# THE TRANSMISSION OF LIGHT THROUGH A LOW-ABSORBING THIN COATING

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The transmission of light waves through a low- absorbing dielectric thin coating on semi-infinite dielectric substrate is considered. It is shown that in this system under certain conditions may be realized the effect of antireflection of light. Expressions for selective thin film thickness and wavelengths in which occur non-reflective absorption and transmission of light have been obtained.

The theory of optical design of multilayers is quite complex and tedious, and has been presented for some simple cases [1-3]. For microwave region of spectrum the effect of reflectionless absorption in layered medium both theoretically and experimentally was investigated in [4, 5]. This method of investigation was extended to include optical wavelengths in [6].

In this paper, we have presented the approximative method for calculating transmittance, reflectance and absorptance of optical waves by low – absorbing dielectric coating. We start by considering the reflection and transmission of light in an absorbing dielectric film on semi-infinite non-absorbing dielectric substrate. Incident light is considered normally and plane polarized. In order to find expressions for the reflectance and transmittance of a dielectric film illuminated by a parallel beam of light at wavelength $\lambda$, we must consider the multiple reflections of light at each surface of the film and perform a multiple beam summation. Thus, reflected and transmitted complex amplitudes are given by [2]

$$\tilde R = \frac{\tilde r_1 + \tilde r_2 e^{-2ikl}}{1+\tilde r_1 \tilde r_2 e^{-2ikl}} , \qquad \tilde T = \frac{\tilde t_1 \tilde t_2 e^{-ikl}}{1+\tilde r_1 \tilde r_2 e^{-2ikl}}, \tag{1}$$

where $\tilde r_1 = r_1 e^{i\varphi_1}$ ; $\tilde r_2 = r_2 e^{i\varphi_2}$ ; $r_1, r_2, \varphi_1, \varphi_2$ are complex amplitudes, modules and phases of reflection coefficients, respectively, $\kappa$ is complex wave number. In these equations the subscripts 1and 2 refer to the first and second surface of the layer. The layer is assumed to be plane and parallel sided of thickness $l$ and complex refractive index $\tilde n$ and is bounded by semi-infinite layers of indices 1 and $n_1$.

The Fresnel coefficients of reflection and transmission are

$$\tilde r_1 = \frac{1-\tilde n}{1+\tilde n},\ \tilde r_2 = \frac{\tilde n - n_1}{\tilde n + n_1},\ \tilde t_1 = \frac{2}{1+\tilde n},\ \tilde t_2 = \frac{2\tilde n}{n_1+\tilde n} \tag{2}$$

In formula (1), propagation constant $k$ of the wave traveling in the covering material is

$$\kappa = \frac{2\pi}{\lambda}\left(n - i\chi\right) = \frac{2\pi}{\lambda_d}(1-iy) \tag{3}$$

where $\lambda_d = \lambda/n$ , $y = \chi/n$, $n$ is the refractive index of the layer material, $\chi$ is the extinction coefficient of the material.

For the considered layered structures modules and phases of reflection coefficients are given by the well-known relations

$$r_1 = \sqrt{\frac{(1-n)^2+\chi^2}{(1+n)^2+\chi^2}} , \qquad \varphi_1 = arctg\frac{2\chi}{1-n^2-\chi^2}$$
$$r_2 = \sqrt{\frac{(n-n_1)^2+\chi^2}{(n+n_1)^2+\chi^2}} , \qquad \varphi_2 = arctg\frac{2n_1\chi}{n_1^2-n^2-\chi^2} \tag{4}$$

Let introduce $x = l/\lambda_d$ .The expressions for energy reflectance and transmittance may be obtained from equation (1)

$$R^2 = \frac{\left(r_1 - r_2 e^{-4\pi xy}\right)^2 + 4r_1 r_2 e^{-4\pi xy}\cos^2\left(2\pi x + \frac{\varphi_1-\varphi_2}{2}\right)}{\left(1 - r_1 r_2 e^{-4\pi xy}\right)^2 + 4r_1 r_2 e^{-4\pi xy}\cos^2\left(2\pi x - \frac{\varphi_1+\varphi_2}{2}\right)} \tag{5}$$

$$T^2 = \frac{(1-r_1)(1-r_2)e^{-4\pi xy}}{\left(1-r_1 r_2 e^{-4\pi xy}\right)^2 + 4r_1 r_2 e^{-4\pi xy}\cos^2\left(2\pi x - \frac{\varphi_1+\varphi_2}{2}\right)} \tag{6}$$

