THERMOELECTRIC EFFECTS IN QUANTUM WELL

F.M. HASHIMZADE, Kh.A. HASANOV, B.H. MEHDIYEV
G.M. Abdullayev Institute of Physics of National Academy of Sciences, AZ-1143, H. Javid ave., 33, Baku, Azerbaijan

In this paper we have calculated the thermomagnetic tensor components for the current density in a quantum well for any degree of degeneration. In our work, we are examinated the diffusion component under the assumption that elastic scattering is dominated, and show that the magneto-thermo-e.m.f. is not determined by the entropy only, as is the case for three dimensions. Elastic electron scattering by acoustic phonons is considered. The magnetic field is directed across of confinement direction, i.e. it is located in the plane of a two-dimensional electron gas. When temperature gradient is along the direction of confinement, magneto-thermoelectric power has a nonmonotonic dependence on magnetic fields. For the magnetic fields less than 4 T magneto-thermoelectric power increase with the magnetic field, and decreased in higher magnetic field. The relative decrease magneto-thermoelectric power achieves 20% at the minimum, which is a significant change and can be easily detected in an experiment. When temperature gradient is along the direction of the free motion, magneto-thermoelectric power is monotonically increasing with magnetic field. For reference, are shown dependence of the non-dissipative magneto-thermoelectric power S/(e n) on magnetic field.

The theory of the quantum thermomagnetic effects in size-quantized systems was studied in [1-8]. In [1,2] the case of a strongly degenerate electronic gas was considered and a focus was placed on the oscillation phenomena. In [3] the thermopower in quantum well structures has been calculated, and the size dependence of thermopower in a quantum limit for different mechanisms of electronic scattering has been considered. The authors used the kinetic equation method and the density matrix approach. In the latter case, the scattering was entered into the equation of motion for the density matrix through the lifetime of a quantum state. In [4] the magneto-thermoelectric power of a two-dimensional electron gas (2DEG) was investigated in the regime of the quantum Hall effect at values of a magnetic field where thermopower is proportional to the entropy of the two-dimensional electron gas. In [5] the magneto-thermoelectric power of a two-dimensional electron gas has been investigated theoretically within the framework of the Boltzman kinetic equation for different mechanisms of electronic scattering taking into account phonon-drag contributions.

Hicks and Dresselhaus [6] predicted that the thermoelectric figure of merit for two-dimensional quantum wells and one-dimensional quantum wires should be substantially enhanced relative to the corresponding bulk materials. A theoretical study of this effect has been undertaken for a bismuth nanowire [7].

The theory of thermopower in quantum dots was developed in [8]. In this work it has been shown that there is an opportunity to create an appreciable temperature difference in a nanostructure and to measure the potential difference induced by this temperature gradient. The paper provides theoretical calculations of magnetothermoelectric power in quantum wells and quantum wires.

In this paper we have calculated the thermomagnetic tensor components for the current density in a quantum well for any degree of degeneration. It is common knowledge that the thermoelectric tensor has two contributions - diffusion and phonon drag, which are linearly additive ones. In our work, we are given an examination the diffusion component under the assumption that elastic scattering is dominated, and show that the magneto-thermo-e.m.f. is not determined by the entropy only, as is the case for three dimensions. Elastic electron scattering by acoustic phonons is considered. The magnetic field is directed across the confinement direction, i.e. it is located in the plane of a two-dimensional electron gas. Thus, two cases for the relative arrangement of the current direction and the confinement direction are possible. In the case where the current is located in a plane of a two-dimensional electron gas it is sufficient to confine ourselves to the relaxation time approximation and to use the kinetic equation. In a case when the current is along the direction of confinement it is necessary to use the density matrix approach obtained in [9-11] for calculation of the diagonal conduction tensor components.

We consider a simple model for the quantum well, in which a two-dimensional electron gas is confined in the x-direction and a homogenous static magnetic field B parallel to the z-axis, with the vector-potential A (0,x B,0) in the Landau gauge. Then the one-particle Hamiltonian is given by

$$\widehat{H}=\frac{\widehat{p}_x^{\,2}}{2m^*}+\frac{1}{2m^*}\left(\widehat{p}_y+\frac{e}{c}xB\right)^2+\frac{\widehat{p}_z^{\,2}}{2m^*}+U(x) \quad (1)$$

where $p=(p_x,p_y,p_z)$ and m^*, respectively, are the momentum operator and the effective mass of a conduction electron. U(x) is the confining potential in the x-direction which is characterized by the parabolic potential:

$$U(x)=\frac{m^*\omega_0^2x^2}{2} \quad (2)$$

The eigenvalues and eigenfunctions of the Schrodinger equation with Hamiltonian (1) are determined by the expressions

$$\varepsilon_\alpha=\left(\frac{1}{2}+N\right)\omega\hbar+\left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar^2k_y^2}{2m^*}+\frac{\hbar^2k_z^2}{2m^*} \quad (3)$$

$$\phi_\alpha(x,y,z)=\frac{1}{2\pi}\varphi_N(x-x_\alpha)e^{i(k_yy+k_zz)} \quad (4)$$

where $\omega=\sqrt{\omega_0^2+\omega_c^2}$ is the "hybrid" frequency, $\omega_c=\frac{eB}{m^*c}$ is the cyclotron frequency of electrons and N - is the oscillation quantum number. The expression

$$\varphi_N(x-x_\alpha)=\frac{1}{\sqrt[4]{\pi}\sqrt{R}\sqrt{2^N N!}}\mathbf{exp}\left(-\left(\frac{x-x_\alpha}{\sqrt{2}R}\right)^2\right)H_N\left(\frac{x-x_\alpha}{R}\right) \quad (5)$$

represents the wave function of a harmonic oscillator, $x_\alpha=-\frac{\omega_c}{\omega}R^2k_y$ - is the oscillator center, and $R=\sqrt{\frac{\hbar}{m\omega}}$ - is the magnetic length , $H_N(\xi)$ is the Hermite polynomial, $\alpha=(N,k_y,k_z)$ is a set of quantum numbers that determine the electron states in a magnetic field.

