DETERMINATION OF THE THICKNESS OF AN ABSORBING FILM ON ANGULAR DEPENDENCE OF THE ELECTROMAGNETIC RADIATION REFLECTION

R.A. KARAMALIYEV
Baku State University, Azerbaijan, AZ-1148, Baku, Z. Khalilov str., 23

A.Ch. IZMAILOV, Ch.O. QAJAR
H.M. Abdullayev Institute of Physics of Azerbaijan National Academy of Sciences, Az-1143, Baku, H. Javid ave., 33

T.Sh. EFENDIEV
B.I. Stepanov Institute of Physics, NAS of Belarus,Belarus, 220072 Minsk, Nezavisimosti Ave, 68

We theoretically elaborated the comparatively simple method of measurement of the thickness of an absorbing film (on a transparent substrate) on features of the dependence of the intensity of the reflected electromagnetic radiation (with a fixed frequency) on the angle of its incidence on the film. The suggested method is based on determination of intervals between extrema in oscillations of the given angular dependence. Possibilities of this method are shown on example of sufficiently exact finding of the thickness of an oil film on a motionless water surface.

1. Introduction

Often in practice highly thin films or surface layers (on substrates) take place, whose thickness may be measured only on reflection of the electromagnetic radiation [1,2]. It concerns, for example, to coatings of optical technique, sprayed surface layers, and also oil films on a water surface.

Resultant reflected electromagnetic wave from the layered system of the type film-substrate is the superposition of two waves reflected from interfaces between air and the film and also between the film and the substrate respectively (fig.1). Therefore the intensity of such a resultant wave essentially depends on the phase difference δ between the given interfering waves. In case of a transparent film (with the thickness h and the refraction index n_2) this value δ has the form [3]:

$$\delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda_0} , \qquad (1)$$

where λ_0 is the wavelength (in the vacuum) of the incident electromagnetic radiation, and θ_2 is the refraction angle of the given radiation in the film (fig.1). Dependence of the intensity of the reflected radiation on the phase difference δ (1) is oscillating [1-3]. Intervals between extrema of given oscillations are defined by values divisible by π It allows to find the thickness h of a film on the basis of the relationship (1) at known values n_2, λ_0 and θ_2 even for two such extrema. Smooth change of the phase difference δ (1) can be carried out, in particular, by scanning of the frequency (i.e. the wavelength λ_0) of the electromagnetic radiation at the fixed angle of its incidence on the film. By present time the corresponding method of measurement of the film thickness on intervals between extrema, arising in oscillations of the reflected radiation intensity at change of its wavelength, has been sufficiently well approved [1,2] . However for realization of this method, the source of optical radiation with the corresponding spectrometer and the detector of radiation scanned in a sufficiently wide range of frequencies are necessary. At the same time, according to the formula (1), similar extrema can be observed also in oscillations of the dependence of the intensity of the reflected radiation with the fixed frequency on the angle of its incidence on the film under investigation.

Therefore in the present work we theoretically analyze possibilities of the method of finding of thickness of a weakly absorbing film (on a transparent substrate) on oscillations of the angular dependence of the reflected electromagnetic radiation. The special attention is given the application of this method for an oil film on a motionless water surface since definition of such film sizes is important for ecology.

2. Relationships for the reflection coefficient

Let's consider the layered system (fig.1), consisting of transparent media 1 and 3 and also the plane-parallel film 2 (with the thickness h) concluded between them, which are characterized by refraction indexes n_1, n_2 and n_3, respectively. It is assumed that the plane electromagnetic wave incidents from the media 1 on the film under the angle θ_1, which partially penetrates into the film and then into the medium 3 under refraction angles θ_2 and θ_3 (Fig.1) respectively. The part of this radiation undergoes mirror reflection from given interfaces.

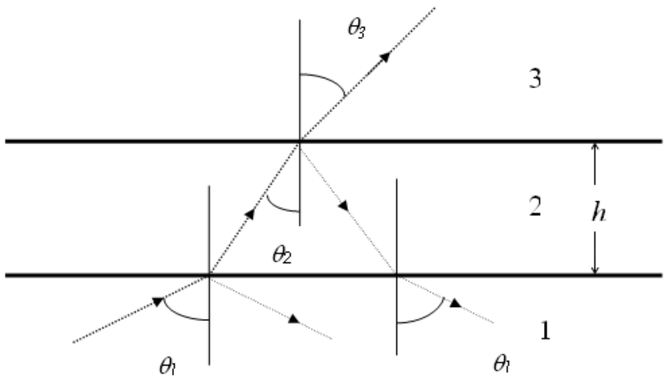

Fig.1. The scheme of transmission and mirror reflection of the electromagnetic wave incident from the medium 1 on the plane-parallel film 2 (with the thickness h) and then on the medium 3.

