1)$$

$$-EC_2-\frac{Pk_-}{\sqrt{6}}C_4+\sqrt{\frac{2}{3}}Pk_zC_5+\frac{Pk_+}{\sqrt{2}}C_6+\frac{Pk_-}{\sqrt{3}}C_7-\frac{Pk_z}{\sqrt{3}}C_8=0 \quad (2)$$

$$-\frac{Pk_+}{\sqrt{2}}C_1-(E+E_g)C_3=0 \quad (3)$$

$$\sqrt{\frac{2}{3}}Pk_zC_1-\frac{Pk_+}{\sqrt{6}}C_2-(E+E_g)C_4=0 \quad (4)$$

$$\sqrt{\frac{2}{3}}Pk_z C_2 + \frac{Pk_-}{\sqrt{6}} C_1 - (E + E_g)C_5 = 0 \quad (5)$$

$$\frac{Pk_-}{\sqrt{2}} C_2 - (E + E_g)C_6 = 0 \quad (6)$$

$$\frac{Pk_z}{\sqrt{3}} C_1 + \frac{Pk_+}{\sqrt{3}} C_2 - (\Delta + E + E_g)C_7 = 0 \quad (7)$$

$$\frac{Pk_-}{\sqrt{3}} \cdot C_1 - \frac{Pk_z}{\sqrt{3}} C_2 - (\Delta + E + E_g)C_8 = 0 \quad (8)$$

Here P is the Kane parameter, E_g - the band gap energy, Δ - the value of spin-orbital splitting and $k_\pm = k_x \pm ik_y$, $\vec{k} = -i\vec{\nabla}$, C_i are envelope functions.

2. Theory

The model that is considered in the present study is composed of an electron confined to move in a cylindrical semiconductor quantum wire under the influence of a magnetic field in the z-direction, which is non-zero except with a cylinder of radius r_0. The magnetic field is described by

$$\vec{B}(\vec{r}) = \begin{cases} B\hat{z} & (r > r_0) \\ 0 & (r < r_0) \end{cases} \quad (9)$$

Than the vector potential will be as follows;

$$\vec{A} = \frac{1}{2}\left(1 - \frac{r_0^2}{r^2}\right)B(-y, x, 0) \quad (10)$$

$k_\pm$ have forms

$$k_\pm \to k_\pm \pm i\frac{1}{2}\lambda_H\left(1 - \frac{r_0^2}{r^2}\right)r_\pm \quad (11)$$

where

$$r_\pm = x \pm iy\,,\ \lambda_H = \frac{eH}{\hbar c} \quad (12)$$

One can now express the envelope functions $C_3, C_4, ...C_8$ by the functions C_1 and C_2 respectively, and substitute them into the first and second equations, we finally obtain the following decoupled equations for $C_{1,2}$:

$$\left(-\varepsilon - \frac{P^2}{3}\left(\frac{2}{\varepsilon + \varepsilon_g} + \frac{1}{\varepsilon + \varepsilon_g + \Delta}\right)\Delta_3\right)C_{1,2} = 0 \ (r < r_0) \quad (13)$$

where Δ_3 is three dimensional Laplacian.

When there is magnetic field

$$\left\{\begin{matrix} -\varepsilon + \dfrac{P^2}{3}\left(\dfrac{2}{\varepsilon + \varepsilon_g} + \dfrac{1}{\varepsilon + \varepsilon_g + \Delta}\right)\left\{-\nabla^2 + 2\lambda_H \hbar^{-1}\left(1 - r_0^2 r^{-2}\right)L_z + \lambda_H^2\left(1 - r_0^2 r^{-2}\right)^2 r^2\right\} \pm \\ 2\lambda_H \dfrac{P^2}{3}\left(\dfrac{1}{\varepsilon + \varepsilon_g} - \dfrac{1}{\varepsilon + \varepsilon_g + \Delta}\right) \end{matrix}\right\} C_{1,2} = 0 \ (r > r_0) \quad (14)$$

where L_z, z component of angular momentum operator L and $r^2 = x^2 + y^2$.