For the magnetic field directed along the z - axis the current density components can be written in the form [12]

$$j_x=\sigma_{xx}E_x+\sigma_{xy}E_y-\beta_{xx}\nabla_xT-\beta_{xy}\nabla_yT$$
$$j_y=\sigma_{yx}E_x+\sigma_{yy}E_y-\beta_{yx}\nabla_xT-\beta_{yy}\nabla_yT \quad (6)$$

where σ_{ik} and β_{ik} are the conduction tensor components, E_k is the components of the electric field and ∇_kT is the temperature gradient.

From the conditions $j_x=j_y=0$, $\nabla_kT=0$ we obtain the thermoelectric power in a transverse magnetic field

$$\alpha_{yy}=\frac{E_y}{\nabla_yT}=\frac{\beta_{yy}\sigma_{xx}-\beta_{xy}\sigma_{yx}}{\sigma_{xx}\sigma_{yy}-\sigma_{xy}\sigma_{yx}} \quad (7)$$

Putting $j_x=j_y=0$, $\nabla_yT=0$ we obtain from Eqs. (6)

$$\alpha_{xx}=\frac{E_x}{\nabla_xT}=\frac{\beta_{xx}\sigma_{yy}-\beta_{yx}\sigma_{xy}}{\sigma_{xx}\sigma_{yy}-\sigma_{xy}\sigma_{yx}} \quad (8)$$

For the calculation of kinetic coefficients α_{xx}, α_{yy} it is necessary to calculate both diagonal and non-diagonal conduction tensor components σ_{ik} and β_{ik} [2,13].

Note that in bulk semiconductors $\sigma_{yx}>>\sigma_{xx}$, $\beta_{yx}>>\beta_{xx}$, $\sigma_{xy}>>\sigma_{yy}$ and $\beta_{xy}>>\beta_{yy}$. It is related to the fact that a decrease in scattering potential results in the diagonal electric conductivity tensor components tending to zero, while the non-diagonal components stay finite[12]. In our case, as we will show later, it is not true.

The average value of the current density components carried by the electrons is defined by the expression

$$j_i=-eTr\left(\hat{\rho}\,\hat{v}^{(i)}\right)=-e\sum_{\alpha\alpha'}\rho_{\alpha\alpha'}v^{(i)}_{\alpha'\alpha}\ ,\quad i=(x,y,z) \quad (9)$$

where $\hat{\rho}$ is the density matrix, and $\hat{\mathbf{v}}$ - is the velocity operator.

The matrix elements of the density matrix are evaluated from the solution of Liouville's equation

$$i\hbar\frac{\partial\hat{\rho}}{\partial t}=\left[H_t,\rho\right] \quad (10)$$

where $\hat{H}_t$ is the total Hamiltonian of the system $\hat{H}_t=\hat{H}+V+F$ which consists of the Hamiltonian (1), the scattering potential V, and the electron-electric field interaction $F=e(\mathbf{E}\cdot\mathbf{r})$.

The matrix elements of the components of the velocity operator in the representation (4) can be written as

$$\hat{v}^x_{\alpha\alpha'}=i\,\omega R\,\delta_{k_y,k_y'}\delta_{k_z,k_z'}\left(\delta_{N',N-1}\sqrt{\frac{N}{2}}-\delta_{N',N+1}\sqrt{\frac{N+1}{2}}\right) \quad (11)$$

$$\hat{v}^y_{\alpha\alpha'}=\omega_cR\,\delta_{k_y,k_y'}\delta_{k_z,k_z'}\left(\delta_{N',N+1}\sqrt{\frac{N+1}{2}}-\delta_{N',N-1}\sqrt{\frac{N}{2}}\right)+\frac{\omega_0^2\,\hbar k_y}{m^*\omega^2}\delta_{\alpha,\alpha'} \quad (12)$$

Using Eqns. (11)- (12) in Eq.(9) and performing the summation over $\alpha'=(N',k_y',k_z')$ we obtain the following expressions for the current density components:

$$j_x=-ie\omega R\sum_\alpha\left(\rho_{N-1,N}\sqrt{\frac{N}{2}}-\rho_{N+1,N}\sqrt{\frac{N}{2}}\right) \quad (13)$$

$$j_y=-e\,\omega_cR\sum_\alpha\left(\rho_{N+1,N}\sqrt{\frac{N+1}{2}}+\rho_{N-1,N}\sqrt{\frac{N}{2}}\right)-\frac{e\hbar}{m^*}\left(\frac{\omega_0}{\omega}\right)^2\sum_\alpha k_y\,\rho_{\alpha,\alpha} \quad (14)$$

