B-BARYON POLARIZATION IN SEMI-INCLUSIVE REACTIONS $\nu_\mu(\bar{\nu}_\mu)N \Rightarrow \mu^-(\mu^+)BX$, $\mu^-(\mu^+)N \Rightarrow \nu_\mu(\bar{\nu}_\mu)BX$

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The common expressions for effective cross-sections of semi-inclusive reactions $\nu_\mu(\overline{\nu_\mu}) + N \Rightarrow \mu^-(\mu^+) + B + X$ and $\mu^-(\mu^+) + N \Rightarrow \nu_\mu(\overline{\nu_\mu}) + B + X$ in the limits of quark-parton model are obtained. The longitudinal polarization degree of Λ^0-hyperon is defined. It is shown, that longitudinal polarization of Λ^0-hyperon is only *z* function and doesn't depend on x and y variables, if it is possible to disregard the antiquark contribution in fragmentation functions. The dependence of longitudinal polarization degree of Λ^0-hyperon on *z* variable in the different variants of fragmentation function choice of polarized quark into polarized Λ^0-hyperon is investigated.

As it is known, the weak neutral currents (WMC) were firstly observed in the processes of deep-inelastic scattering (DIS) by neutrino (antineutrino) nucleons $\nu_\mu(\bar{\nu}_\mu) + N \Rightarrow \nu_\mu(\bar{\nu}_\mu) + X$. The study of these and another lepton-nucleon processes allows us to obtain the information about structure of WMC leptons and hadrons and about distribution functions of quarks and gluons in nucleons.

Last years the new class of processes, which are semi-inclusive hadron creation in DID of polarized leptons on polarized nucleons [1-4], the study of which is the information source about distribution and fragmentation functions of polarized quarks and gluons, has been well discussed.

Here B-baryon polarization in semi-inclusive reactions is considered

$$\nu_\mu + N \Rightarrow \mu^- + B + X\,, \qquad (1)$$

$$\overline{\nu_\mu} + N \Rightarrow \mu^+ + B + X\,, \qquad (2)$$

$$\mu^- + N \Rightarrow \nu_\mu + B + X\,, \qquad (3)$$

$$\mu^+ + N \Rightarrow \overline{\nu_\mu} + B + X\,, \qquad (4)$$

where the initial nucleon is longitudinal polarized (or unpolarized), neutrino is always polarized (neutrino spirality is $\lambda_\nu = -1$, and antineutrino helicuty is $\lambda_{\bar{\nu}} = +1$), B is emphasized inclusive baryon with measured longitudinal polarization h_B.

The differential cross-section of semi-inclusive reactions (1)-(4) in the framework of quark-parton model can be written in the following form:

$$\frac{d\sigma}{dxdydz} = \sum_{q,h_q} \sum_{q',h'_q} f^{N(h_N)}_{q(h_q)}(x) \frac{d\hat{\sigma}}{dy} D^{B(h_B)}_{q'(h'_q)}(z) \qquad (5)$$

.Here $f^{N(h_N)}_{q(h_q)}(\mathrm{x})$ is distribution function of polarized quark in polarized nucleon; $D^{B(h_B)}_{q'(h'_q)}(z)$ is fragmentation function of polarized quark in polarized baryon B; $\frac{d\hat{\sigma}}{dy}$ is differential cross-section of parton subprocess; *x*, *y* and *z* are ordinary kinematic variables of DIS.

The negative muon forms in semi-inclusive reaction (1) at neutrino scattering that's why the parton electric charge (quark or antiquark) should increase on the one unit. The positive muon forms at antineutrino scattering and therefore the parton electric charge should decrease on the one unit. Thus, neutrino can interact with *d*- and *s*-quarks, which transform to u-quark or with $\bar{u}$ -quark, which transforms to $\bar{d}$ - or antiquark (only contributions of *u*-, *d*- and *s*-quarks are taken into consideration for simpleness):

$$\nu_\mu + q \Rightarrow \mu^- + q'\text{, where } q = d\,;\ s\,;\ q' = u\,,$$
$$\nu_\mu + \bar{q} \Rightarrow \mu^- + \bar{q}'\text{, where } \bar{q} = u\,;\ \bar{q}' = d\,;\ \bar{s}\,. \qquad (6)$$

