

ON PION CORRECTION TO QUARK MASS IN NEXT-LEADING ORDER OF MEAN-FIELD EXPANSION IN NJL MODEL

R.G. JAFAROV

*Institute of Physics Problems, Baku State University
Z.Khalilov, str., 23, AZ 1148 Baku, Azerbaijan*

The correction to quark mass are calculated in Nambu - Jona-Lasinio model with 4 – dimensional cutoff regularization and with dimensional-analytical regularization in the next-to-leading order of mean-field expansion. The analytical calculations show that the pion correction to quark mass is equal to zero. Comparing the results in both regularization one can signify that the zero value of the pion correction to quark mass is the regularization-independent fact of Nambu - Jona-Lasinio model.

1. Introduction

The chiral-symmetrical Nambu - Jona-Lasinio (NJL) model [1] with the quark content [2] is one of the most successful theoretical laboratories for investigation of the phenomenon of the spontaneous breakdown of chiral symmetry and for study of the light hadrons in the non-perturbative region [3,4]. The NJL model was intensively investigated also at finite temperature and density [5] and with various external fields [6].

The nonrenormalizability of the NJL model implies a suitable choice of regularization. The most common regularizations for NJL model traditionally entail a four-dimensional cutoff (FDC) regularization [3,7,8] or three-dimensional momentum cutoff regularization [3,4]. Other regularization schemes are also used for NJL model [9].

Scalar meson contributions in the chiral quark condensate in the framework of the dimensional-analytical regularization (DAR) have been calculated in [10,11]. These contributions for physical values of parameters were found to be significant and should be taken into account in choice of the parameters values. The improved fit of parameters has been carried out in $SU_V(2) \times SU_A(2)$ - NJL model.

In [8] systematical comparison of the dimensional-analytically regularized NJL model with the NJL model with FDC regularization has been done. Apart from the corrections to chiral condensate, the corrections to quark mass in both regularizations were also calculated. The numerical calculations at two characteristic values of condensate showed that the pion contributions to quark mass in both regularizations are equal to zero. Also it was supposed in [8] that an absence of the pion corrections to quark mass is a regularization-independent fact of NJL model.

The present work, which essentially based on results of [8], is devoted to analytical calculations of the pion correction to quark mass.

2. NJL model with dimensional-analytical regularization and with four-dimensional cutoff. Meson contributions in chiral condensate

The model under consideration contains up and down quarks fields $\psi(x)$, each with n colors. The $SU_V(2) \times SU_A(2)$ - invariant Lagrangian of this model is:

$$L = \bar{\psi} i \partial \psi + \frac{g}{2} \left[(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \tau^\alpha \psi)^2 \right],$$

where τ^α -are Pauli matrices normalized by $\text{tr}(\tau^a \tau^b) = 2\delta^{ab}$. For formulating of the mean-field expansion we use an iteration scheme of the solution of Schwinger-Dyson equation with the fermion bifocal source, which has been developed in [12].

The unique connected Green's function in the leading mean-field approximation is a single-particle Green's function (quark propagator):

$$S^{(0)} = (m - p)^{-1},$$

where the dynamical quark mass m is a solution of the gap equation:

$$1 = -8 \text{ign}_c \int \frac{d\tilde{q}}{m^2 - q^2} \quad (1)$$

where $d\tilde{q} \equiv d^4 q / (2\pi)^4$.

The solution of the gap equation (1) in DAR and FDC regularization leads to [8]:

$$1 = k \Gamma(\xi) \left(\frac{M^2}{m^2} \right)^{1+\xi}, \quad (2)$$

$$1 = k_\Lambda \left(1 - \frac{1}{x} \log(1+x) \right). \quad (3)$$

respectively. Here, $k_\Lambda = \frac{gn_c \Lambda^2}{2\pi^2}$, $k = \frac{gn_c m^2}{2\pi^2}$, $x = \frac{\Lambda^2}{m^2}$;

ξ and Λ are regularizations parameters in DAR and in FDC regularization correspondingly.

Meson contribution to chiral condensate can be calculated in the next-to-leading term of the mean-field expansion.

Ratio of the first iteration condensate $\chi^{(1)}$ to leading-approximation condensate $\chi^{(0)}$ in the pion channel has the form [8,10]:

$$r_\pi = -\frac{24 \text{ign}_c}{1 - 8 \text{ign}_c J} \int d\tilde{p} d\tilde{q} \frac{m^2 - p^2 + 2pq}{(m^2 - p^2)^2 [m^2 - (p-q)^2]} A_\pi(q), \quad (4)$$

where $J = \int d\tilde{p} \frac{m^2 + p^2}{(m^2 - p^2)^2}$

In (4) the pseudoscalar amplitude A_π is [10]:

$$A_\pi = \frac{ig}{1 + L_p} \quad (5)$$

where $L_p(p) = ig \int d\tilde{q} \text{tr} S^{(0)}(p+q) \gamma_5 S^{(0)}(q) \gamma_5$ - pseudoscalar quark loop.

