FARADAY EFFECT IN HOLLOW QUANTUM CYLINDR OF FINITE THICKNESS

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The interband Faraday rotation (FR) in hollow quantum cylindr of finite thickness is theoretically investigated. FR in the dependence on incident light energy for different values of cylindr thickness. It is seen that the resonance peaks appear on FR curve. The rules of selection are obtained.

Introduction.

Nowadays Faraday effect in semiconductor low-dimensional systems is the subject of many theoretical and experimental investigations [1-5]. The interest to this effect is caused by its applications in physics, optics and nano-electronics. The obtaining of charge carrier effective masses or their densities, life time of nano-equilibrium charge carriers in semiconductors and low-dimensional structures on their base, amplitude modulation of laser radiation for optical links, magneto-optical recording and information reproducing in both special and domestic aims.

The nano-technology development makes possible the production of surfaces of different curvatures from heterostructural layers [6] and in particular the cylindrical surfaces the physical properties of which reveal the interesting peculiarities [7,8]. These structures can be described with the help of parabolic potential model.

In the present paper the interband Faraday effect in hollow semiconductor quantum cylinder with finite wall thickness has been considered. This quantum system is nano-structure the confinement of which can be modeled by parabolic potential and moreover, electron-electron interaction doesn't influence on optical properties, i.e. it isn't considered [9]. The analytical expression for rotation angle in the dependence on incident light energy and magnetic field is obtained. The rules of selection are established. The numerical results of FR for hollow cylinder GaAs/AlGaAs are presented.

Energy spectrum and wave functions.

In quantum nano-structures the confinement of which can be modeled by parabolic potential in the correspondence with generalized *K*on theorem the electron-electron interaction usually doesn't influence on system optical properties. In [10] it is shown that parabolic potential is equivalent to infinite layer potential with equally distributed positive charge and in this case the system optical properties don't depend on both electron-electron interaction and electron number in the layer.

The parabolic potential model is useful for description of physical properties of quantum wires. Nowadays the production technology of almost ideal quantum wires with confinement parabolic potential has been developed [11].

We will model the quantum cylinder with thin walls by the following way. Let's consider the thin film with two-dimensional electronic gas with confinement parabolic potential. If one carry out the periodic boundary condition in the one of directions, let's this direction will be x one and then we have [9]:

$$\psi(x,y)=\psi(x+L_x,y) \quad , \tag{1}$$

where L_x is film cross length. Thus we obtain the hollow cylindr with finite thickness in which $x\in[-\frac{L_x}{2},\frac{L_x}{2}]$ is circular coordinates and can be expressed by $\varphi=2\pi x/L_x$ angle variable. Meanwhile the cylindr effective length will coincide with L_y film thickness.

Shrödinger's equation for electron in conduction band in approximation of effective mass can be witten in the following form:

$$\{\frac{1}{2m_{0c}}(\vec{p}-\frac{e}{c}\vec{A})^2+\frac{m_{0c}\omega_{0c}^2y^2}{2}+g_c\mu_B Bm_s\}\psi(\vec{r})=E\psi(\vec{r}), \tag{2}$$

here ω_{0c} is frequency corresponding to holding parabolic potential on y axis; m_{0c} is electron effective mass; $\vec{p}$ is electron impulse, $g_c\mu_B Bm_s$is member taking under consideration the spin; g_c is effective electron g-factor in conduction band, m_s is spin quantum number ; $\mu_B=\frac{e\hbar}{2m_0}$ is Bohr magneton.

The calibration of following vector potential

$$\vec{A}=(-By,0.0). \tag{3}$$

with taking under consideration the periodicity condition (1) the solution of equation (2) we find in the following form:

$$\psi(x,y,z)=\frac{e^{im\frac{x}{R}}}{\sqrt{2\pi R}}\frac{e^{ik_z z}}{\sqrt{L_z}}\phi(y) \tag{4}$$

Here $L_x=2\pi R$ is cylinder perimeter of circle, R is cylinder average radius, $m=0,\pm1,...$ is quantum number connecting with projection of angular momentum, k_z is wave number in the direction of z axis.

