DEFINITION OF RESULTING ANGLE OF THE DEVIATION AFTER THE PASSAGE OF NEUTRONS THROUGH THE CRYSTAL

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The present paper is dedicated to the consideration of passage of neutrons in the crystal. The expression for intensity distribution is obtained by the method of the multiple scattering theories. The received results are applied to calculation angle of the deviation of neutrons by nucleus of the crystal. Here is shown that at the large angle of the deviation, the intensity of distribution decreases with reduction of angle much more slowly, than till Gaussian` law. The received expression for the angle of the deviation of allows defining of the impulse of the neutron.

1. Introduction

At the analysis of angular distribution of the thermal neutrons disseminated by a layer of substance, there is a problem of finding resulting angle of the deviation after consecutive collisions with nucleus of substance. Many authors considered this problem approximately [1-3]. In works [1-2] unitary scattering is considered and it was not required any approach. In work [3] passage charged through substance is investigated and the theory for small angles and only for screen Coulomb's fields is resulted. On the basis of the theory of multiple Watson scattering [4], we deduced a basic formula for distribution of intensity and we defined angle of the deviation after passage of the neutron waves through the crystal.

2. Formula for definition of the angle of deviation

At the analysis of angular distribution of coherent neutron scattering, there is a problem to finding the resulting angle ϑ of the deviation after *n* consecutive collisions of a neutron by nucleons of a nucleus of substance. The probability of that in one act of collision a deviation of a particle will lay in a solid, angle $d\Omega = \sin\vartheta d\vartheta d\varphi$ is equal:

$$W_1(\vartheta,\varphi)d\Omega = \frac{\sigma(\vartheta)}{\sigma}d\Omega \qquad (1)$$

where $\sigma(\vartheta)d\Omega$ and σ are differential and full cross-section, accordingly. We expand (1) over Legendre's polynom:

$$W_1(\vartheta,\varphi)d\Omega = \frac{1}{4\pi}\sum_l (2l+1)\,g_l P_l(\cos\vartheta)d\Omega \qquad (2)$$

where:

$$g_l = \frac{1}{\sigma}\int P_l(\cos\vartheta)\sigma(\vartheta)d\Omega \qquad (3)$$

After the second act of scattering (on other nucleon) the particle moves in a direction, are defined by polar angles (ϑ_2,φ_2) concerning a direction of (ϑ_1,φ_1) occurring once scattering bunch or angles (ϑ,φ) concerning falling bunch ,i.e:

$$\cos\vartheta_2 = \cos\vartheta_1\cos\vartheta + \sin\vartheta_1\sin\vartheta\cos(\varphi_1-\varphi). \qquad (4)$$

To find probability of that after two consecutive, independent impacts the deviation of a particle will lay in a solid angle $d\Omega = \sin\vartheta d\vartheta d\varphi$, it is necessary to integrate product on $W_1(\vartheta_1,\varphi_1)W_1(\vartheta_2,\varphi_2)$ all ϑ_1,φ_1 at the fixed values ϑ and φ. Decomposing on $P_l(\cos\vartheta_2)$ attached Legendre's polynomial:

$P_l^m(\cos\vartheta_1)\exp(im\varphi_1)$ and
$P_l^m(\cos\vartheta)\exp(im\varphi)$,

For probability it is found:

$$W_2(\vartheta,\varphi)d\Omega = \frac{1}{4\pi}\sum_l (2l+1)(g_l)^2 P_l(\cos\vartheta)d\Omega \qquad (5)$$

Similarly, for probability of a deviation in a solid angle $d\Omega = \sin\vartheta d\vartheta d\varphi$ after *n* collisions it is had:

$$W_n(\vartheta,\varphi)d\Omega = \frac{1}{4\pi}\sum_l (2l+1)(g_l)^n P_l(\cos\vartheta)d\Omega \qquad (6)$$

Let's designate through *W(n)* probability of that passing through substance the neutron will test on the average *n* collisions. Then the probability of that a neutron will deviate on a corner lying between ϑ And $\vartheta+\delta\vartheta$, is equal:

$$I(\vartheta)\sin\vartheta d\vartheta = \frac{1}{2}\sum_l (2l+1)\left[\sum_n W(n)(g_l)^n\right]P_l(\cos\vartheta)\sin\vartheta d\vartheta \qquad (7)$$

The average of collisions ν at passage through a film the thickness *t*, containing *N*, nucleus in unit of volume, will be written down as:

$$\nu = Nt\int\sigma(\vartheta)d\Omega. \qquad (8)$$

or $\nu >> 1$ for *W(n)* it is possible to apply Poisson's distribution:

$$W(n) = \frac{e^{-\nu}\nu^{n}}{n!} \tag{9}$$

Thus, we get:

$$\sum_{n=0}^{\infty} W(n)(g_l)^{n} \approx exp[-\nu(1-g_l)] \tag{10}$$

Considering last expressions, for distribution of intensity at multiple scattering we receive:

$$I(\vartheta) = \frac{1}{2}\sum_{l}(2l+1)exp\left\{-2\pi Nt\int \sigma(\theta)[1-P_l(cos\,\theta)]sin\,\theta d\theta\right\}P_l(cos\,\vartheta)\ . \tag{11}$$