In a zero-order approximation with respect to the scattering potential V the matrix elements of the density matrix $\rho_{\alpha,\alpha'}$ have the form

$$\rho_{\alpha',\alpha}=e\left(E_x\,x_{\alpha'\alpha}+E_y\,y_{\alpha'\alpha}\right)\frac{f_{\alpha'}-f_\alpha}{\varepsilon_{\alpha'}-\varepsilon_\alpha} \quad (15)$$

where $x_{\alpha'\alpha}$ and $y_{\alpha'\alpha}$ are the matrix elements of the x and y coordinates, respectively. In Eq.(15) $f_\alpha = f\left(\varepsilon_\alpha\right)$ is the equilibrium electron distribution function (Fermi- Dirac function)

$$f\left(\varepsilon_\alpha\right)=\left(1+\mathbf{exp}\left(\frac{\varepsilon_\alpha-\zeta}{k_0 T}\right)\right)^{-1} \tag{16}$$

where ζ is the chemical potential of the electrons, and k_0 is the Boltsmann constant.

Substituting Eq.(15) into Eqs.(13)-(14) and calculating the matrix elements of the coordinates we obtain

$$j_y=\sigma_{yx}E_y,\ \sigma_{yx}=\frac{\omega_c e^2}{m^*\omega^2}\sum_\alpha f_\alpha=\frac{\omega_c e^2 n}{m^*\omega^2}, \tag{17}$$

$$j_x=0$$

where $\sum_\alpha f_\alpha = n$ is the areal density of the two-dimensional electron gas and

$$n=\frac{k_0 T m^*}{2\pi\hbar^2}\frac{\omega}{\omega_0}\sum_N \ln\left(1+e^{\eta-x_N}\right), \tag{18}$$

where

$$\eta=\frac{\zeta}{k_0 T},\quad x_N=\frac{\hbar\omega}{k_0 T}\left(N+\frac{1}{2}\right). \tag{19}$$

In the limit of strong magnetic fields, $\omega_0 \ll \omega_c$, or equivalently, in the bulk case, when $\omega_0 \to 0$ the energy spectrum (3) equals that of an electron in a magnetic field. In this case the expression for σ_{yx} in (17) coincides with that for the non-diagonal component of the conductivity tensor of the bulk semiconductor material.

In order to find the explicit form of the non-diagonal component β_{xy} *(B)* we will take advantage of the Onsager reciprocal relation

$$\beta_{xy}(B)=\frac{1}{T}\gamma_{yx}(-B) \tag{20}$$

where $\gamma_{ik}(B)$ is the coefficient in the formula of *i*-th heat flux density transported by the electrons $W_i=\gamma_{ik}E_k$.

In ref. [14] it was shown explicitly that in the presence of a magnetic field it is necessary to take into account the contribution to the current of electrons the edge current $-c\,\nabla\times\mathbf{M}$ due to magnetization $\mathbf{M}$. In this case the coefficient γ_{yx} can be represented as

$$\gamma_{yx}=\gamma_{xy}^{(0)}-cM \tag{21}$$

where $\gamma_{yx}^{(0)}$ is the coefficient in the heat flux density in the absence of scattering which defined by [12, 1]

$$W_y^0=\frac{1}{2}\sum_{\alpha\alpha'}\rho_{\alpha'\alpha}v_{\alpha\alpha'}^y\left(\varepsilon_\alpha+\varepsilon_{\alpha'}-2\zeta\right) \tag{22}$$

The magnetization M is defined by the relationship $M=-\left(\frac{\partial\Omega}{\partial B}\right)_{T,\zeta}$, where $\Omega=-k_0 T\sum_\alpha \mathbf{ln}\left(1+\mathbf{exp}\left(\frac{\zeta-\varepsilon_\alpha}{k_0 T}\right)\right)$ is the Gibbs thermodynamic potential.

Substitution of the Eq.(15) into Eq.(22) yields the following expression for $\gamma_{\mathbf{yx}}^{(0)}$

$$\gamma_{yx}^{(0)}=-\frac{e\omega_c R^2}{\hbar\omega}\left(\bar{\varepsilon}-\zeta n+\sum_\alpha \hbar\omega\left(N+\frac{1}{2}\right)f_\alpha\right) \tag{23}$$

where $\bar{\varepsilon}=\sum_\alpha \varepsilon_\alpha f_\alpha$ is the average energy of the system.

The magnetization can be written as

$$M=-\frac{\omega_c^2}{\omega^2}\frac{\Omega}{B}-\frac{1}{B}\frac{\omega_c^2}{\omega^2}\sum_\alpha \hbar\omega\left(N+\frac{1}{2}\right)f_\alpha \tag{24}$$

where

$$\Omega=-\left(k_0 T\right)^2\frac{\omega}{\omega_0}\frac{m}{2\pi\hbar^2}\sum_N\int_{x_N}^{\infty}\mathbf{ln}(1+\mathbf{exp}(\eta-x))dx \tag{25}$$

is the thermodynamic potential per unit area.