Taking under consideration (3) and (4) we obtain the energy spectrum in the following form:

$$E_c = \hbar\Omega_c(n+\frac{1}{2}) + \lambda m^2 + \frac{\hbar^2 k_z^2}{2m_{0c}} + g_c\mu_B B m_s \quad , \tag{5}$$

where $\Omega_c = \sqrt{\omega_c^2 + \omega_{oc}^2}$, $\omega_c = \frac{eB}{m_{0c}}$ is cyclotron frequency $\lambda_c = \frac{2\pi^2\hbar^2\omega_{oc}^2}{m_{0c}L_x^2\Omega_c^2}$ is energy of geometric confinement, $n=0,1,2,\ldots$ is principal quantum number.

The wave function corresponding to this spectrum can be written in the form:

$$\psi_c = \frac{e^{ik_z z}}{\sqrt{L_z}}\frac{e^{im\frac{x}{R}}}{\sqrt{2\pi R}}\phi_n\left[\frac{1}{l_c}(y + y_{oc})\right], \tag{6}$$

Here $l_c = \sqrt{\frac{\hbar}{m_{0c}\Omega_c}}$, $y_{oc} = \frac{\omega_c l_c^2}{\Omega_c R} m$ is oscillator center, $\phi_n(y)$ are oscillator functions defined as follows

$$\phi_n(y) = \frac{1}{\sqrt{l_c}} A_n^{-\frac{1}{2}} exp\left(-\frac{(y+y_{0c})^2}{2l_c^2}\right) H_n\left(\frac{y+y_{0c}}{l_c}\right) \tag{7}$$

H_n are Hermitian polynomials, A_n is normalization coefficient:

$$A_{nm} = \int_{\eta_1}^{\eta_2} [e^{-\eta^2/2} H_n(\eta)]^2 d\eta \, , \tag{8}$$

where $\eta_1 = \frac{y_1 + y_{0c}}{l_c}$, $\eta_2 = \frac{y_2 + y_{0c}}{l_c}$, $y_1=-L_y/2$, $y_2=L_y/2$.

The energy spectrum and wave function for electrons in valency band can be written in the form:

$$E_v = -E_g - \hbar\Omega_v(n'+\frac{1}{2}) - \lambda m'^2 - \frac{\hbar^2 k_z'^2}{2m_{0v}} + g_v\mu_B B m_s' \, , \tag{9}$$

$$\psi_v = \frac{e^{ik_z' z}}{\sqrt{L_z}}\frac{e^{im'\frac{x}{R}}}{\sqrt{2\pi R}}\phi_{n'}\left[\frac{1}{l_v}(y + y_{ov})\right], \tag{10}$$

Here E_g is forbidden band width in volume material.

3. FARADEY ROTATION ANGLE.

The general expression for FR has the form [21]:

$$\Theta = CE^2 \sum_{c,v} \frac{(E_c - E_v)^2 - E^2}{\left[(E_c - E_v)^2 - E^2\right]^2 + 4E^2\Gamma^2} \frac{1}{(E_c - E_v)^2}\left[\left|\left\langle c\middle|\vec{e}_+\vec{P}\middle|v\right\rangle\right|^2 - \left|\left\langle c\middle|\vec{e}_-\vec{P}\middle|v\right\rangle\right|^2\right], \tag{11}$$

where $E = \hbar\omega$ is light energy, E_c and E_v are electron energy in conduction and valency bands correspondingly, Γ is spectral widening half-width of $v \to c$ transitions. $\vec{e}_\pm = \frac{1}{\sqrt{2}}(\vec{e}_x \pm i\vec{e}_y)$ corresponds to right and left light polarizations, $C = \frac{\hbar k^{1/2} e^2 L_z}{cn\varepsilon_0 m_0^2 V}$. $V = \pi(R_{out}^2 - R_{in}^2)L_z$ is normalization volume of hollow quantum cylinder and rest designations have the usual values. The interband matrix elements $\langle c|\vec{e}_\pm\vec{P}|v\rangle$ can be written in the form [2]:

$$\langle c|\vec{e}_\pm\vec{P}|v\rangle = \langle\psi_c|\psi_v\rangle \cdot \langle u_{c0}|\vec{e}_\pm\vec{P}|u_{v0}\rangle \, . \tag{12}$$