Analogically, neutrino can interact with *u*-quark wchich transforms to d- and s-quark, and also with $\bar{d}$ - and $\bar{s}$ -antiquarks, which transform to $\bar{u}$ -antiquark:

$$\overline{\nu_\mu} + q \Rightarrow \mu^+ + q'\text{, where } q = u\,;\ q' = d\,; s\,,$$
$$\overline{\nu_\mu} + \bar{q} \Rightarrow \mu^+ + \bar{q}'\text{, where } \bar{q} = \bar{d}\,; \bar{s}\,;\ \bar{q}' = \bar{u}\,. \qquad (7)$$

As quark spirality keeps in disregard of their masses, then elementary subprocesses (6) and (7) are defined by only one spiral amplitude: $\hat{M}_{\lambda_\mu,\lambda_{q'};\lambda_\nu,\lambda_q}$

$$\hat{M}^{\nu_\mu q \Rightarrow \mu^- q'}_{L,L;L,L} = \hat{M}^{\bar{\nu}_\mu \bar{q} \Rightarrow \mu^+ \bar{q}'}_{R,R;R,R} = \frac{4\pi\alpha \cdot U_{qq'}}{x_w} \cdot \frac{xs}{xys + M_W^2}\ , \qquad (8)$$

$$\hat{M}^{\nu_\mu \bar{q} \Rightarrow \mu^- \bar{q}'}_{L,R;L,R} = \hat{M}^{\bar{\nu}_\mu \bar{q} \Rightarrow \mu^+ \bar{q}'}_{R,L;R,L} = \frac{4\pi\alpha \cdot U_{qq'}}{x_w} \cdot \frac{xs(1-y)}{xys + M_W^2} \quad , \tag{9}$$

where $x_w = sin^2\theta_w$, $U_{ud} = cos\theta_C$, $U_{us} = sin\theta_C$; θ_w is Weinberg angle; θ_C is Cabibbo angle; M_w is mass of $W^\pm$ - Bose particle, s is square of sum energy of initial particles in c.m.system.

Let's give the cross-sections of parton processes at definite spiralities of initial and final particles:

$$\frac{d\hat{\sigma}(\nu_L q_L \Rightarrow \mu_L^- q_L')}{dy} = \frac{d\hat{\sigma}(\bar{\nu}_R \bar{q}_R \Rightarrow \mu_R^+ \bar{q}_R')}{dy} = \pi\alpha^2 xsF_{LL}^2 ,$$
$$\frac{d\hat{\sigma}(\nu_L \bar{q}_R \Rightarrow \mu_L^- \bar{q}_R')}{dy} = \frac{d\hat{\sigma}(\bar{\nu}_R q_L \Rightarrow \mu_R^+ q_L')}{dy} = \pi\alpha^2 xsF_{LL}^2 \cdot (1-y)^2 , \tag{10}$$

where the designation $F_{LL} = \frac{U_{qq'}}{x_W}(xys + M_W^2)^{-1}$ is introduced.

From formula (10) it is seen, that neutrino and antineutrino interact only with right polarized quarks and left polarized antiquarks.

For differential cross-sections of semi-inclusive processes (1) and (2) on the base of formulae (5) and (10) we have the following expressions:

$$\frac{d\sigma(\nu_\mu N)}{dxdydz} = \frac{\pi\alpha^2 xs}{2}\sum_{q,q'} F_{LL}^2 \Big\{ f_q^N D_{q'}^B + f_{\bar{q}}^N D_{\bar{q}'}^B (1-y)^2 - h_N \Big[\Delta f_q^N D_{q'}^B - \Delta f_{\bar{q}}^N D_{\bar{q}'}^B (1-y)^2\Big] -$$
$$- h_B \Big[f_q^N \Delta D_{q'}^B - f_{\bar{q}}^N \Delta D_{\bar{q}'}^B (1-y)^2\Big] + h_N h_B \Big[\Delta f_q^N \Delta D_{q'}^B + \Delta f_{\bar{q}}^N \Delta D_{\bar{q}'}^B (1-y)^2\Big] \Big\}, \tag{11}$$