The wave functions in cylindrical coordinates are separable;

$$C_{1,2} = e^{im\phi + ik_z z}\phi_{1,2}^1(r) \qquad (r < r_0) \quad (15)$$

$$C_{1,2} = e^{im\phi + ik_z z}\phi_{1,2}^2(r) \qquad (r > r_0) \quad (16)$$

where m is the angular momentum quantum number. The equation of the radial part is written as;

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \left(\frac{2m_n E_1'}{\hbar^2} - \frac{m^2}{r^2}\right)\right]\phi_{1,2}^{(1)}(r) = 0 \qquad r < r_0 \quad (17)$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \left(\frac{2m_n E_2'}{\hbar^2} - \frac{(m-s)^2}{r^2} - r^2 - \lambda_H(m-s) - \frac{e^2H^2}{4\hbar c^2}r^2\right)\right]\phi_{1,2}^{(2)}(r) = 0 \qquad r > r_0 \quad (18)$$

Here $E_1^{'} = \frac{\hbar^2}{2m_n}\left(\frac{3}{P^2}\frac{\varepsilon(\varepsilon+\varepsilon_g)(\varepsilon+\varepsilon_g+\Delta)}{(3\varepsilon+3\varepsilon_g+2\Delta)} - k_z^2\right)$, and

$$E_2^{'} = \frac{\hbar^2}{2m_n}\left[\frac{3}{P^2}\frac{\varepsilon(\varepsilon+\varepsilon_g)(\varepsilon+\varepsilon_g+\Delta)}{(3\varepsilon+3\varepsilon_g+2\Delta)} \mp \lambda_H \frac{\Delta}{3\varepsilon+3\varepsilon_g+2\Delta}\right]$$

$\phi_{1,2}^{(1)}(r)$ is expressed as $\phi_{1,2}^{1}(r) = C_1 J_{|m|}\left(\sqrt{\chi} r\right)$ where the function J_m is the Bessel function of order *m*, $\phi_{1,2}^{(2)}(r) = C_2 r^{|m-s|/2} e^{-\frac{r^2}{2}} U(a,b,x)$ where $U(a,b,x)$ is the confluent hypergeometric function [17].

In here

$$\chi^2 = \frac{2m_0}{\hbar^2}\frac{3}{\varepsilon_P}\left(\frac{\varepsilon(\varepsilon+\varepsilon_g)(\varepsilon+\varepsilon_g+\Delta)}{(3\varepsilon+3\varepsilon_g+2\Delta)}\right) - k_z^2 \tag{19}$$

$$a = \frac{1}{2} + \frac{|m-s|+m-s}{2} - \frac{3\varepsilon(\varepsilon+\varepsilon_g)(\varepsilon+\varepsilon_g+\Delta)}{\hbar\omega_0(3\varepsilon+3\varepsilon_g+2\Delta)\varepsilon_p} \pm \frac{\Delta}{2(3\varepsilon+3\varepsilon_g+2\Delta)} \tag{20}$$

$$b = |m-s| + 1 \tag{21}$$

where $\varepsilon_p = \frac{2m_0}{\hbar^2}P^2$, $x = \frac{r^2}{2l_H^2}$ and the magnetic length is $l_H = \sqrt{\frac{\hbar c}{eH}}$.

It's expressed the whole quantities in dimensionless units by letting $\hbar\omega_0$ ($\omega_0 = \left(eH_0 / 2m_0 c\right)$ is the Larmor frequency) and the inverse length $\beta\left(= \sqrt{m_0\omega_0 / \hbar}\right)$ be 1. In these units, $\hbar^2 / m_0 = \hbar\omega_0 / \beta^2 \to 1$ and $r_0 = \sqrt{s}$ so that $s = H_0 \pi r_o^2 / \phi_0$ is the only relevant parameter. Here *s* is a scale parameter which represents the number of missing flux quanta within the wire [9,10,11] and $\phi_0 \left(= hc / e\right)$ the flux quantum. Since there is no magnetic field inside the magnetic wire, the magnetic edge states may not enclose the magnetic flux, resulting in missing flux quanta; these are absent in the edge states formed by electrostatic confinements. The energies are easily determined from the continuity of the wave functions and their derivatives at the boundary of the wire.

In the calculations of the electron energy spectra for narrow gap InSb cylindrical wires we choose the semiconductor band structure parameters for InSb: energy gap *Eg*=0.2368 eV, spin orbit splitting is Δ=*0.810 eV*, $\varepsilon_p = \frac{2m_0}{\hbar^2}P^2 = 23.42$ eV and m_0 is the free-electron mass [18].

The effective edge g-factor can be determined from the Zeeman splitting of subbands:

$$g(\varepsilon) = \frac{\varepsilon\uparrow - \varepsilon\downarrow}{\mu_B H} \tag{22}$$

Here $\varepsilon\uparrow$ and $\varepsilon\downarrow$ are the electron energy for spin +*z* and –*z* directions respectively.

