ABOUT LIMITATION OF ELEMENTERY PARTICLES MASS

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The simple examples of spontaneous breaking of various symmetries for the scalar theory with fundamental mass have been considered. Higgs' generalizations on "fundamental mass" that was introduced into the theory on a basis of the five-dimensional de Sitter space are found.

The concept of mass having its root in great antiquity still remains fundamental. Every theoretical and experimental research in classical physics and the quantum physics, related to mass - a step to an insight of the nature. Besides mass, other fundamental constants, such as Planck's constant $\hbar$ and speed of light c, also play the most important role in modern theories. The first one is related to quantum mechanics, and the second one is related to the theory of relativity.

Characteristics and interactions of elementary particles (EP) can be described more or less in terms of local fields (LP) which in their turn regard to low representations of corresponding compact groups of symmetry. Concept of LP essentially is a synonym of concept EP. At present elementary particles are such kind of particles (real and hypothetical), characteristics and interactions of which could be adequately described in terms of LP. As we know, mass of E P m is Kazimir's operator of noncompact Poincare group, and those representations of the given group which are used in the quantum field theory (QFT), can take any values in an interval $0 \le m < \infty$. Two particles today mentioned as EP can have masses different from each other on many orders. Formally standard QFT remains logically irreproachable circuit in cases when masses of particles can be comparable to masses of macromatters. Modern QFT does not forbid such physically nonsensical extrapolation. Probably it is the basic defect of the theory?

In 1965 M.A. Markov has put forward a hypothesis [1] according to which the spectrum of masses of EP should break on «planck mass»

$$m < m_{Planck.} = \sqrt{\hbar c / G} \approx 10^{19}\,Gev \ , \qquad (1)$$

Here $\hbar$, c are known universal constants and G is a gravitational constant. The particles of limiting mass $m = m_{Planck.}$ named by M.A. Markov as "maximons" are called to play a special role in the world of elementary particles. The concept of "maximon" is assumed as a basis of Markov's script of the early universe [2]. It is significant that in relation to QFT Markov's restriction (1) acts as an additional phenomenological condition. It does not affect structure of this theory in any way, and even for the description of maximon the standard theoretical-field device is used. New version of QFT, in basis of which the postulate-M.A.Markov's principle about limitation of mass of elementary particles (1) is put alongside with traditional quantum and relativistic postulates, has been worked out by V.G. Kadyshevsky [3]. The key role in the approach developed by him belongs to 5-dimensional configuration representation. Remaining inherently four-dimensional, the theory assumes the original local Langrangian formulation in which dependence of fields on auxiliary fifth coordinate also is found as local. Internal symmetries in this formalism generate the gauge transformations localized in the same 5-dimensional configuration space. Thus Markov's condition is written down as $m \le M$, considering limiting mass M simply as a certain new universal constant of the theory, so-called «fundamental mass» (FM). EP with $m = M$ are still called as maximons. In the limit $M \to \infty$ new QFT coincides with the usual field theory in which the spectrum of particles is unlimited. On a strict mathematical basis new parameter FM is entered in QFT which. Together with parameters of the standard quantum theory this parameter will play an essential role in high energy physics [4]. In work [5] geometrical interpretation of effect of spontaneous breaking of symmetry which plays a key role in standard model is advanced. This approach is related to an effective utilization in device QFT of 4-pulse de Sitter and anti-de Sitter's spaces with constant curvature. In our works [6] simple examples of spontaneous breakings of various symmetries for the scalar theory with FM have been considered and Higgs' generalizations on FM are cited.

In the given work we shall continue research on the basis of simple examples of spontaneous breakings of various symmetries for the scalar theory with ΦM. For this purpose we use Lagrangian formalism from works [3, 4].

Formulation of QFT with FM, discussed in work [4], is based on the quantum version of the de Sitter's equation, that is on the 5-dimensional equation of a field:

$$\left[\frac{\partial^2}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial^2}{\partial x_5^2} - \frac{M^2 c^2}{\hbar^2}\right]\Phi(x, x^5) = 0 \qquad {}_{\mu=0,1,2,3}\,. \quad (2)$$

To every field in 5-space a wave function $\Phi(x, x^5)$ submitting with the equation (2) is compared. This is equivalent to the statement that $\Phi(x, x^5)$ the field in usual space-time is described by wave function with the double number of components:

$$\Phi(x,x^5)\leftrightarrow\begin{pmatrix}\Phi(x,0)\\ \dfrac{\partial\Phi(x,0)}{\partial x^5}\end{pmatrix}\equiv\begin{pmatrix}\Phi(x)\\ \chi(x)\end{pmatrix}. \qquad (3)$$

Typical for this circuit the doubling of number of field degrees of freedom disappears at $M\to\infty$. At finite M the analogue of a usual field variable should be considered $\Phi(x)=\Phi(x,0)$, and function $\chi(x)=\dfrac{\partial\Phi(x,0)}{\partial x^5}$ is auxiliary.