$$\frac{d\sigma(\bar{\nu}_\mu N)}{dxdydz} = \frac{\pi\alpha^2 xs}{2}\sum_{q,q'} F_{LL}^2 \Big\{ f_q^N D_{q'}^B (1-y)^2 + f_{\bar{q}}^N D_{\bar{q}'}^B - h_N \Big[\Delta f_q^N D_{q'}^B (1-y)^2 - \Delta f_{\bar{q}}^N D_{\bar{q}'}^B\Big] -$$
$$- h_B \Big[f_q^N \Delta D_{q'}^B (1-y)^2 - f_{\bar{q}}^N \Delta D_{\bar{q}'}^B\Big] + h_N h_B \Big[\Delta f_q^N \Delta D_{q'}^B (1-y)^2 + \Delta f_{\bar{q}}^N \Delta D_{\bar{q}'}^B\Big] \Big\}, \tag{12}$$

where

$$f_q^N = f_{q(+1)}^{N(+1)} + f_{q(-1)}^{N(+1)},\ \Delta f_q^N = f_{q(+1)}^{N(+1)} - f_{q(-1)}^{N(+1)},$$
$$D_{q'}^B = D_{q'(+1)}^{B(+1)} + D_{q'(-1)}^{B(+1)},\ \Delta D_{q'}^B = D_{q'(+1)}^{B(+1)} - D_{q'(-1)}^{B(+1)}.$$

Here the longitudinal polarization degree of inclusive B baryon interests us:

$$P_B(h_N) = \frac{d\sigma(h_N; h_B = 1) - d\sigma(h_N; h_B = -1)}{d\sigma(h_N; h_B = 1) + d\sigma(h_N; h_B = -1)} , \tag{13}$$

which can be measured on angular distribution of decay products in $B \Rightarrow N + \pi$ reaction. Summing on quark flavor, for longitudinal polarization degree of baryon we have the expressions:

1) In semi-inclusive reaction $\nu_\mu + N \Rightarrow \mu^- + B + X$:

$$P_B^{\nu_\mu N}(h_N) = \Big\{\Big[- f_d^N - Rf_s^N + h_N(\Delta f_d^N + R\Delta f_s^N)\Big]\Delta D_u^B + (1-y)^2 (f_{\bar{u}}^N + h_N \Delta f_{\bar{u}}^N)$$
$$\times (\Delta D_{\bar{d}}^B + R\Delta D_{\bar{s}}^B)\ \Big\} \times \Big\{\Big[f_d^N + Rf_s^N - h_N(\Delta f_d^N + R\Delta f_s^N)\Big]D_u^B + (1-y)^2 (f_{\bar{u}}^N + h_N \Delta f_{\bar{u}}^N)$$
$$\times (D_{\bar{d}}^B + RD_{\bar{s}}^B)\ \Big\}^{-1}; \tag{14}$$

2) In semi-inclusive reaction $\bar{\nu}_\mu + N \Rightarrow \mu^+ + B + X$:

$$P_B^{\bar{\nu}_\mu N}(h_N) = \Big\{(-f_u^N - h_N \Delta f_u^N)(\Delta D_d^B + R\Delta D_s^B)(1-y)^2 +$$
$$\Big[f_{\bar{d}}^N + Rf_{\bar{s}}^N + h_N(\Delta f_{\bar{d}}^N + R\Delta f_{\bar{s}}^N)\Delta D_{\bar{u}}^B\Big]\ \Big\} \times \Big\{(f_u^N - h_N \Delta f_u^N)(D_d^B + RD_s^B)(1-y)^2 +$$
$$\Big[f_{\bar{d}}^N + Rf_{\bar{s}}^N + h_N(\Delta f_{\bar{d}}^N + R\Delta f_{\bar{s}}^N)D_{\bar{u}}^B\Big]\ \Big\}^{-1}, \tag{15}$$

where $R \equiv tg^2\theta_C \cong 0.056$.