3. Results

In this section, it's calculated exactly and discussed the single-electron eigenstates and transport properties of a magnetic quantum wire by taking into account the real band structure of InSb type materials; narrow energy gap and strong spin-orbit interaction. Figure 1 represents the estimated dependence of the energy eigenvalues in the magnetic quantum wire on the angular momentum *m*. The solid curves correspond to the spin up case $\sigma = +1/2$ and the spin down case $\sigma = -1/2$ respectively for different *m* values. To allow comparison with the works of [9,10,11], in this figure and the following one, the energy is in units of $\hbar\varpi_0 = 1$ at $r_0 = 500$ Å and $H_0 = 2.633T$ (corresponding the *s=5* in references [9,10,11]).

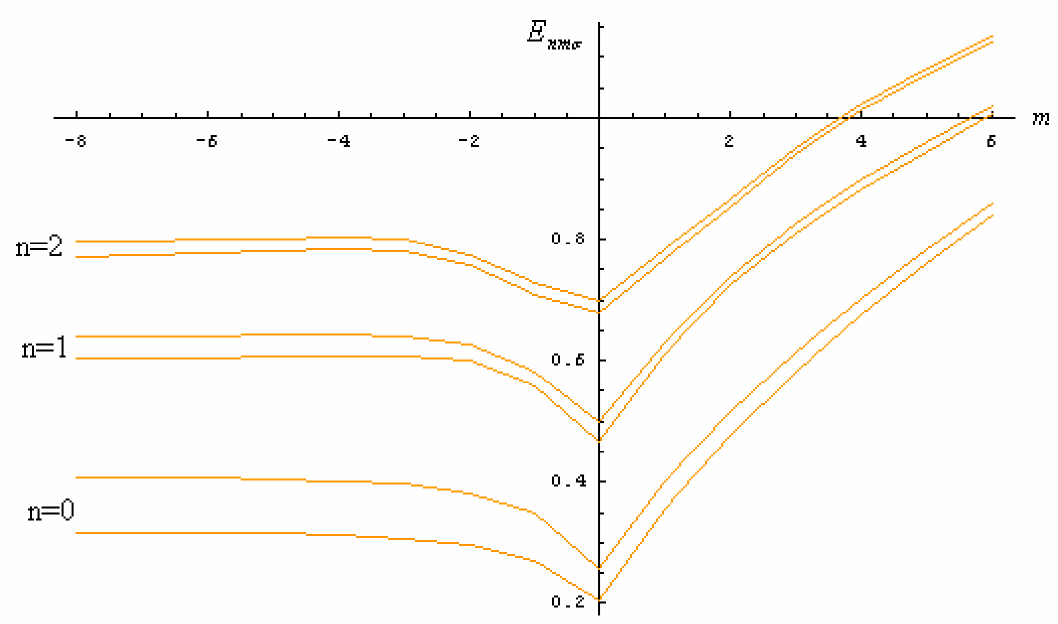

Fig. 1. The dependence of the energy eigenvalues $E_{nm\,\sigma}$ in the quantum wire on the angular momentum *m* for $r_0 = 500$ Å, $H_0 = 2.633T$, $k_z = 0$.

At a first look in Figure 1, we notice the Landau level degeneracy is broken. As it can be seen the lowest energy state occurs at *m=0.* This result indicates that the inhomogeneity of the magnetic field perturbs mostly the states near the boundary of the quantum wire, and this perturbation is caused by the missing flux quanta *s*. The energy levels increase slowly with the decrease of angular momentum *m* for *m<0*, while they increase rapidly with the increasing of *m* for *m>0* as in references [9,10,11]. The splitting, due to the spin between the energy levels are decreased with the increasing of n, which is the main different result in comparison with the ref. [11] and it's because of the interaction between the valence and conduction band.