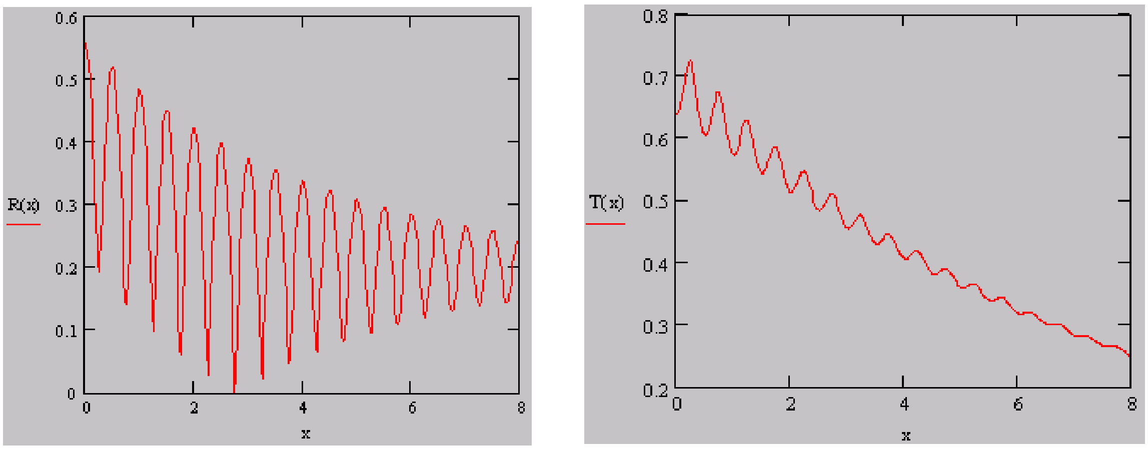


*Fig.1.* Absolute values of reflection *R(x)* and transmission *T(x)* coefficients .vs. thickness of the covering material in the two-layer dielectric-dielectric structure: $r_1$=0.2; $r_2$=0.4; *n*=1.5; *y*=0.02.

Fig.1 represents dependence of reflection coefficient amplitude R and transmission coefficient amplitude *T* on absorbing layer thickness when the coating is low-absorbing. One can see that these coefficients have damping oscillations. Thus, a standing of extremes of curves $R(l)$ are realized at thickness of an absorbing layer of distinct from quantities of multiple $\lambda_d/4$.There are two regions in the $R(l)$ dependence which differ by character of changing of extrema in increasing of thickness of the layer. The lower absorption coefficient is the higher the number of minimum which takes place without reflection.

It is well – known that in its simplest form (when $\chi = 0$) anti- reflecting coating is homogeneous, its thickness is a quarter wavelength and its refractive index equals the geometric mean of the indices of the two adjoining media:

$$n = \sqrt{n_1} \tag{7}$$

In this connection we shall introduce the suggestion that the specified zero minimum of function $R(l)$ is realized at thickness of a layer of the substance a little differing from quantities of multiple $\lambda_d/4$

$$x = \frac{2N-1}{4} + \Delta \tag{8}$$

where $N$ is ordinal number of the zeroth minimum of function $R(l)$; $\Delta$ is the quantity to be determined by optical parameters of coating material.

Substituting equations (3) and (8) into equation (1), we find

$$\ln\frac{r_2}{r_1} = y\left[\pi(2N-1) + \varphi_2 - \varphi_1\right] \tag{9}$$

$$\frac{l}{\lambda} = \frac{1}{n}\left(\frac{2N-1}{4} + \frac{\varphi_2-\varphi_1}{4\pi}\right) \tag{10}$$

$$\Delta = \frac{\varphi_2-\varphi_1}{4\pi} \tag{11}$$

Equations (8)- (11) allow functional relation between incident light wavelength $\lambda$, layer thicknees $l$ and its optical parameters $n$ and $\chi$, for to arise reflectionlees absorption in the coating.