It is possible to introduce the following complex refraction index for the absorbing film:

$$\hat{n}_2 = n_2 + i\chi_2 \tag{2}$$

where n_2 is the real refraction index and χ_2 is the absorption index of the film. It is convenient to put the following:

$$n_2 \cos\theta_2 = u + iv \ , \tag{3}$$

where u and v are real values. According to the Snell's law we have [3]:

$$n_1 \sin\theta_1 = \hat{n}_2 \sin\theta_2 . \tag{4}$$

Taking into account the equality (4) we receive from the formula (3):

$$(u + iv)^2 = \hat{n}_2^2 - n_1^2 \sin^2\theta_1 . \tag{5}$$

Equating separately real and imaginary parts in (5) we find

$$u^2 - v^2 = n_2^2 - \chi_2^2 - n_1^2 \sin^2\theta_1 , \tag{6}$$

$$uv = \chi_2^2 . \tag{7}$$

From here follows, that

$$2u^2 = n_2^2 - \chi_2^2 - n_1^2 \sin^2\theta_1 + \sqrt{\left(n_2^2 - \chi_2^2 - n_1^2 \sin^2\theta_1\right)^2 + 4n_2^2\chi_2^2} \ , \tag{8}$$

$$2v^2 = -n_2^2 + \chi_2^2 + n_1^2 \sin^2\theta_1 + \sqrt{\left(n_2^2 - \chi_2^2 - n_1^2 \sin^2\theta_1\right)^2 + 4n_2^2\chi_2^2} \ . \tag{9}$$

The complex amplitude reflection coefficient $\hat{\rho}$ of the considered layered system is expressed by the formula [3]:

$$\hat{\rho} = \frac{\hat{r}_{12} + \hat{r}_{23} \exp\left[2i\hat{k}h\cos(\theta_2)\right]}{1 + \hat{r}_{12}\hat{r}_{23} \exp\left[2i\hat{k}h\cos(\theta_2)\right]}, \tag{10}$$

where h is the film thickness, $\hat{r}_{12}$ and $\hat{r}_{23}$ are Fresnel amplitude reflection coefficients from surfaces 1-2 and 2-3 (Fig.1), respectively. The wave number $\hat{k}$ has the form:

$$\hat{k} = \frac{2\pi(n_2 + i\chi_2)}{\lambda_0}, \tag{11}$$

where λ_0 is the wavelength of the incident radiation in the vacuum.

Let us consider the case of the wave, whose polarization vector is perpendicular with respect to the incidence plane (s-wave). Then we have following relationships for complex amplitude coefficients $\hat{r}_{12}$ and $\hat{r}_{23}$ [3]:

$$\hat{r}_{12} = \rho_{12} \exp(i\varphi_{12}) = \frac{n_1\cos\theta_1 - (u + iv)}{n_1\cos\theta_1 + (u + iv)}, \tag{12}$$

where the amplitude module ρ_{12} and phase difference φ_{12} are determined by formulas

$$\rho_{12} = \sqrt{\frac{(n_1\cos\theta_1 - u)^2 + v^2}{(n_1\cos\theta_1 + u)^2 + v^2}} \ , \qquad tg\varphi_{12} = \frac{2vn_1\cos\theta_1}{u^2 + v^2 - n_1^2\cos^2\theta_1} \tag{13}$$

and by analogy

$$\hat{r}_{23} = \rho_{23} \exp(i\varphi_{23}) , \tag{14}$$

$$\rho_{23} = \sqrt{\frac{(n_3\cos\theta_3 - u)^2 + v^2}{(n_3\cos\theta_3 + u)^2 + v^2}} \ , \qquad tg\varphi_{23} = \frac{2vn_3\cos\theta_3}{u^2 + v^2 - n_3^2\cos^2\theta_3} . \tag{15}$$

After introduction of the denotation

$$\eta = \frac{2\pi h}{\lambda_0} \tag{16}$$

we have the following

$$\frac{2\pi h\hat{n}_2\cos\theta_2}{\lambda_0} = (u + iv)\eta . \tag{17}$$

By substituting relationships (12) - (17) in formula (10), we receive expression for the amplitude reflection coefficient:

$$\hat{\rho} = \frac{\rho_{12}\exp(i\varphi_{12}) + \rho_{23}\exp(-2v\eta)\exp\left[i(\varphi_{23} + 2u\eta)\right]}{1 + \rho_{12}\rho_{23}\exp(-2v\eta)\exp\left[i(\varphi_{12} + \varphi_{23} + 2u\eta)\right]} . \tag{18}$$