Using (24) we obtain the following expression for $\gamma_{yx}^{(0)}$ instead of Eq.(23):

$$\gamma_{yx}^{(0)}=-\frac{e\omega_c R^2}{\hbar\omega}\left(\bar{\varepsilon}-\zeta n-\frac{\omega^2}{\omega_c^2}MB-\Omega\right) \tag{26}$$

On the other hand, according to the definition of the thermodynamic potential

$$\Omega=\bar{\varepsilon}-\zeta n-TS \tag{27}$$

where $S=-\left(\frac{\partial\Omega}{\partial T}\right)_{B,\zeta}$ is the entropy per unit area which has the following form:

$$S=\frac{mk_0^2 T}{2\pi\hbar^2}\frac{\omega}{\omega_0}\sum_N\left(\left(x_N-\eta\right)\mathbf{ln}\left(1+e^{\eta-x_N}\right)-2Li_2\left(-e^{\eta-x_N}\right)\right) \tag{28}$$

where $Li_\nu(\xi)$ is the polylogarithmic function of order *ν*.

Substituting (27) into (26) and using (21) we obtain

$$\gamma_{yx}=-\frac{e\omega_c}{m\omega^2}TS \tag{29}$$

Finally, for β_{xy} we obtain

$$\beta_{xy}=\frac{e\omega_c}{m\omega^2}S \tag{30}$$

Similar expression was obtained in Ref.[1] for a quantum wire.

For the strong magnetic field case, $\omega_c \gg \omega_0$, Eqn. (30) is reduced to $\beta_{xy}^{(bulk)} = cS/B$, which was obtained in Ref.[14] for bulk specimens. At zero temperature, the transport coefficient β_{xy}, consequently, the entropy vanishes as required by the third law of thermodynamics.

For the calculation of the diagonal components of tensors α_{xx} and β_{xx} when the electric field or the temperature gradient are perpendicular to the plane of two-dimensional electron gas we will make use of the expressions obtained in [9] and [12]:

$$\beta_{xx} = -\frac{e}{\Omega_0 T}\sum_{\alpha\alpha'}\left(-\frac{\partial f\left(\varepsilon_\alpha\right)}{\partial \varepsilon_\alpha}\right)\frac{\left(x_{\alpha'}-x_\alpha\right)^2}{2}\left(\varepsilon_\alpha-\zeta\right)W_{\alpha\alpha'} \tag{31}$$

$$\sigma_{xx} = \frac{e^2}{\Omega_0 T}\sum_{\alpha\alpha'}\left(-\frac{\partial f\left(\varepsilon_\alpha\right)}{\partial \varepsilon_\alpha}\right)\frac{\left(x_{\alpha'}-x_\alpha\right)^2}{2}W_{\alpha\alpha'} \tag{32}$$

where Ω_0 is the volume of the system, $W_{\alpha\alpha'}$ is the electron transition probability from state $\alpha = \left(N, k_y, k_z\right)$ to state $\alpha' = \left(N', k_y^{'}, k_z^{'}\right)$ caused by the effect of the scattering potential.

The scattering mechanism explicitly considered in the present paper is the acoustic phonon deformation potential (DPA scattering). Acoustic-phonon scattering via piezoelectric coupling could also be considered, but this will have a similar temperature and magnetic-field dependence to DPA scattering and so is not included separately. because will not qualitatively affect the results. Other scattering mechanisms, as the interface roughness mechanism, playes a negligible role in heterojunctions, because with the current crystal growth methods, high crystalline quality with atomically sharp resolution is easily achieved, that is interface are not especially rough. In addition, impurity scattering arising from background impurities in the quantum well or remote ionized donors it was to be expected, however in high magnetic fields the magnetic length will be much smaller than the scale of fluctuations due the remote impurities, so remote impurity scattering can be treated by the short-range point defect approach. In this case, the scattering from the point defects has the same functional form as for scattering from the DPA. Only, the temperature and electron concentration dependence will be different.

The transition probability due to the carrier scattering by acoustic phonons has the form

$$W_{\alpha\alpha'} = \sum_{\vec{q}} w(q)\,|\left\langle \alpha \,|\, e^{iq_x x + iq_y y + iq_z z} \,|\, \alpha' \right\rangle|^2 \times$$
$$\times\left(N_q \delta_{k_y^{'},k_y+q_y}\delta_{k_z^{'},k_z+q_z}\delta\left(\varepsilon_{\alpha'}-\varepsilon_\alpha-\hbar\omega_q\right)+\left(N_q+1\right)\delta_{k_y^{'},k_y-q_y}\delta_{k_z^{'},k_z-q_z}\delta\left(\varepsilon_{\alpha'}-\varepsilon_\alpha+\hbar\omega_q\right)\right) \tag{33}$$

where

$$w(q) = \frac{\pi E_1^2}{\rho s \Omega_0} q \tag{34}$$

Here s is the speed of sound, ρ is the density of the material, E_1 is the constant of the acoustic phonon deformation potential, q is the phonon wave vector, and $N_q = \left(\mathbf{exp}\left(\frac{\hbar\omega_q}{k_0 T}\right)-1\right)^{-1}$ is the occupation number (the Planck function) for phonons with frequency $\omega_q = s\,q$.