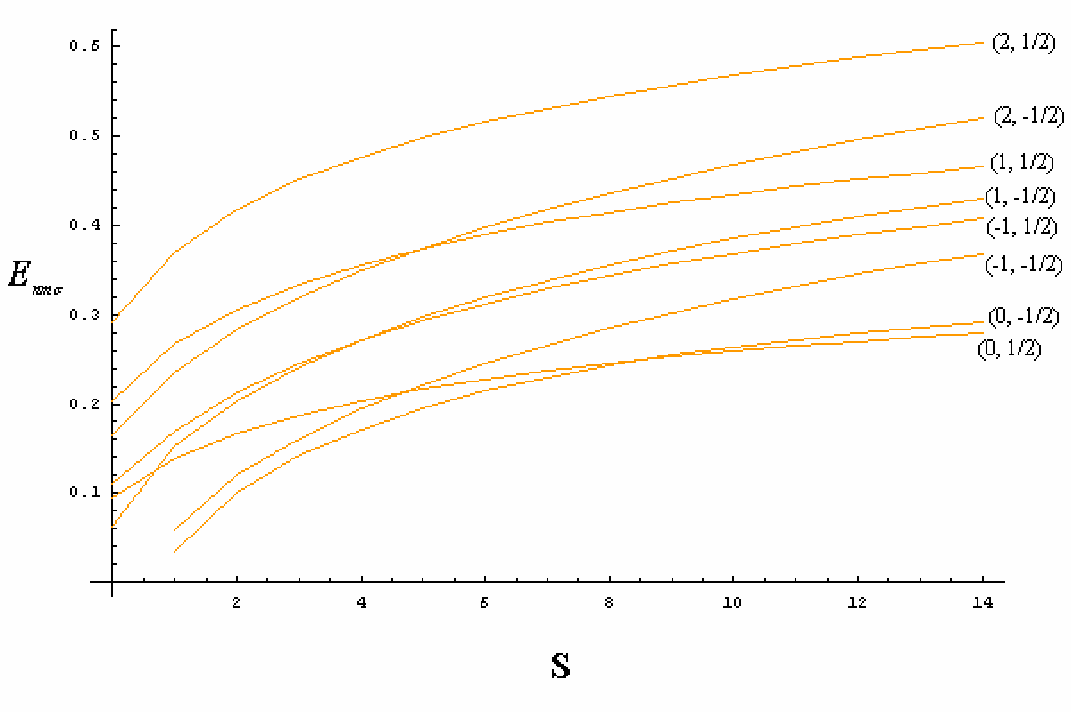

Fig. 2. Energy spectra as a function of S *n=0* and *m=-1 to +2* , where $\pm 1/2$ is spin up and spin down values. $r_0 = 500$ Å, $k_z = 0$.

In Figure 2, we represent the energy spectra as a function of the number of missing magnetic flux quanta *S* ($S = H\pi r^2 / \phi_0$). In this figure it's shown only *n=0* and *m=-1* to *+2* states to avoid the complexity. The energy units $\hbar\omega_0$

are set to one at *s=5* and r_0 is fixed. The different energy levels are labeled with the corresponding quantum numbers (m,σ). Since the spin effect is taken into account, each energy curve splits into two curves. The crossover point of energy curves for *m=0* state with opposite spins is obtained around the value of *S=8*. It's noted that some of the crossover points in the same angular quantum number may be located outside the magnetic region presented.

In Figure 3 the magnetic field change of edge g factor for electrons at the ground state (*m=0*) is illustrated. It's seen that edge g-factor increases with the increasing of the magnetic field.