Let, further the $q(\theta)$ attitude of cross-section $\sigma(\theta)$ scattering of a neutron on nucleus of a crystal to section on a free nucleus σ_0, i.e:

$$q(\theta) = \frac{\sigma(\theta)}{\sigma_0} \tag{12}$$

Designating $y = sin\frac{\theta}{2}$ expression (11) will become:

$$I(\vartheta) = \frac{1}{2}\sum_{l} l(l+1)\int_0^1 q(\theta)\frac{dy}{y} - \sum_{2}^{l}(-1)^{k}\frac{(l+k)!}{(l-1)!(k!)^2}\int_0^1 q(\theta)y^{2k-3}dy \tag{13}$$

Function $q(y)$ possesses following properties:

$$q(0) = 0,\quad q(y) = 1 \quad \text{for} \quad y > y_0, \tag{14}$$

Whence at scattering on angle θ, greater of some angle θ_0, influence of other nucleus a little. If in integral $\int_0^1 q(y)y^{2k-3}dy$, $k \geq 2$ to approximate *q(y)* expression:

$$q(y) = 1 - exp\left(-\frac{y}{y_0}\right) \tag{15}$$

and to put:

$$P_l(cos\,\vartheta) \approx J_0[(l+1/2)\vartheta] = J\left(\frac{\vartheta}{\theta_0}\right) \tag{16}$$

It is possible to replace summation on *l* in (13) integration. With this purpose we use Euler-Maclaurin's formula:

$$\sum_{l} f(n+1/2) = \int_0^{\infty} f(x)dx + \frac{1}{24}f'(0) + \ldots \tag{17}$$

After enough bulky calculations, in the first Born's approach we receive:

$$J(\vartheta) \approx \int_0^{\infty}\left(1+\frac{2\theta_0}{\beta}(1-\beta)\right)\left(ln\frac{\theta_0^2}{4} - ln\frac{\theta^2}{4}\right)J\left(\frac{\vartheta}{\theta_0}\right)d\theta = exp\left(-\frac{\vartheta^2}{\overline{\vartheta}^2}\right). \tag{18}$$

where:

$$\beta = \frac{v}{c}\ ,\ \overline{\vartheta}^2 = \theta_0^2\left(1+\frac{2\theta_0}{\beta}\right)ln\frac{\theta_0^2}{4} \tag{19}$$

At greater angles $\vartheta/\theta_0\sqrt{ln\frac{\theta_0^2}{4}} \geq 4$ function $J(\vartheta)$ decreases with reduction ϑ much more slowly, than on Gaussian` law, and following members in (17), arising in the second and higher Born's approximation, become essential. Expression (19) allows defining an impulse of a neutron from data about an average square a scattering of angle $\overline{\vartheta}^2$.

4. Conclusion

The theory of multiple scattering has served in work as a starting point of calculation. (18) corresponds flat wave to pulse approach in which scattering on separate particles is considered independently, thus we completely ignored influence of other particles. In more perfect variant by consideration of scattering on a separate particle, the account of influence of the others is reduced to introduction of distortion of wave functions of a disseminated particle. As scattering the ideal harmonious crystal has been considered. For thermal neutrons because of small depth of their penetration into a wall of a crystal it is necessary to consider carefully influence of structure of the surface limiting disseminating environment on behavior of the neutron wave received. In the first Boron's approach expression for the angle of scattering allows to define a parameter of refraction of a neutron wave on a crystal and this to serve as object for the further researches.

[1] *E.J. Williams.* Phys. Rev. 1988, 58, p. 292.
[2] *S.A. Goudsmit, J.L. Saunderson.* Phys. Rev. 1990, 57, p. 24.
[3] *H. Snyder, W. Scott.* Phys. Rev. 1989, 76, p. 220.
[4] *K.M. Watson.* Phys. Rev., 1967, 105, p. 1388.

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NEYTRONLARIN KRISTALDAN KEÇMƏSİ ZAMANI SON DÖNMƏ BUCAĞININ TƏYİNİ

İş neytronların kristallardan keçməsinin tədqiqinə həsr edilmişdir. Çoxdəfəli səpilmə nəzəriyyəsi əsasında intensivliyin paylanma funksiyası üçün ifadə alınmışdır. Alınmış ifadə neytronların kristaldan keçərkən kristalın nüvələrindən səpilməsinin son dönmə bucağının hesablanmasına tətbiq edilmişdir. Müəyyən edilmişdir ki, böyük bucaq altında dönmə zamanı intensivlik bucağın azalması ilə Qauss qanuna nəzərən az azalır. Dönmə bucağı üçün alınmış ifadə zərrəciyin impulsunu təyin etməyə imkan verir.

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ОПРЕДЕЛЕНИЕ РЕЗУЛЬТИРУЮЩЕГО УГЛА ОТКЛОНЕНИЯ ПОСЛЕ ПРОХОЖДЕНИЯ НЕЙТРОНА ЧЕРЕЗ КРИСТАЛЛ

Настоящая работа посвящена рассмотрению прохождения нейтронов через кристалл. На основе многократного рассеяния было получено выражение для функции распределения интенсивности. Полученные результаты применяются к вычислению угла отклонения частицы на ядрах кристалла. Показано, что при больших углах отклонения интенсивность распределения уменьшается с уменьшением угла значительно медленнее, чем по гауссовому закону. Полученное выражение для угла отклонения позволяет определить импульс нейтрона.

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