Using the wavefunctions from Eqn.(4), one can write the matrix elements of the electron - phonon interaction as

$$\left\langle \alpha \,|\, e^{i\mathbf{q}\cdot\mathbf{r}} \,|\, \alpha' \right\rangle|^2 = \left|J_{NN'}\left(q_x, q_y\right)\right|^2 \delta_{k_y^{'},k_y+q_y}\delta_{k_z^{'},k_z+q_z} \tag{35}$$

where

$$\left|J_{NN'}\left(q_x,q_y\right)\right|^2 = \frac{N!}{N'!}\mathbf{exp}\left(-\frac{R^2\left(q_x^2+q_y^2\frac{\omega_c^2}{\omega^2}\right)}{2}\right)\left(\frac{R^2\left(q_x^2+q_y^2\frac{\omega_c^2}{\omega^2}\right)}{2}\right)^{N'-N}\left(L_N^{N'-N}\left(\frac{R^2\left(q_x^2+q_y^2\frac{\omega_c^2}{\omega^2}\right)}{2}\right)\right)^2 \tag{36}$$

Here $L_n^m(\xi)$ is the associated Laguerre polynomial.

Further we will focus on the extreme situation, namely, the quantum limit in which the scattering of electrons is confined within the $N = N' = 0$ level. For the quantum well in a magnetic field the quantum limit criterion is $\hbar\omega > k_0 T$.

Above 20 K the available acoustic phonon energies will be small compared to k_0T. Since the electron scattering by the acoustic phonons is elastic, it is possible to neglect the phonon energy $\hbar\omega_q$ in the arguments of the δ-functions in (33). In addition, as $\hbar\,\omega_q < k_0 T$, therefore it is possible to expand the Plank function. Thus we obtain

$$2N_q + 1 \approx \frac{2k_0 T}{\hbar s q} \tag{37}$$

Taking Eq. (37) into account we can rewrite the expression for $\sigma_{\mathbf{xx}}$ in the form

$$\sigma_{xx} = \frac{e^2}{\Omega_0}\frac{w_0}{2}\left(\frac{\omega_c}{\omega}R^2\right)^2 \sum_{q_x}\sum_{k_z,k_y}\sum_{k_y',k_z'}\left(-\frac{\partial f}{\partial \varepsilon}\right)\left(k_y'-k_y\right)^2 e^{-\frac{R^2\left(q_x^2+\left(\frac{\omega_c}{\omega}\right)^2\left(k_y'-k_y\right)^2\right)}{2}}\times$$
$$\times\delta\left(\frac{\hbar^2 k_z'^2}{2m^*}+\left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar^2 k_y'^2}{2m^*}-\frac{\hbar^2 k_z^2}{2m^*}-\left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar^2 k_y^2}{2m^*}\right), \quad (38)$$

here

$$w_0 = \frac{4\pi T E_1^2 k_0}{s^2\rho\Omega_0\hbar} \quad (39)$$

Transforming the sum over q, k_y, k_y', k_z and k_z' in Eq.(38) into an integral form in a usual way we introduce new, deformed coordinates,

$$k_y' = \frac{\omega}{\omega_0}\tilde{k}_y',\ k_y = \frac{\omega}{\omega_0}\tilde{k}_y,\ k_z' = \tilde{k}_z',\ k_z = \tilde{k}_z \quad (40)$$

Using the momentum conservation law $\tilde{k}' = \tilde{k}$ and integrating over the angle between vectors $\vec{\tilde{k}}'$ and $\vec{\tilde{k}}$ we obtain:

$$\sigma_{xx} = \frac{1}{\tau_0}\frac{3e^2 n}{m^*\omega_0^2}\left(\frac{\omega_c}{\omega_0}\right)^2\sqrt{\frac{\omega_0}{\omega}}\frac{1}{\ln\left(1+e^{\tilde{\eta}}\right)}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)x\,{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8ax\right)dx. \quad (41)$$

Similarly, for β_{xx} we obtain

$$\beta_{xx} = -\frac{k_0}{e}\frac{1}{\tau_0}\frac{3e^2 n}{m^*\omega_0^2}\left(\frac{\omega_c}{\omega_0}\right)^2\sqrt{\frac{\omega_0}{\omega}}\frac{1}{\ln\left(1+e^{\tilde{\eta}}\right)}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)x\left(x-\tilde{\eta}\right){}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8ax\right)dx \quad (42)$$

In Eqns.(41) and (42) the following notations were used

$$\tau_0 = \frac{\sqrt{2\pi}\ s^2\rho\hbar^{7/2}}{m^{*3/2}\ E_1^2 k_0 T\sqrt{\omega_0}} \quad (43)$$

$$a = \frac{k_0 T\omega_c^2}{2\omega\hbar\omega_0^2} \quad (44)$$

$$f_0 = \left(1+\exp\left[x-\tilde{\eta}\right]\right)^{-1},\ \tilde{\eta} = \eta - x_0 \quad (45)$$

and ${}_2F_2\left(a_1,a_2;b_1,b_2;z\right)$ is the generalized hypergeometric function [15].

For the case of the electric field and the temperature gradient directed along the plane of two-dimensional electron gas, we use the solution of the kinetic equation to calculate the diagonal components of the tensors α_{yy} and β_{yy}. These are given by

$$\sigma_{yy} = \frac{e^2}{\Omega_0}\sum_\alpha\left(-\frac{\partial f\left(\varepsilon_\alpha\right)}{\partial\varepsilon_\alpha}\right)\tau_\alpha v_{k_y}^2 \quad (46)$$

$$\beta_{yy} = -\frac{e^2}{\Omega_0 T}\sum_\alpha\left(-\frac{\partial f\left(\varepsilon_\alpha\right)}{\partial\varepsilon_\alpha}\right)\left(\varepsilon_\alpha-\xi\right)\tau_\alpha v_{k_y}^2 \quad (47)$$

where

$$v_{k_y} = \frac{1}{\hbar}\partial_{k_y}\varepsilon = \left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar k_y}{m^*} \quad (48)$$

and τ_α is the relaxation time of electrons.