Particularly, if the initial nucleon isn't polarized then the longitudinal polarization degrees (14) and (15) have the form:

$$P_B^{\nu_\mu N}(h_N) = -\frac{(f_d^N + Rf_s^N)\Delta D_u^B - (1-y)^2 f_{\bar u}^N(\Delta D_{\bar d}^B + R\Delta D_s^B)}{(f_d^N + Rf_s^N)D_u^B + (1-y)^2 f_{\bar u}^N(D_{\bar d}^B + RD_s^B)}, \tag{16}$$

$$P_B^{\bar\nu_\mu N}(h_N) = -\frac{f_u^N(\Delta D_d^B + R\Delta D_s^B)(1-y)^2 - (f_{\bar d}^N + Rf_{\bar s}^N)\Delta D_{\bar u}^B}{f_u^N(D_d^B + RD_s^B)(1-y)^2 + (f_{\bar d}^N + Rf_{\bar s}^N)D_{\bar u}^B}. \tag{17}$$

The above mentioned formulae for longitudinal polarization degree (14)-(17) are true in the case of creation of arbitrary baryon B with spin ½. For example, let's consider the semi-inclusive creation of Λ^0-hyperon in $\nu_\mu + N \Rightarrow \mu^- + \Lambda^0 + X$ and $\bar\nu_\mu + N \Rightarrow \mu^+ + \Lambda^0 + X$ reactions. In this case we can neglect the fragmentation functions of antiquarks in Λ^0-hyperon, as the contribution is small at big x and z.

Then we obtain the simple expressions for longitudinal polarization degree of Λ^0-hyperon:

$$P_\Lambda^{\nu_\mu N}(h_N = \pm 1) = P_\Lambda^{\nu_\mu N} = -\frac{\Delta D_u^\Lambda(z)}{D_u^\Lambda(z)} \tag{18}$$

$$P_\Lambda^{\bar\nu_\mu N}(h_N = \pm 1) = P_\Lambda^{\bar\nu_\mu N} = \frac{\Delta D_d^\Lambda(z) + R\Delta D_s^\Lambda(z)}{D_d^\Lambda(z) + RD_s^\Lambda(z)}. \tag{19}$$

As it is seen, the longitudinal polarization of Λ^0-hyperon is only z function and doesn't depend on variables x and y. The study of this polarization can give the valuable information about fragmentation function of polarized quark in polarized baryon.

The analogous results are obtained in semi-inclusive processes $\mu^- + N \Rightarrow \nu_\mu + B + X$, $\mu^+ + N \Rightarrow \bar\nu_\mu + B + X$. The formulae:

$$\begin{aligned}
&\mu^- + q \Rightarrow \nu_\mu + q', \text{ where } q = u,\ q' = d, s;\\
&\mu^- + \bar q \Rightarrow \nu_\mu + \bar q', \text{ where } \bar q = \bar d, \bar s,\ \bar q' = \bar u;\\
&\mu^+ + q \Rightarrow \bar\nu_\mu + q', \text{ where } q = d, s,\ q' = u;\\
&\mu^+ + \bar q \Rightarrow \bar\nu_\mu + \bar q', \text{ where } \bar q = \bar u,\ \bar q' = \bar d, \bar s.
\end{aligned} \tag{20}$$

are parton subprocesses of these reactions.

The differential cross-sections of these subruns are defined by the same expressions as the subprocess cross-sections are defined $\nu_\mu q \Rightarrow \mu^- q'$, $\nu_\mu \bar q \Rightarrow \mu^- \bar q'$, $\bar\nu_\mu q \Rightarrow \mu^+ q'$, $\bar\nu_\mu \bar q \Rightarrow \mu^+ \bar q'$. Then we obtain the expression for longitudinal polarization of baryon:

1) in $\mu^- + N \Rightarrow \nu_\mu + B + X$ reaction

$$\begin{aligned}
P_B^{\mu^- N}(h_N) = &\Big\{(-f_u^N + h_N\Delta f_u^N)(\Delta D_d^B + R\Delta D_s^B) +\\
&+ (1-y)^2\Delta D_{\bar u}^B\left[f_{\bar d}^N + Rf_{\bar s}^N + h_N(\Delta f_{\bar d}^N + R\Delta f_{\bar s}^N)\right]\Big\} \times \Big\{(f_u^N - h_N\Delta f_u^N)(D_d^B + RD_s^B) +\\
&+ (1-y)^2 D_{\bar u}^B\left[f_{\bar d}^N + Rf_{\bar s}^N + h_N\Delta f_{\bar d}^N + R\Delta f_{\bar s}^N)\right]\Big\}^{-1};
\end{aligned} \tag{21}$$