Making use of the gap equation (1) we obtain the following form for amplitude A_π in momentum space:
 $L_s(p) = ig \int d\tilde{q} \text{tr} S^{(0)}(p+q) S^{(0)}(q)$

$$A_\pi = -\frac{1}{4n_c p^2 I_0(p^2)}, \quad (6)$$

where

$$I_0(p^2) = \int \frac{d\tilde{q}}{(m^2 - (p+q)^2)(m^2 - q^2)} \quad (7)$$

The calculation of the integral (7) in both regularizations (in DAR and in FDC regularization) leads to:

$$I_0^{DAR}(p^2) = \frac{i}{(4\pi)^2} \frac{\xi}{k} F\left(1 + \xi, 1; 3/2; \frac{p^2}{4m^2}\right), \quad (8)$$

$$I_0^{FDC}(p^2) = \frac{i}{(4\pi)^2} \left[\log(1+x) - \frac{x}{1+x} F\left(1, 1; 3/2; \frac{p^2}{4m^2(1+x)}\right) - \frac{p^2}{6m^2(1+x)} F\left(1, 1; 5/2; \frac{p^2}{4m^2(1+x)}\right) + \frac{p^2}{6m^2} F\left(1, 1; 5/2; \frac{p^2}{4m^2}\right) \right]. \quad (9)$$

Here $F(a, b; c; z)$ is the Gauss hypergeometric function.

The pseudoscalar amplitude A_π^{pole} according to the pole ($p^2=0$) representations of the formulas (8) and (9) and the gap

equation (2) in both regularizations (at $n=3$) takes the following forms:

$$(A_\pi^{pole})^{DAR} = \frac{1}{12p^2 I_0^{DAR}(0)} = -\frac{2igm^2}{\xi p^2}, \quad (10)$$

$$(A_\pi^{pole})^{FDC} = \frac{1}{12p^2 I_0^{FDC}(0)} = -i \frac{4\pi^2}{3 \left(\log(1+x) - \frac{x}{1+x} \right) p^2}. \quad (11)$$

As a measure of quantum fluctuations of the chiral condensate in pion channel, it is used the first iteration condensate to the leading-approximation condensate in pole approximation of amplitude (10) in DAR [10]:

$$\tau_\pi^{DAR} = \frac{1}{8O} \quad (12)$$

Using the pole approximation of amplitude (11) in calculation of the ratio r_π^{FDC} in Euclidean momentum space in FDC regularization, we obtain for ratio of the first iteration condensate to the leading approximation condensate:

$$\tau_\pi^{FDC} = -\frac{\log(1+x)}{8 \left(\log(1+x) - \frac{x}{1+x} \right)}. \quad (13)$$

3. Pion correction to quark mass

In [8] it has been obtained the formula for the correction to quark mass in next-to-leading approximation of the mean-field expansion. We'll use that formula having the following form:

$$\frac{\delta m_\pi}{m} \cong b_\pi^{(1)}(m^2) - a_\pi^{(1)}(m^2), \quad (14)$$

where $a_\pi^{(1)}$ and $b_\pi^{(1)}$ are the first order mass functions. These functions are defined by the following equations:

$$p^2 a_\pi^{(1)}(p^2) = -3 \int \frac{d\tilde{q}}{m^2 - (p-q)^2} A_\pi^{pole}(q) \quad (15)$$

$$b_\pi^{(1)}(p^2) = r_\pi - 3 \int \frac{d\tilde{q}}{m^2 - (p-q)^2} A_\pi^{pole}(q). \quad (16)$$

Using in (15) and (16) the leading singularity approximation for $(A_\pi^{pole})^{DAR}$ (10) and $(A_\pi^{pole})^{FDC}$ (11) and calculating the integrals in DAR and FDC regularization (taking also into consideration (14)) we obtain for pion corrections to quark mass the following expressions:

$$\left(\frac{\delta m_\pi}{m} \right)^{DAR} = r_\pi^{DAR} - \frac{3}{8\xi}, \quad (17)$$

$$\left(\frac{\delta m_\pi}{m}\right)^{FDC} = r_\pi^{FDC} + \frac{\log(I+x)}{8\left(\log(I+x) - \frac{x}{I+x}\right)}. \quad (18)$$

From (17) and (18) according to (12) and (13) it follows that the pion contribution to quark mass is equal to zero. It means that the zero value of the pion correction to quark mass is independent from the regularization choice in NJL model.

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R.Q. Cəfərov

NİL MODELİNDƏ ORTA SAHƏ PAYLANMASININ BAŞ HƏDDİNDƏN SONRAKI HƏDDƏ PIONUN KVARKIN KÜTLƏSİNƏ DÜZƏLİŞLƏRİNƏ DAİR

NİL modelində orta sahə paylanmasının baş həddindən sonrakı həddə dördölçülü kəsmə requlyarlaşmasında və ölçülü-analitik requlyarlaşmada kvarkın kütləsinə düzəlişlər hesablanmışdır. Analitik hesablamalar göstərir ki, kvarkın kütləsinə pionun düzəlişləri sifira bərabərdir. Hər iki requlyarlaşmadakı hesablaşmaların müqayisəsi göstərir ki, kvarkın kütləsinə pionun düzəlişlərinin sifra bərabərliyi NİL modelində requlyarlaşma seçiminə asılı olmayan faktır.

Р.Г. Джафаров

К ПОПРАВКАМ ПИОНА В МАССУ КВАРКА В СЛЕДУЮЩЕМ ЗА ГЛАВНЫМ ПОРЯДКЕ РАЗЛОЖЕНИЯ СРЕДНЕГО ПОЛЯ В МОДЕЛИ НИЛ

Вычислены поправки в массу кварка в модели НИЛ в регуляризации с четырехмерным обрезанием и в размерно-аналитической регуляризации в следующем за главным порядке разложения среднего поля. Аналитические вычисления показывают, что пионные поправки в массу кварка равны нулю. Сравнение результатов вычислений в обеих регуляризациях дает возможность утверждать, что равенство нулю пионной поправки в массу кварка является фактом модели НИЛ, независимым от выбора регуляризации.

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