According to dispersion theory of light the real and imaginary parts of dielectric constant may be expressed as [2]

$$n^2 - \chi^2 = n_\infty^2 + \frac{4\pi N_0 q^2}{m}\frac{\omega_1^2-\omega^2}{\left(\omega_1^2-\omega^2\right)^2+\gamma^2\omega^2}; \quad 2n\chi = \frac{4\pi N_0 q^2}{m}\frac{\gamma\omega}{\left(\omega_1^2-\omega^2\right)^2+\gamma^2\omega^2} \tag{12}$$

where $n_\infty$ is refractive index far from the resonance; q, m are charge and mass of electron, $N_0$ is concentration, $\gamma$ is damping coefficient, $\omega$ is frequency. Resonance frequency $\omega_1$ according to Lorentzs field in condensed matter may be expressed by frequency $\omega_0$ and concentration

$$\omega_1^2 = \omega_0^2 - \frac{4\pi N_0 q^2}{m} \qquad (13)$$

It is easy in the first approximation from (12) to have the relation between $n$ and $\chi$

$$\left(n - n_\infty\right)^2 + \left(\chi - d\right)^2 = d^2 \qquad (14)$$

where

$$d = \frac{\pi N_0 q^2}{m\gamma\omega_1 n_\infty} \qquad (15)$$

Solution of equations (9) and (14) allows us to find optical parameters of absorbing dielectric layer.

To determine the frequency which the effect of reflectionless absorption takes place may be used next equation

$$\frac{\chi}{n - n_\infty} = \frac{\gamma}{2(\omega_1 - \omega)} \qquad (16)$$

We have been presented basic expressions to solve the problem of reflection and transmission in absorbing coating. But in learn this problem by practice often we deal with low – absorbing layers $\left(\chi << n\right)$ when $\chi << n$ from equations (4) may be obtained

$$r_1 = \frac{n-1}{n+1}\,, r_2 = \frac{n_1 - n}{n_1 + n}\,; \varphi_2 - \varphi_1 = \frac{2n_1\chi}{n_1^2 - n^2 - \chi^2} - \frac{2\chi}{1 - n^2 - \chi^2} \qquad (17)$$

From equations (9) and (17) in the first approximation one can obtain the next relation

$$\chi = \frac{1}{\pi\left(2N-1\right)} \frac{2n\left(n_1 - n^2\right)}{\left(n-1\right)\left(n+n_1\right)} \qquad (18)$$

Combined solution of equations (4) and (18) allows us to determine selective quantities of $n$ and $\chi$ for low – absorbing dielectric layer.

In Fig.2 the dependence of selective values of $\chi$ on $n$ calculated from equation (18) for low-absorbing dielectric thin film applied on dielectric substrate is presented. There also is given the dependence between $\chi$ and $n$ for rhodamine layer with concentration of dye molecules $N_0 = 6.10^{21} cm^{-3}$. The crossing points of straight lines and circle allow to find selective values of $n$ and $\chi$. Then using equations (16) and (8) we have calculated selective wavelength $\lambda_0$ and thickness of coating $l_0$, respectively. The results of these calculations are given in Table. One can see from Table the values of $n$ and $\chi$ of coating for given N and appropriate values of $\lambda_0$ and $l_0$ for to realise the effect of antireflection of light.

The energy absorbed in low – absorbing layer in the reflectionless absorption may be expressed by

$$W = 1 - n_1\left|f\right|^2 = n_1\sqrt{\frac{(1-n^2)^2 + 2\chi^2(1+n^2)}{(n_1^2 - n^2)^2 + 2\chi^2(n_1^2 + n^2)}} \qquad (19)$$

Optical parameters $n_1, n$ and $\chi$ in (19) are quantities which satisfied condition of reflectionless absorption and related in equation (9).

The reflection and transmission of light by absorbing two-layer dielectric- dielectric structure is theoretically considered. Expressions for selective layer thickness and frequencies in which occur reflectionless absorption of light in this system have been obtained. The conditions for to realize anti-reflecting coatings in optical medium with resonance type dispersion have been found.

Propagation of light in low-absorbing layered system has been analyzed. The energy transmitted from low-absorbing layer when reflectionless absorption takes place was calculated.