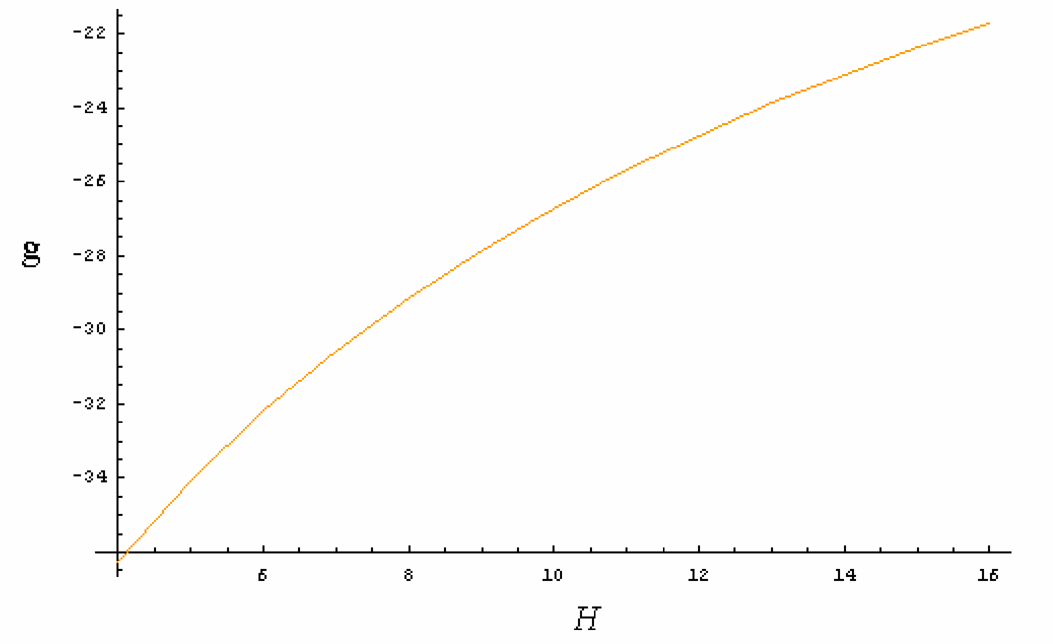

Fig. 3. Edge *g*-factor versus magnetic field, for electrons of InSb at the ground state. $r_0 = 500$ Å, $k_z = 0$.

Figure 4 represents the radius change of edge g factor for electrons at the ground state (*m=0*). It can be seen that the edge g-factor approaches to the bulk value with the increasing of the radius.

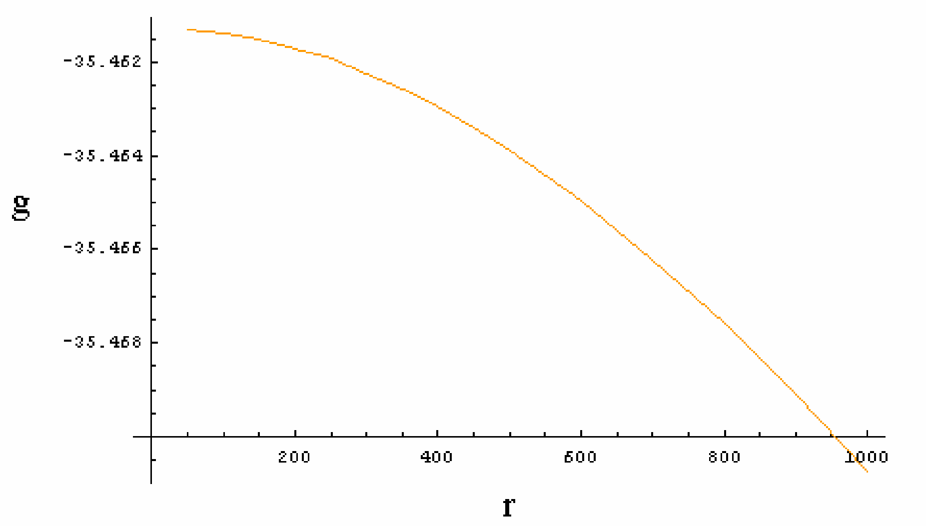

Fig. 4. Edge g factor versus radius for electrons of InSb at the ground state. *H=1 T*, $k_z = 0$.

4. Conclusion

We have investigated the energy spectra of a magnetic quantum wire by taking into account the real band structure of InSb type semiconductors. Energy spectra of quantum wire shows deviated structures from the bulk Landau levels. It's shown that the different behaviors of the edge states depend on the amount of the flux in a magnetic quantum wire. The magnetic field dependence of edge g-factor is also investigated and found that edge g-factor increases with the increasing of the magnetic field. In addition to this, it is shown that the edge g-factor approaches to the bulk value with the increasing of the radius.

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KEYN TİPLİ YARİMKEÇİRİCİ KVANT TELLƏRİNDƏ KƏNAR HALLAR

InSb tipli maqnit kvant tellərində elektronların enerji spektrləri yarimkeçiricinin real zona quruluşu nəzərə alınmaqla hesablanmışdır. Elektronların kənar hallarının effektiv *g*-faktorunun kvant telinin radiusundan və xarici magnit sahəsindən asılılığı öyrənilmişdir.

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КРАЕВЫЕ СОСТОЯНИЯ В КВАНТОВОЙ ПРОВОЛОКЕ КЕЙНОВСКОГО ПОЛУПРОВОДНИКА

Исследован энергетический спектр носителей тока в полупроводниковой цилиндрической квантовой проволоке во внешнем, неоднородном магнитном поле, $B=0$,при $r<r_0$ and $B\neq 0$ в остальном пространстве, с учетом реальной зонной структуры узкощелевых полупроводников типа InSb с сильным спин-орбитальным взаимодействием. Найдено, что энергетический спектр критически зависит от числа квантов магнитного потока. С учетом спина каждая энергетическая кривая расщепляется на две. Для состояния *m=0* кривые с противоположными значениями спина пересекаются около *S=8*. Изучена зависимость от магнитного поля и радиуса проволоки краевого g- фактора.

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