For the elastic scattering by acoustic phonons the relaxation time is given by a simple expression:

$$\frac{1}{\tau_\alpha} = \sum_{\alpha'}W_{\alpha\alpha'}\left(1-\frac{\mathbf{k}'}{\mathbf{k}}\right) \quad (49)$$

Taking into account expression (33) for the transition probability and proceeding the same was as above in the calculation of the transverse diagonal components of the transport coefficients, the following expression for τ obtains in the quantum limit:

$$\tau = \tau_0\left(\frac{\omega_0}{\omega}\right)^{3/2}\frac{1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-\frac{4\,\omega_c^2}{\omega\hbar\omega_0^2}\left(\varepsilon-\frac{\hbar\omega}{2}\right)\right)} \quad (50)$$

Substitution of (49) into (46) and (47) and summation over α yields

$$\sigma_{yy} = \frac{e^2 n}{m^*\ln\left(1+e^{\tilde{\eta}}\right)}\tau_0\left(\frac{\omega_0}{\omega}\right)^{7/2}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)\frac{1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8ax\right)}x\,dx \quad (51)$$

$$\beta_{yy} = -\frac{k_0}{e}\frac{e^2 n}{m^* \ln\left(1+e^{\tilde{\eta}}\right)}\tau_0\left(\frac{\omega_0}{\omega}\right)^{7/2}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)\frac{\left(x-\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8ax\right)}x\,dx \qquad (52)$$

Combining Eqs.(17), (30), (41), (42), (51) and (52) with Eqs. (7) and (8) and taking into account the symmetry of the conductivity tensor, we obtain the following expressions:

$$\alpha_{yy} = -\frac{1}{en}\frac{3k_0\, n K_1 K_4 + S\ln^2\left(1+e^{\tilde{\eta}}\right)}{3K_1 K_3 + \ln^2\left(1+e^{\tilde{\eta}}\right)}, \qquad (53)$$

$$\alpha_{xx} = -\frac{1}{en}\frac{3k_0\, n K_2 K_3 + S\ln^2\left(1+e^{\tilde{\eta}}\right)}{3K_1 K_3 + \ln^2\left(1+e^{\tilde{\eta}}\right)}, \qquad (54)$$

where

$$K_1 = \int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right){}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8ax\right)x\,dx \qquad (55)$$

$$K_2 = \int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)x\left(x-\tilde{\eta}\right){}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8ax\right)dx \qquad (56)$$

$$K_3 = \int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)\frac{1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8ax\right)}x\,dx \qquad (57)$$

$$K_4 = \int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)\frac{\left(x-\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8ax\right)}x\,dx \qquad (58)$$

Eqs.(57)-(60) is applicable for any degree of degeneracy of two-dimensional electron gas.

For the degenerated electron gas, when $\eta = \zeta/k_0T > 1$, the expressions for the transport coefficients can be considerably simplified. In this case we replace $\left(-\frac{\partial f(\varepsilon)}{\partial\varepsilon}\right)$ by the delta function $\delta(\in-\zeta)$. Then we obtain

$$n = \frac{m^* k_0 T}{2\pi\hbar^2}\frac{\omega}{\omega_0}\tilde{\eta}, \qquad (59)$$

$$S^{(d)} = \frac{\pi m^* T k_0^2\omega}{6\hbar^2\omega_0}, \qquad (60)$$

$$\sigma_{\mathbf{yx}}^{(d)} = \frac{\omega_c e^2 n}{m^*\omega^2}, \qquad (61)$$

$$\beta_{yx}^{(d)} = -\frac{k_0}{e}\frac{\pi^2}{3\tilde{\eta}}\sigma_{yx}^{(d)} \qquad (62)$$

$$\sigma_{xx}^{(d)} = \frac{1}{\tau_0}\frac{3e^2 n}{m\omega_0^2}\left(\frac{\omega_c}{\omega_0}\right)^2\sqrt{\frac{\omega_0}{\omega}}\;{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8a\tilde{\eta}\right) \qquad (63)$$

$$\beta_{xx}^{(d)} = -\frac{k_0}{e}\frac{\pi^2}{3\tilde{\eta}}\sigma_{\mathbf{xx}}^{(d)}\left(1-\frac{35}{6}\frac{a\tilde{\eta}\;{}_2F_2\left(\frac{9}{4},\frac{11}{4};\frac{5}{2},3;-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2,-8a\tilde{\eta}\right)}\right) \qquad (64)$$

$$\sigma_{yy}^{(d)} = \frac{e^2 n}{m}\tau_0\left(\frac{\omega_0}{\omega}\right)^{7/2}\frac{1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8a\tilde{\eta}\right)} \qquad (65)$$

$$\beta_{yy}^{(d)} = -\frac{k_0}{e}\frac{\pi^2}{3\tilde{\eta}}\sigma_{yy}^{(d)}\left(1+\frac{3a\tilde{\eta}\;{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2,-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8a\tilde{\eta}\right)}\right) \qquad (66)$$