From the relationship (18) we receive the formula for the energetic reflection coefficient R $(R \le 1)$ of the s-wave, which may be directly recorded at experiments:

$$R = |\hat{\rho}|^2 = \frac{[\rho_{12}\exp(vh) - \rho_{23}\exp(-vh)]^2 + 2\rho_{12}\rho_{23}[1+\cos(\varphi_{23} - \varphi_{12} + 2u\eta)]}{\exp(2vh) + \rho_{12}^2\rho_{23}^2\exp(-2vh) + 2\rho_{12}\rho_{23}\cos(\varphi_{12} + \varphi_{23} + 2u\eta)}. \quad (19)$$

3. Discussion of results

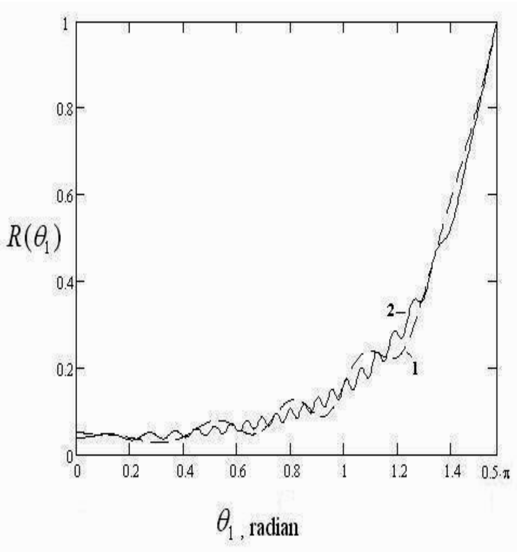

Fig.2. Dependences of the reflection coefficient R of the light wave on the angle θ_1 of its incidence on the oil film with thickness $h=5\lambda_0$ (dotted curve 1) and $25\lambda_0$ (continuous curve 2) at optical constants of the oil (n_2=1.485, χ_2=0.0029) and water (n_3=1.339) characteristic for the light wavelength λ_0=0.6µm.

Let us consider, for example, incidence of the optical wave from the air on the weakly absorbing oil film on the motionless water surface. At numerical calculations we used known characteristic optical parameters of light kinds of the oil $(n_2 = 1.485, \chi_2 = 0.0029)$ and water $(n_3 = 1.339)$ for the wavelength λ_0=0.6 µm of the incident radiation [2].

Fig.2 presents dependences of the reflection coefficient R (19) on the angle θ_1 of its incidence (Fig.1) for two fixed values of the film thickness $h = 5\lambda_0$ and $25\lambda_0$.

In given dependences oscillations are obviously displayed which are caused, mainly, by the cosine in the numerator of the reflection coefficient R (19). The interval between neighboring extrema of the dependence $R(\theta_1)$ is determined by the value π. Hence, after a finding of incidence angles $\theta_1^{(1)}$ and $\theta_1^{(2)} > \theta_1^{(1)}$ for two such extrema, we can determine the film thickness h on the following formula which directly follows from the cosine argument in the numerator of the expression (19):

$$\frac{h}{\lambda_0} = \frac{\pi(N+1) + [\varphi_{12}(\theta_1^{(1)}) - \varphi_{23}(\theta_1^{(1)})] - [\varphi_{12}(\theta_1^{(2)}) - \varphi_{23}(\theta_1^{(2)})]}{4\pi[u(\theta_1^{(1)}) - u(\theta_1^{(2)})]}, \quad (20)$$

where N is the number of extrema between points $\theta_1^{(1)}$ and $\theta_1^{(2)}$ in the angular dependence $R(\theta_1)$. Other values in the formula (20) can be calculated on known relationships (8), (13) and (15).

In case of a sufficiently weak absorption of the film (when $\chi_2 << n_2$), it is possible to neglect a difference $|\varphi_{12}-\varphi_{23}| << \pi$ in the expression (20) and to put $u \approx n_2\cos\theta_2$ (8), where according to the law (4), $\theta_2 \approx \arcsin[(n_1/n_2)\sin\theta_1]$. As a result, instead of (20) we receive the simple approximate formula:

$$\frac{h}{\lambda_0} = \frac{(N+1)}{4n_2}\left\{\cos\left[\arcsin\left(\frac{n_1}{n_2}\sin\theta_1^{(1)}\right)\right] - \cos\left[\arcsin\left(\frac{n_1}{n_2}\sin\theta_1^{(2)}\right)\right]\right\}^{-1}. \quad (21)$$