2) in $\mu^+ + N \Rightarrow \bar\nu_\mu + B + X$ reaction

$$\begin{aligned}
P_B^{\mu^+ N}(h_N) = &\Big\{-(1-y)^2\Delta D_u^B\left[f_d^N + Rf_s^N - h_N(\Delta f_d^N + R\Delta f_s^N)\right] +\\
&+ (\Delta D_{\bar d}^B + R\Delta D_{\bar s}^B)(f_{\bar u}^N + h_N\Delta f_{\bar u}^N)\Big\} \times \Big\{(1-y)^2 D_u^B\left[f_d^N + Rf_s^N - h_N(\Delta f_d^N + R\Delta f_s^N)\right] +\\
&+ (D_{\bar d}^B + RD_{\bar s}^B)(f_{\bar u}^N + h_N\Delta f_{\bar u}^N)\Big\}^{-1}.
\end{aligned} \tag{22}$$

If nucleon isn't polarized, then longitudinal polarization degree of baryon has the form:

$$P_B^{\mu^- N} = \frac{-f_u^N(\Delta D_d^B + R\Delta D_s^B) + (1-y)^2\Delta D_{\bar u}^B(\Delta f_{\bar d}^N + R\Delta f_{\bar s}^N)}{f_u^N(D_d^B + RD_s^B) + (1-y)^2 D_{\bar u}^B(f_{\bar d}^N + Rf_{\bar s}^N)}, \tag{23}$$

$$P_B^{\mu^+ N} = \frac{-(1-y)^2\Delta D_u^B(f_d^N + Rf_s^N) + f_{\bar u}^N(\Delta D_{\bar d}^B + R\Delta D_{\bar s}^B)}{(1-y)^2 D_u^B(f_d^N + Rf_s^N) + f_{\bar u}^N(D_{\bar d}^B + RD_{\bar s}^B)}. \tag{24}$$

The obtained expressions for longitudinal polarization degree of baryon are essentially simplified, if we can neglect the antiquark contribution in fragmentation functions:

$$P_{\Lambda}^{(\mu^- N)}(h_N = \pm 1) = P_{\Lambda}^{(\mu^- N)} = \frac{-\Delta D_d^{\Lambda}(z) - R\Delta D_s^{\Lambda}(z)}{D_d^{\Lambda}(z) + RD_s^{\Lambda}(z)}, \quad (25)$$

$$P_{\Lambda}^{(\mu^+ N)}(h_N = \pm 1) = P_{\Lambda}^{(\mu^+ N)} = \frac{-\Delta D_u^{\Lambda}(z)}{D_u^{\Lambda}}. \quad (26)$$

It is seen, that baryon longitudinal polarization doesn't depend on x and y variables in this approximation.

According to [4,5], fragmentation functions of polarized quarks in polarized Λ^0-hyperon can be parametrized in the form:

$$\Delta D_s^{\Lambda}(z,Q^2) = z^{\alpha} D_s^{\Lambda}(z,Q^2),$$
$$\Delta D_u^{\Lambda}(z,Q^2) = \Delta D_d^{\Lambda}(z,Q^2) = N_u \Delta D_s^{\Lambda}(z,Q^2), \quad (27)$$

α and N_u parameters are chosen as follows:

Parameter	Variant 1	Variant 2
α	0.27	1.66
N_u	-0.2	1

The dependence of longitudinal polarization degree of Λ^0-hyperon (18) and (19) on z variable is given on the fig.1. As it is seen, the longitudinal polarization degree of Λ^0-hyperon is positive one in variant 1 and monotonically increases when z variable increase (in this variant the main contribution in polarization of Λ^0-hyperon is made by s-quark, the contribution of u- and d-quarks in polarization is about 20%). The polarization of Λ^0-hyperon in variant 2 is negative one, increases on module when z increases and achieves 1 value in the spectrum end (in this variant the contributions in polarization of Λ^0-hyperon of u-, d- and s-quarks are similar: $\Delta D_u^{\Lambda} = \Delta D_d^{\Lambda} = \Delta D_s^{\Lambda}$). Note, that in this case the polarization of Λ^0-hyperon is similar in all reactions.