For the case of strongly degenerate electron gas the thermopower can be written as

$$\alpha_{yy} = -\frac{\pi^2 k_0}{e\tilde{\eta}}\frac{\Psi_1\left(1+3a\tilde{\eta}\Psi_1\right)+1}{\left(3\Psi_1+1\right)} \qquad (67)$$

$$\alpha_{xx} = -\frac{\pi^2 k_0}{e\tilde{\eta}}\frac{\Psi_1\left(1-\frac{35}{6}a\tilde{\eta}\Psi_2\right)+\frac{1}{3}}{\left(3\Psi_1+1\right)} \qquad (68)$$

where

$$\Psi_1 = \frac{{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2,-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8a\tilde{\eta}\right)} \qquad (69)$$

$$\Psi_2 = \frac{{}_2F_2\left(\frac{9}{4},\frac{11}{4};\frac{5}{2},3;-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8a\tilde{\eta}\right)} \qquad (70)$$

In the case of strongly degenerate electron gas and weak fields, $\tilde{\eta}$ depends on the magnetic field only weakly, and the observed changes in the transport coefficients are due to parameter a

$$\alpha_{xx} = -\frac{k_0\pi^2}{3\left(e\tilde{\eta}\right)}+\frac{35k_0\pi^2 a}{24e} \qquad (71)$$

$$\alpha_{\mathbf{yy}} = -\frac{k_0\pi^2}{3\left(e\tilde{\eta}\right)}-\frac{3\left(k_0\pi^2\right)a}{4e} \qquad (72)$$

one can see that α_{xx} decreases by absolute value, whereas absolute value α_{yy}, on the contrary, increases. Coefficients α_{xx} and α_{yy} are proportional to the temperature. In a strong magnetic field the situation is similar to the quantum limit for the bulk case.

We present a numerical calculations for the thermopower for GaAs/Al_xGa_{1-x}As parabolic quantum well. We use the following set of physical parameters $m^* = 0.066\,m_0$, where m_0 is the free electron mass. The parameter of the parabolic potential is $\omega_0 = 1.4\times10^{13}\ s^{-1}$. The value of the deformation-potential constant is as $E_1 = 10\ eV$. The density of the material and the speed of sound are taken as $\rho = 5\times10^3\ kg/m^3$ and $s = 5400\ m/s$. Notice that numerical calculations for the quantum limit criterion must be carried out when the Fermi level is between the first and second subbands $\hbar\omega/2 \le \zeta < 3\hbar\omega/2$.

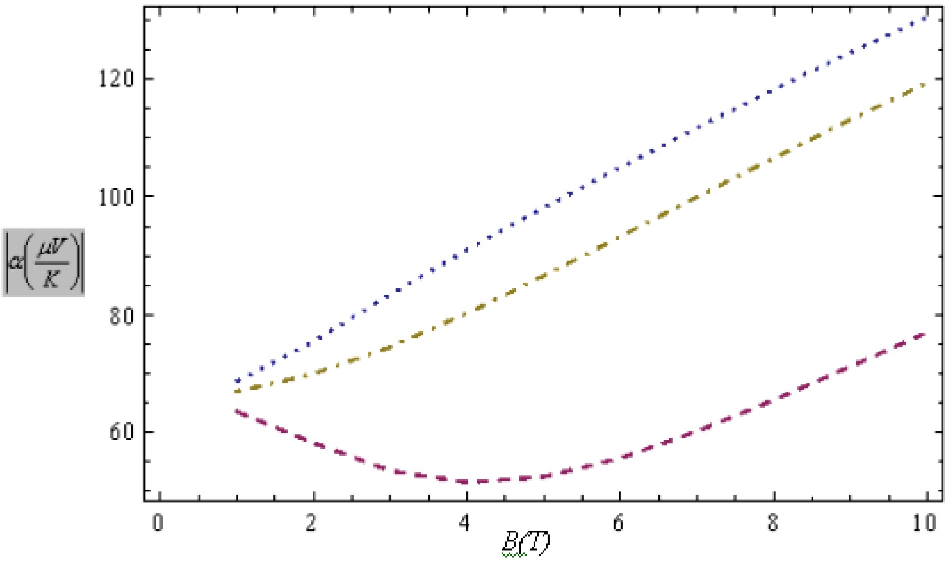

Fig.1. Absolute value transversal magneto-thermoelectric power of two-dimensional electron gas versus the magnetic field. T=20K, n=10^{14} m^{-2};α_{yy} – dotted, α_{xx} – dashed, $S/(e\,n)$ – dotdashed.

The dependence of absolute value the magneto-thermoelectric power on magnetic field are shown in Fig.1 for T=20 K, $n = 10^{14}$ m^{-2}. When temperature gradient is along the direction of the confinement, magneto-thermoelectric power α_{xx} has a nonmonotonic dependence on magnetic fields. For the magnetic fields less than $4T\alpha_{xx}$ increase with the magnetic field, and decreased in higher magnetic field. As one can see from Fig. 1, the relative decrease α_{xx} achieves 20% at the minimum, which is a significant change and can be easily detected in an experiment. When temperature gradient is along the direction of the free motion magneto-thermoelectric power α_{yy} is monotonically increasing with magnetic field. For reference, in Fig.1 are shown dependence of the non-dissipative magneto-thermoelectric power $S/(e\,n)$ on magnetic field [16].