Taking under consideration (7), (10) and electron spin in (12) we obtain following rules of selection:

$$\Delta k_z = 0 \, , \; |\Delta n| = 0,1,2,\ldots \, , \Delta m = \pm 1 \, .$$

$$\Delta m_s = \pm 1 \, \left(m_s = \pm\frac{1}{2}\right). \tag{13}$$

(+) and (-) correspond to right- and left-polarized light.
The matrix element can be led to form:

$$\langle c|\vec{e}_\pm\vec{P}|v\rangle = |P_{cv}|\alpha^{\frac{1}{2}}(A_{nm}A_{n'm'})^{-\frac{1}{2}} I_{nm,n'm'}\delta_{k_z k_z'}\delta_{m,m'\pm 1} \tag{14}$$

Here $P_{cv} = \langle S|P_z|Z\rangle$ is impulse matrix element between Blohov functions at κ=0. S ,Z are functions transforming similar to s and p atomic functions.

$$I_{n\,m;n'm'} = \int_{\eta_1}^{\eta_2} \exp[-\frac{1}{2}\eta^2 - \frac{1}{2}(\eta + Y)^2\alpha^2]H_n(\eta)H_{n'}((\eta + Y)\alpha)d\eta\,, \tag{15}$$

$$Y = \frac{y_{0v} - y_{0c}}{l_c}\,,$$

$$\alpha = \frac{l_c}{l_v}\quad .$$

Taking under consideration (14) after integration in (11) over k_π we obtain the following expression for rotation angle:

$$\theta = C'\alpha E^2 \sum_{n,m}\sum_{n',m'}\left(F^{+}_{nm;n'm'} - F^{-}_{nm;n'm'}\right) \times \left|\left(A_{nm}\cdot A_{n'm'}\right)^{\frac{1}{2}} I_{nm;n'm'}\cdot\delta_{k_z k'_z}\cdot\delta_{m,m'\pm 1}\right|^2\,, \tag{16}$$

$$C' = \frac{k^{1/2}e^2\left|P_{cv}\right|^2\sqrt{2\mu}L_z}{4\pi^2 cn\varepsilon_0 m_0^2(R_{out}^2 - R_{in}^2)}\,, \tag{17}$$

$$\mu = \frac{m_{0c}m_{0v}}{m_{0c} + m_{0v}}\,,$$

$$F^{\pm}_{nm;n'm'} = \int_0^\infty \frac{(E^{\pm}_{nm;n'm'} + E_z)^2}{[(E^{\pm}_{nm;n'm'} + E_z)^2 - E^2]^2 + 4\Gamma^2 E^2}\cdot\frac{dE_z}{(E^{\pm}_{nm;n'm'} + E_z)^2\sqrt{E_z}}\,, \tag{18}$$

$$E^{\pm}_{nm;n'm'} = E_g + E_{nm} + E_{n'm'} \pm \frac{1}{2}(g_c + g_v)\mu_B B, \tag{19}$$

($\pm$) are related to right- and left-polarized light.

$$E_{nm} = \hbar\Omega_c(n + \frac{1}{2}) + \lambda_c m^2\,, \tag{20}$$

$$E_{n'm'} = \hbar\Omega_v(n' + \frac{1}{2}) + \lambda_v m'^2\,, \tag{21}$$

$$E_z = \frac{\hbar^2 k_z^2}{2\mu}\,,$$

$$\Omega_{c(v)} = \sqrt{\omega^2_{c(v)} + \omega^2_{0c(v)}}\,. \tag{22}$$

$\omega_{0c(v)}$ frequencies are defined as follows:

$$\omega_{0c(v)} = \frac{1}{L_y}\sqrt{\frac{2\Delta_{c(v)}}{m_{c(v)}}}\,, \tag{23}$$

where $\Delta_{c(v)}$ are barrier heights in conduction and valency bands.

Faraday effect follows from difference between polarizabilities of quantum system in the fields of right and left circularly polarized waves. This follows from the fact that transition energies and matrix elements of right- and left-polarized circularly waves are different ones. In calculations we have neglected the field dependence of matrix elements as a result of which their values for right- and left-polarized waves are equal ones. As a result the rotation of polarization plane is defined by only Zeeman splitting of energy levels.