Now we shall consider simple examples of spontaneous breakings of various symmetries for the scalar theory with FM.

The Lagrangian of the real scalar field in frameworks of QFT with FM has the form [4]:

$$L_0(x,M)=\frac{1}{2}\left[\left[\frac{\partial\Phi(x)}{\partial x_n}\right]^2+m^2\Phi^2(x)+M^2\left[\chi(x)-\cos\mu\Phi(x)\right]^2\right]. \qquad (4)$$

Taking into account interaction in (3), we can (4) write following:

$$L(x,M)=\frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_\mu}\right)^2-\frac{1}{2}m^2\varphi^2(x)-\frac{M^2}{2}\left(\chi(x)-\cos\mu\varphi(x)\right)^2-\chi(x)U(\varphi(x)), \qquad (5)$$

here $\cos\mu\equiv\sqrt{1-\dfrac{m^2}{M^2}}$, where m - mass of the particles, described by field φ, and χ is the auxiliary field, playing a role in interaction, M - fundamental mass and $U(\varphi)$ is the unknown function describing interactions of particles.

Whether is possible to choose the interaction $L_{\text{int}}=\chi(x)U(\varphi(x))$ between fields $\varphi(x)$ and $\chi(x)$ that Higg's potential for a field $\varphi(x)$ exists at exception of a field $\chi(x)$? Free Lagrangian (5) is invariant under transformation $\varphi\to-\varphi$ and $\chi\to-\chi$. But thus is necessary to demand, that $U(-\varphi)=-U(\varphi)$, that $U(\varphi)$ is an odd function of φ. Action for (5) is possible to be written as:

$$S=\int\{\frac{1}{2}[(\frac{\partial\varphi(x)}{\partial x_\mu})^2-m^2\varphi^2(x)-M^2(\chi(x)-\cos\mu\varphi(x))^2]+\chi(x)U(\varphi(x))\}d\varphi(x). \qquad (6)$$

If we differentiate (6) on $\chi(x)$, we find:

$$\chi(x)=\cos\mu\varphi(x)+\frac{U(\varphi(x))}{M^2} \qquad (7)$$

Substituting (7) in (6), we have:

$$L_{tot}(\varphi)=\frac{1}{2}[(\frac{\partial\varphi}{\partial x_\mu})^2-m^2\varphi^2+\frac{U^2(\varphi)}{M^2}+2U(\varphi)\cos\mu\varphi], \qquad (8)$$

that is invariant to $\varphi\to-\varphi$.

From breakings of discrete symmetry for usual scalar field it is known that Higg's potential looks like:

$$V(\varphi)=-\frac{1}{2}m^2\varphi^2+\frac{1}{4}\lambda^2\varphi^4, \qquad (9)$$

where λ is the dimensionless constant describing interaction between particles.

Let find a kind of $U(\varphi)$ function that in (8) potential Higgs to appear. We shall consider the Lagrangian (8) at $m\to im$, then $\cos\mu=\sqrt{1-\dfrac{m^2}{M^2}}\to ch\mu'=\sqrt{1+\dfrac{m^2}{M^2}}$.

Potential energy (9) shall look like:

$$V(\varphi)=-\frac{1}{2}m^2\varphi^2-\frac{U^2}{2M^2}-U(\varphi)ch\mu'\varphi. \qquad (10)$$

Comparing (10) and (9), for $U(\varphi)$ we have two different roots (real and imaginary) at $\varphi^2<\dfrac{2M^2ch^2\mu'}{\lambda^2}$ and one $U(\varphi)=-M^2ch\mu'\varphi$ at $\varphi^2=\dfrac{2M^2ch^2\mu'}{\lambda^2}$. It results to that $L_{tot}(\varphi)_{Higgs}=L^0_{\max imon}(\varphi)$, i.e.

$$L^0_{\max imon}(\varphi)=\frac{1}{2}(\frac{\partial\varphi}{\partial x_\mu})^2-\frac{1}{2}M^2\varphi^2. \qquad (11)$$

Now we shall consider a case when $L_{\text{int}}(\varphi)=-\dfrac{\lambda^2}{4}\varphi^2\chi^2$. At $m\to im$ Lagrangian (5) shall look like:

$$L_{tot}(\varphi)=\frac{1}{2}[(\frac{\partial\ \varphi}{\partial\ \mathrm{x}_{\mu}})^2+M^2sh^2\mu^{'}\varphi^2-M^2(\chi-ch\mu^{'}\varphi)^2-\frac{\lambda^2}{2}\varphi^2\chi^2],\qquad(12)$$

where $M\,sh\mu^{'}=m$.