[1] *M.D. Blokh,* Phys. Solid State 896, 1975. (in Russian).
[2] *G.R. Aizin and V.A. Volkov*, Zh. Eksp. Teor. Fiz., 1469, 1984. (in Russian).
[3] *S.S. Kubakaddi, B.G. Mulimani and V.M. Jali*, Phys. Stat. Sol. (b), 683, 1986.
[4] *J.S. Davidson, E.D. Dahlberg,, A.J. Valois and G.Y.Robinson*, Phys. Rev. B, 2941, 1986.
[5] *X. Zianni, P.N. Butcher and M.J. Kearney*, Phys. Rev. B, 7520, 1994.
[6] *L.D. Hicks and M.S. Dresselhaus*, Phys. Rev. B, 12727, 1993.
[7] *T. Koga, T.C. Harman, S.B. Kronin and M.S. Dresselhaus*, Phys. Rev. B, 14286, 1999.
[8] *C.W. Beenakker and A.A. Staring*, Phys. Rev. B, 9667, 1992.
[9] *E.N. Adams and T.D. Holstein*, J. Phys. Chem. Sol., 254, 1959.
[10] *P.S. Zyryanov and G.I. Guseva* , Uspekhi Fizicheskikh Nauk , 565, 1968. [Sov. Phys. Usp., 538, 1969].
[11] *A.I. Anselm and B.M. Askerov*, Phys. Solid State, 31, 1967. (in Russian).
[12] *B.M. Askerov,* Electron Transport Phenomena in Semiconductors (World Scientific, Singapore, 1994).
[13] *E.P. Sinyavskii and R.A. Khamidullin*, Semiconductors, 924, 2002.
[14] *Yu. N. Obraztsov*, Fiz. Tverd. Tela (Leningrad), 414, 1964 [Sov. Phys.-Solid State, 331 (1964)].
[15] A.*P. Prudnikov, Yu.A. Brychkov, and O.I. Marichev*, Integrals and Series, Vol., More spesial Functions (Fizmatlit, Moscow, 2003).
[16] *F.M. Gashimzade, A.M. Babaev and Kh.A. Gasanov,* Semiconductors and Dielectrics, Vol.43, N10, pp.1850-1852, 2001.

F.M. Haşımzadə, X.A. Həsənov, B.H. Mehdiyev

KVANT ÇUXURUNDA TERMOELEKTRİK EFFEKTLƏR

Bu məqalədə ixtiyari cırlaşma halında kvant çuxurunda cərəyan sıxlığının termomaqnit tenzorunun komponentləri hesablanmışdır. Biz diffuziya komponentlərini elastiki səpilmə halında hesablayaraq göstərmişik ki, maqnit termoelektrik hərəkət qüvvəsi üçölçülü halda olduğu

kimi ancaq entropiya ilə təyin edilmir. Elektronların akustik fononlardan elastiki səpilməsinə baxılmışdır. Maqnit sahəsi ikiölçülü elektron qazının müstəvisi üzərində yerləşmişdir. Elektron qazı müstəvisinə normal istiqamətdə qradient temperaturu yaradıldıqda maqnit termo-ehq maqnit sahəsindən qeyri-monoton asılı olur. Maqnit sahəsi 4T-dən kiçik olduqda maqnit termoehq sahədən asılı olaraq artır, maqnit sahəsinin yuxarı qiymətlərində isə azalır. Minimum halda maqnit termoehq-nin nisbi azalması 20% təşkil edir və eksperimentdə asanlıqla aşkar edilə bilər. Temperatur qradiyenti sərbəst hərəkət istiqamətində yönəldikdə maqnit termoehq maqnit sahəsindən asılı olaraq artır. Müqayisə üçün qeyri-dissipativ maqnit termoehq-nin *S/(en)* maqnit sahəsindən asılılığı göstərilmişdir.

Ф.М. Гашимзаде, Х.А. Гасанов, Б.Г. Мехдиев

ТЕРМОЭЛЕКТРИЧЕСКИЕ ЭФФЕКТЫ В КВАНТОВОЙ ЯМЕ

В данной статье мы рассчитали компоненты термомагнитного тензора для плотности тока в квантовой яме с произвольной степенью вырождения. В нашей работе мы проверили диффузионную компоненту в предположении доминирующей роли упругого рассеяния и показано, что магнетотермоэдс определяется не только энтропией, как в трехмерном случае. Рассмотрено упругое рассеяние электронов акустическими фононами. Магнитное поле расположено в плоскости двумерного электронного газа. Когда создается температурный градиент вдоль направления ограничения, магнитотермоэдс немонотонно зависит от магнитного поля. Для магнитного поля меньшего, чем 4T, магнетотермоэдс увеличивается с ростом магнитного поля, но уменьшается при высоких значениях магнитного поля. Относительное уменьшение магнетотермоэдс достигает 20% при минимуме, который является существенным изменением и может быть легко обнаружен при эксперименте. Когда температурный градиент направлен вдоль направления свободного движения, магнетотермоэдс монотонно растет с ростом магнитного поля. Для сравнения показана зависимость недиссипативной магнетотермоэдс *S/(en)* от магнитного поля.

Received:18.03.08