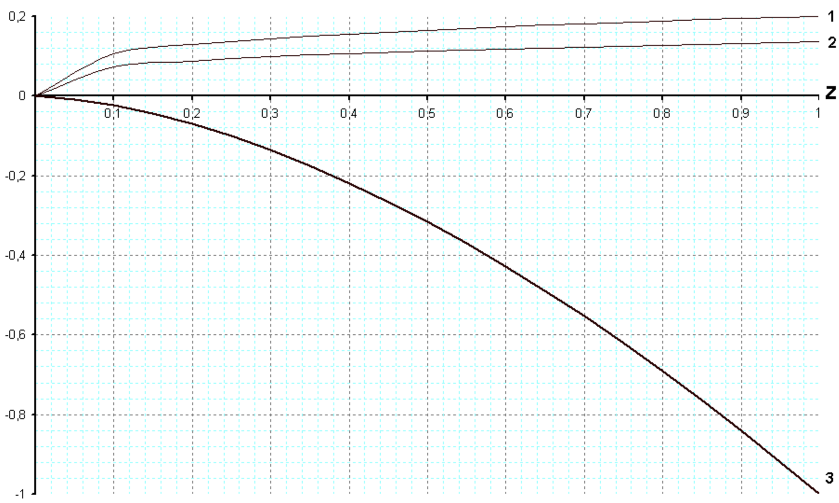

Fig.1. The dependence of Λ^0-hyperon on z variable in $\nu_\mu N \Rightarrow \mu^- \Lambda^0 X$ (curves 1 and 3) and $\overline{\nu_\mu} N \Rightarrow \mu^+ \Lambda^0 X$ (curves 2 and 3).

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YARIMİNKLÜZİV $\nu_\mu(\overline{\nu_\mu})N \Rightarrow \mu^-(\mu^+)BX$, $\mu^-(\mu^+)N \Rightarrow \nu_\mu(\overline{\nu}_\mu)BX$ PROSESLƏRİNDƏ B-BARİONUN POLYARİZASİYASI

Kvark-parton modeli çərçivəsində yarıminklüziv $\nu_\mu(\overline{\nu_\mu}) + N \Rightarrow \mu^-(\mu^+) + B + X$ və $\mu^-(\mu^+) + N \Rightarrow \nu_\mu(\overline{\nu_\mu}) + B + X$ proseslərinin effektiv kəsikləri üçün ümumi ifadələr alınmışdır. Λ^0-hiperonun uzununa polyarizasiya dərəcəsi təyin edilmişdir. Göstərilmişdir ki, fragmentasiya funksiyalarına antikvarkların payı nəzərə alınmazsa, Λ^0-hiperonun uzununa polyarizasiya dərəcəsi yalnız z dəyişənin funksiyası olub, x və y dəyişənlərindən asılı deyildir. Polyarizasiya olunmuş kvarkların polyarizasiya olunmuş Λ^0-hiperona fragmentasiya funksiyalarını müxtəlif şəkildə seçməklə, Λ^0-hiperonun uzununa polyarizasiya dərəcəsinin z dəyişənindən asılılığı tədqiq edilmişdir.

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ПОЛЯРИЗАЦИЯ B-БАРИОНА В ПОЛУИНКЛЮЗИВНЫХ РЕАКЦИЯХ $\nu_\mu(\bar{\nu}_\mu)N \Rightarrow \mu^-(\mu^+)BX$, $\mu^-(\mu^+)N \Rightarrow \nu_\mu(\bar{\nu}_\mu)BX$

В рамках кварк-партонной модели получены общие выражения для эффективных сечений полуинклюзивных реакций $\nu_\mu(\overline{\nu_\mu}) + N \Rightarrow \mu^-(\mu^+) + B + X$ и $\mu^-(\mu^+) + N \Rightarrow \nu_\mu(\overline{\nu_\mu}) + B + X$. Определена степень продольной поляризации Λ^0-гиперона. Показано, что если пренебречь вкладом антикварков в функции фрагментации, то продольная поляризация Λ^0-гиперона является функцией только z, а от переменных x и y она не зависит. Исследована зависимость степени продольной поляризации Λ^0-гиперона от переменной z в различных вариантах выбора функций фрагментации поляризованного кварка в поляризованный Λ^0-гиперон.

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