If we differentiate (12) on χ, we find: $\chi=\dfrac{M^2ch\mu'\varphi}{M^2+\dfrac{\lambda^2\varphi^2}{2}}$. Now (12) shall look like:

$$L_{tot}(\varphi)=\frac{1}{2}\left[(\frac{\partial\ \varphi}{\partial\ \mathrm{x}_{\mu}})^2+M^2sh^2\mu^{'}\varphi^2-\frac{\lambda^2}{2}\varphi^2(\frac{\lambda^2\varphi^2}{2M^2}+1)\frac{M^4ch^2\mu^{'}\varphi^2}{(M^2+\dfrac{\lambda^2\varphi^2}{2})^2}\right].\qquad(13)$$

This is one of Higg's generalizations on fundamental mass. From (13) at $M\to\infty$ we shall receive the usual Higg's Lagrangian:

$$\lim_{M\to\infty}L_{tot}(\varphi)=\frac{1}{2}\left[(\frac{\partial\ \varphi}{\partial\ \mathrm{x}_{\mu}})^2+m^2\varphi^2-\frac{\lambda^2\varphi^4}{2}\right].\qquad(14)$$

In case of spontaneous breaking of global symmetry $U(1)$ we have:

$$\begin{aligned}L_{tot}(x)&=\left|\frac{\partial\ \varphi}{\partial\ \mathrm{x}}\right|^2-m^2|\varphi|^2-M^2|\chi-\cos\mu\varphi|^2-\frac{\lambda^2}{2}|\chi|^2|\varphi|^2=\\&=\left|\frac{\partial\varphi}{\partial\ \mathrm{x}}\right|^2-M^2|\varphi|^2-M^2|\chi|^2+M^2\cos\mu\,|\chi\overline{\varphi}+\varphi\overline{\chi}|-\frac{\lambda^2}{2}|\chi|^2|\varphi|^2\end{aligned},\qquad(15)$$

at $m\to im$, then

$$L_{tot}(x)=\left|\frac{\partial\ \varphi}{\partial\ \mathrm{x}}\right|^2-M^2|\varphi|^2-M^2|\chi|^2+M^2ch\mu^{'}|\chi\,\overline{\varphi}+\varphi\,\overline{\chi}|-\frac{\lambda^2}{2}|\chi|^2|\varphi|^2.\qquad(16)$$

This Lagrangian differs from (15) by its sign before m^2, but still invariant to group of global transformations:

$$\begin{aligned}&\varphi(x)\to\varphi(x)=e^{ig\varepsilon}\varphi(x),\qquad \varphi^*(x)\to\varphi^*(x)=e^{ig\varepsilon}\varphi^*(x)\\&\chi\to\chi(\mathrm{x})=e^{ig\varepsilon}\chi(x),\qquad \chi^*(x)\to\chi^*(x)=e^{ig\varepsilon}\chi^*(x)\end{aligned}.\qquad(17)$$

Taking a derivative from (16) on χ and $\overline{\chi}$, we shall find the equation of motion for $\overline{\chi}$ and χ accordingly:

$$-M^2\overline{\chi}+\mathrm{M}^2ch\mu^{'}\overline{\varphi}-\frac{\lambda^2}{2}|\varphi|^2\overline{\chi}=0\ \text{ and }\ -M^2\chi+\mathrm{M}^2ch\mu^{'}\varphi-\frac{\lambda^2}{2}|\varphi|^2\chi=0$$

From these equations we find: $\overline{\chi}=\dfrac{ch\mu^{'}\overline{\varphi}}{1+\dfrac{\lambda^2}{2M^2}|\varphi|^2}$ and $\chi=\dfrac{ch\mu^{'}\varphi}{1+\dfrac{\lambda^2}{2M^2}|\varphi|^2}$. Having substituted these values in (16), we shall find:

$$L_{tot}(x)=\left|\frac{\partial\ \varphi}{\partial\ \mathrm{x}}\right|^2-V(\varphi),\qquad(18)$$

where $V(\varphi)$ is Higg's potential $V(\varphi)=M^2|\varphi|^2-\dfrac{M^2ch^2\mu'|\varphi|^2}{1+\dfrac{\lambda^2}{2M^2}|\varphi|^2}$. This potential has the minimum $V_{\min}(|\varphi|) = -\frac{2M^4}{\lambda^2}(ch\mu' - 1)^2$ at $|\varphi|^2 = \frac{2M^2}{\lambda^2}(ch\mu' - 1)$. In a flat limit $M \to \infty$ (18) will have a usual form. If we shall write as $V_{New}(|\varphi|) = V(|\varphi|) - V_{\min}(|\varphi|)$ then we have:

$$V_{New}(|\varphi|) = \frac{\lambda^2}{2} \frac{\left[|\varphi|^2 - \frac{h^2}{2}\right]^2}{1 + \frac{\lambda^2}{2M^2}|\varphi|^2}, \qquad (19)$$

where $\frac{h^2}{2} = \frac{2M^2}{\lambda^2}(ch\mu' - 1)$, this quantity at $M \to \infty$ is equal $\frac{m^2}{\lambda^2}$. At $M \to \infty$ (19) has a usual form $\lim_{M\to\infty} V_{New}(|\varphi|) = \frac{\lambda^2}{2}\left(|\varphi|^2 - \frac{m^2}{\lambda^2}\right)^2 = V$. It is obvious, that function (19) $V_{New}(|\varphi|)$ has a minimum at $|\varphi_0|^2 = \frac{2M^2}{\lambda^2}(ch\mu' - 1)$. It is always possible to choose as vacuum material value $\varphi_0 = \frac{\sqrt{2}M}{\lambda}\sqrt{ch\mu' - 1}$. For $L_{tot}(x)$ we receive expression:

$$L_{tot}(x) = \left|\frac{\partial\varphi}{\partial x}\right|^2 + \frac{\lambda^2}{2} \frac{\left[|\varphi|^2 - \frac{h^2}{2}\right]^2}{1 + \frac{\lambda^2}{2M^2}|\varphi|^2}. \qquad (20)$$

This Lagrangian invariant to global gauge *U(1)* - transformation: $\varphi \to \varphi' = \varphi e^{i\alpha}$. The system described by Lagrangian (20), has spontaneously broken symmetry *U(1)*. Now the point $\varphi(x) = \bar{\varphi}^*(x) = 0$ does not corresponding with a minimum of energy. There any point on a circle of radius $R = \sqrt{2}\frac{M^2}{\lambda}\sqrt{ch\mu' - 1}$ is agree with a minimum of energy. We can choose as stable vacuum any position, situated on a circle of radius *R*, that is all states are equivalent because of change concerning transformation (17). We shall choose value of gauge phases $\alpha = 0$, uniform for all the world, and we shall write down *φ(x)* in the form of real and imaginary parts:

$$\varphi(x) = \frac{1}{\sqrt{2}}(h + \varphi_1(x) + i\varphi_2(x)), \qquad (21)$$

here $\varphi_1(x)$ and $\varphi_2(x)$ are two material fields, describing excitation of system concerning vacuum $\varphi(x) = \frac{h}{\sqrt{2}}$. At transition to stable vacuum *U(1)* invariance is broken, as the phase of function φ is fixed.

In new variables for Lagrangian (20) we have:

$$L_{tot}(\varphi) \Rightarrow L_{tot}(\varphi_1, \varphi_2) = \\ = \frac{1}{2}\left(\frac{\partial\varphi_1(x)}{\partial x_\mu}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi_2(x)}{\partial x_\mu}\right)^2 + \frac{\lambda^2}{8} \frac{\left(\varphi_1^2 + 2h\varphi_1 + \varphi_2^2\right)^2}{1 + \frac{\lambda^2}{4M^2}\left[(h + \varphi_1)^2 + \varphi_2^2\right]}. \qquad (22)$$

As a result of spontaneous breaking of symmetry the goldstone scalar massless particle φ_2 and the real scalar particle φ_1 with mass $m_1 = \frac{\lambda h}{\sqrt{1 + \frac{\lambda^2 h^2}{4M^2}}}$ have appeared. At $M \to \infty$ we have $m_1 = \sqrt{2}m$.

Acknowledgement

R.I. and U.Kh. gratefully acknowledges support by the Volkswagenstiftung.

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ELEMENTAR ZƏRRƏCİKLƏRİN KÜTLƏ MƏHDUDİYYƏTİ HAQQINDA

Fundamental kütləli skalyar nəzəriyyənin köməyi ilə müxtəlif simmetriyaların spontan pozulmasına aid sadə misallar təhlil edilmişdır. “Fundamental kütlə” üçün Higgs ümumiləşməsi tapılmışdır. Bu ümumiləşmə nəzəriyyəyə de-Sitterin beş ölçülü fəzası əsasında daxil edilmişdir.

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ОБ ОГРАНИЧЕНИИ МАСС ЭЛЕМЕНТАРНЫХ ЧАСТИЦ

Рассмотрены простые примеры спонтанного нарушения различных симметрий для скалярной теории с фундаментальной массой. Найдены обобщения Хиггса на “фундаментальную массу”, которые введены в теорию на основе пятимерного пространства де-Ситтера.

Received: 17.01.07