The resonance peaks appear on FR curve when $E = E^{\pm}_{nm;n'm'}$ .FR angle oscillates achieving in $E = E^{+}_{nm;n'm'}$ points of maximal values and in points of $E = E^{-}_{nm;n'm'}$ minimum ones and its places shift to photon big energy region. The maximums are on $\hbar(\Omega_c + \Omega_v)$ distance from each other. $(g_c + g_v)\mu_B B$ is distance between maximums and minimums. The amplitude peaks and distance between them increase with decrease of wall thickness of quantum cylinder. This takes place because of distance increase between discrete energy levels. Note that on FR curve the maximum with least amplitude shifts to of small energy region with decrease of wall thickness. This takes place because of matrix element dependence on wall thickness (fig.2). In the comparison with solid cylinder, the dependences of energy transitions on magnetic field are similar for circularly right- and left- polarized light in the case of hollow cylinder (fig.3,4).

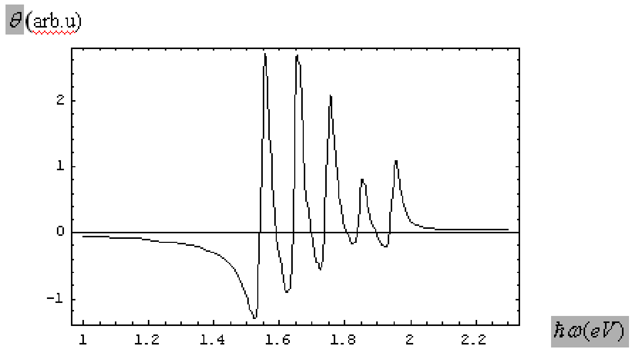

Fig.1. Plot of the FR angle as a function of photon energy calculated from Eq.16 for L_y=100Å. *B=0.5 T.*

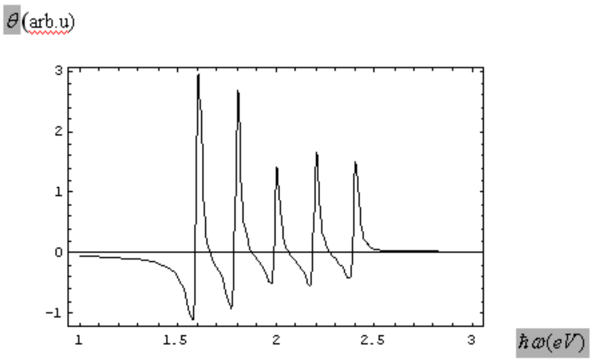

Fig. 2. Same that is on Fig.1 for L_y=50 Å.

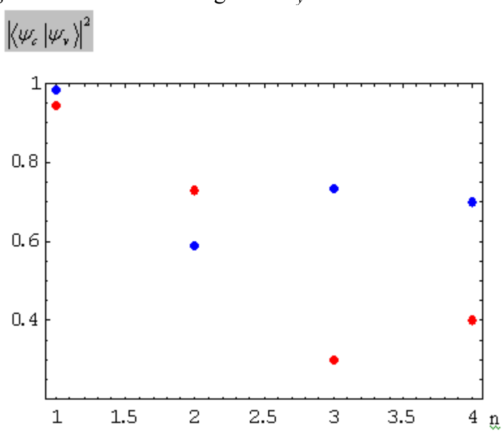

Fig.3. Plot of the overlap integral as a function of quantum number n calculated from Eq.14 for L_y=100Å (red) and L_y=50 Å (blue).

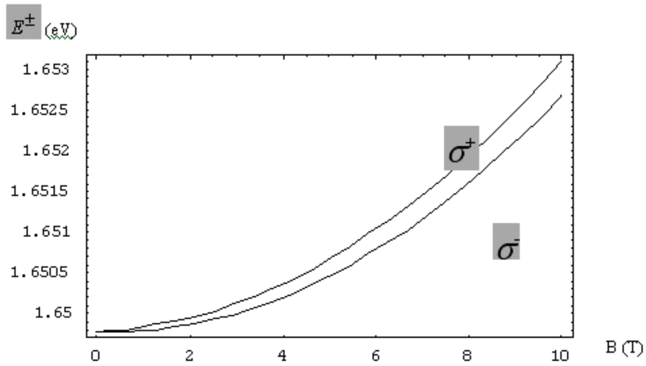

Fig.4. Plot of the transition energy as a function of magnetic field in the case of the hollow quantum cylinder.