Thus, for example, values of angles $\theta_1^{(1)}$=0.3 and $\theta_1^{(2)}$=0.92 correspond to points of extrema for the curve 1 in Fig.2. In the interval of values between these points we observe N=3 others extrema of oscillations. By substituting given values $\theta_1^{(1)}$, $\theta_1^{(2)}$ and N in the relationship (21), we receive $h = 4.965\lambda_0$, that coincides with the real value of the film thickness $5\lambda_0$ for the curve 1 in Fig.2 with an accuracy about 1 %. By analogy, sufficiently exact determination of the film thickness (close to $25\lambda_0$) is possible for the curve 2 in Fig.2. Thickness values received on the basis of the more strict relationship (20) for the considered weakly absorbing film practically coincide with calculation results on much more simple formula (21).

It is necessary to note, that the obtained formulas (20) and (21) are inapplicable for calculation of a too small thickness of a film when $h < 0.5\lambda_0/n_2$ and there are no at least two extrema in oscillations of the dependence $R(\theta_1)$. Moreover oscillation amplitudes in a dependence $R(\theta_1)$ decrease with rise of a thickness of an absorbing film (Fig.2) and become negligible, when $\exp(-4\pi vh/\lambda_0) << 1$ according to the relationship (19). At a small absorption index of a film $\chi_2 << n_2$, the given upper restriction on a measured thickness of a film is reduced to the following inequality:

$$h \le \frac{\lambda_0}{4\pi\chi_2}\cos\left[\arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right)\right]. \qquad (22)$$

In particular, we can see from the relationship (22), that the offered method is most effective in a range of sufficiently small incidence angles θ_1. Really, according to Fig.2, oscillations of dependences $R(\theta_1)$ disappear at approach of the incidence angle θ_1 to the extreme value $0.5\,\pi$.

Though only the case of the s-wave has been considered in this work, similar results are valid also for the p-wave, whose polarization vector is parallel to the incidence plane.

We note that indicated restrictions on possible minimal and maximal measured oil thicknesses are characteristic also for the method based on the scanning of the radiation frequency at a fixed angle of its incidence [1, 2].

4. The conclusion

In the present work we have shown possibility of the sufficiently exact measurement of a thickness of a weakly absorbing film (on a transparent substrate) on features of the angular dependence of the reflected electromagnetic radiation with the fixed frequency. For realization of the given method, sufficiently simple equipment is necessary: a source of the narrow-band radiation with a stable wavelength and the corresponding sensitive detector for record of the reflected radiation intensity. The offered method can be effectively used in particular for finding of a thickness of an oil film on a motionless water surface.

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R.Ə. Kərəməliyev, A.Ç. İzmaylov, Ç.O. Qacar, T.Ş. Əfəndiyev

ELEKTROMAQNİT ŞÜALANMASININ ƏKS OLUNMASININ BUCAQDAN ASILILIĞINA ƏSASƏN UDAN TƏBƏQƏNİN QALINLIĞININ TƏYİNİ

Əks olunan elektromagnit şüalanmasının (sabit tezlikli) intensivliyinin düşmə bucağından asılılığına əsasən udan təbəqənin qalınlığını təyin etmək üçün nisbətən sadə nəzəri metod işlənib hazırlanmışdır. Təklif olunan metod həmin bucaq asılılığı ekstremumları arasındakı intervalın təyin olunmasına əsaslanır. Bu metodun imkanları su səthindəki neft təbəqəsinin qalınlığının dəqiq təyin edilməsi misalında nümayiş etdirilmişdir.

Р.А. Карамалиев, А.Ч. Измайлов, Ч.О. Каджар, Т.Ш. Эфендиев

ОПРЕДЕЛЕНИЕ ТОЛЩИНЫ ПОГЛОЩАЮЩЕЙ ПЛЕНКИ ПО УГЛОВОЙ ЗАВИСИМОСТИ ОТРАЖЕНИЯ ЭЛЕКТРОМАГНИТНОГО ИЗЛУЧЕНИЯ

Теоретически разработан сравнительно простой метод измерения толщины поглощающей пленки (на прозрачной подложке) по особенностям зависимости интенсивности отраженного электромагнитного излучения (с фиксированной частотой) от угла его падения на пленку. Предлагаемый метод основан на определении интервалов между осцилляционными экстремумами данной угловой зависимости. Возможности этого метода продемонстрированы на примере достаточно точного определения толщины нефтяной пленки на неподвижной водной поверхности.

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