Table.

Selective values of incident radiation wavelength $\lambda_0$, refraction index $n$, extinction coefficient $\chi$, rhodamine layer thickness $l_0$ applied on dielectric substrate. $N$ is number of the minimum of curve $R$; $W$ is absorbed energy in the rhodamine layer. Refraction index of the dielectric substrate $n_1 = 4$.

| N | Low-frequency branch | | | | | High-frequency branch | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $N$ | $\chi$ | $\lambda_0 \cdot 10^{-5} cm$ | $l_0 \cdot 10^{-5} cm$ | $W$ | $n$ | $\chi$ | $\lambda_0 \cdot 10^{-5} cm$ | $l_0 \cdot 10^{-5} cm$ | $W$ |
| 1 | - | - | - | - | - | - | - | - | - | - |
| 2 | 1,84 | 0,05 | 5,96 | 2,54 | 0,24 | 1,67 | 0,11 | 5,35 | 2,28 | 0,46 |
| 3 | 1,83 | 0,03 | 6,10 | 4,34 | 0,26 | 1,67 | 0,07 | 5,19 | 3,68 | 0,46 |
| 4 | 1,82 | 0,02 | 6,20 | 6,16 | 0,27 | 1,68 | 0,05 | 5,17 | 5,15 | 0,45 |
| 5 | 1,81 | 0,02 | 6,51 | 8,32 | 0,28 | 1,69 | 0,04 | 5,09 | 6,51 | 0,43 |

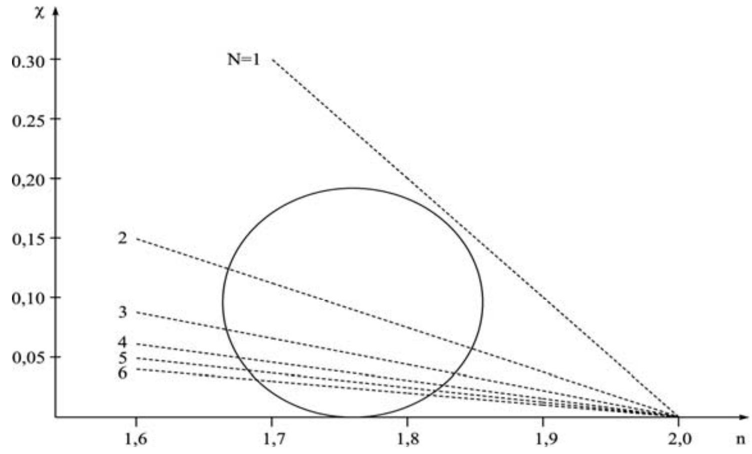


*Fig.2.* Dependences of extinction coefficient $\chi$ on refraction index $n$ of the dielectric thin film:

a) applied on dielectric substrate (-------);

b) according to its resonance type dispersion in the range of optical wavelengths for rhodamine layer with concentration of dye molecules $N_0 = 6.10^{21} cm^{-3}$ (circle).

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## İŞIĞIN ZƏİF UDAN NAZİK ÖRTÜKDƏN KEÇMƏSİ

Yarımsonsuz dielektrik səthinə çəkilmiş zəif udan nazik dielektrik örtükdən işıq dalğalarının keçməsinə baxılmışdır. Göstərilmişdir ki, bu sistemdə müəyyən şərtlər ödənildikdə işığın əks olunmadan udulması hadisəsi mümkündür. İşığın əks olunmadan udulması və keçməsinin baş verməsinə uyğun selektiv örtük qalınlıqları və dalğa uzunluqları üçün ifadələr alınmışdır.

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## ПРОХОЖДЕНИЕ СВЕТА ЧЕРЕЗ СЛАБОПОГЛОЩАЮЩЕЕ ТОНКОЕ ПОКРЫТИЕ

Рассматривается прохождение световых волн через слабопоглощающее тонкое диэлектрическое покрытие на полубесконечной диэлектрической подложке. Показано, что в этой системе при определенных условиях возможен эффект безотражательного поглощения света. Получены выражения для селективных значений толщины тонкой пленки и длин волн, при которых происходит безотражательное поглощение и прохождение света.

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