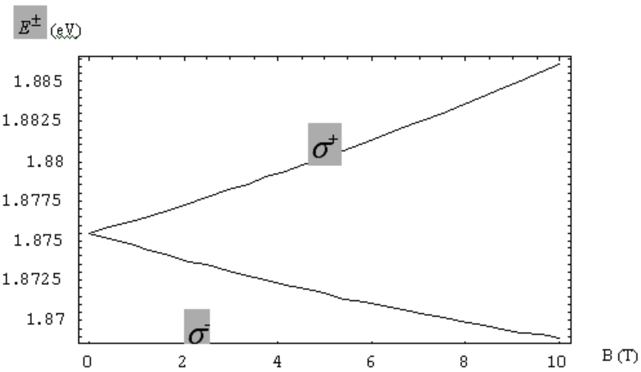

Fig.5. Same that is on Fig.4 for the case of solid quantum cylinder [17].

For calculation we use the following physical parameters in the case of GaAs/AlGaAs hollow quantum cylinder: E_g=1.5eV, m_{0c}=0.067m$_0$, and m_{0v}=0.45m$_0$ (for heavy holes), widening parameter is Γ=40 meV. The barrier heights for electrons and holes are Δ_c = 255meV and Δ_v=170meV [22-23]. The average radius of hollow cylinder is R=1500Å.

FR dependence on incident light energy for two different wall thickness values of quantum cylinder is shown on fig.1 and 2; L_y=100Å and L_y=50Å at magnetic field value B=0.5 *tesla*. The dependences of overlap integral on n quantum number for two different values of quantum cylinder wall thickness are shown on fig.3. The transition energy dependences on magnetic field corresponding to circularly right- and left- polarized light in the cases of hollow and solid quantum cylinders are shown on fig.4,5.

[1] *F.Meier, V.Cerletti, O. Gywat, D. Loss, and D. D. Awschalom*. Phys.Rev.2004, B 69,195315.

[2] *F.Meier, D.D. Awschalom*. Phys. Rev. 2005, B 71, 205315.

[3] *D.Dragoman*. J.Opt. Soc. Am. B 22, 2005, 2697.

[4] *Michael N. Leuenberger*. Phys. Rev., 2006, B 73, 075312.

[5] *Jiang-Tao Liu and Kai Chang*. J.Appl. Phys. Letters, 2007, 90, 061114.

[6] *V.Ya.Prinz, V.A.Seleznev, A.K.Gutakovsky*. Microelectr. Eng.30,1996, 439.

[7] *V.Geyler, V.A.Margulis, A.V.Shoroxov*. JETP 1999, 115,1450.

[8] *L.I.Magaril, A.V.Chaplik*. JETP, 1999, 115, 1478.

[9] *N.G.Galkin, V.A.Margulis, A.V.Shorohov*. Fiz. Tverd.Tela, 2001, 43, 511.

[10] *L.Brey, N.Johnson, B.J.Halperin*. Phys. Rev. B40, 1989, 10647.

[11] *W. Geiselbrecht, U. Sahr, A. Masten, O. Gräbner, Klütz, M. Forkel, G.H. Döhler, K. Campman, A.C. Gossard*. Physica, E2, 106, 1998.
[12] *I.M. Boswarva, R.E. Howard, A.B. Lidiard*. Proc. Roy.Soc. 1962, v.269, p.125..
[13] *T. Moss, G. Barrel, B. Ellis*. Semiconductor opto-electronics. Mir, 1976.
[14] *L.E.Vorobev,E.L.Ivchenko,D.A.Firsov,V.A.Shalygin*. Optical properties of nanostructures. St. Petersburg, Science. 2001.
[15] *J.H.Burnett, H.M.Cheong, R.M.Westervelt, W. Paul, P.F. Hopkins, M.Sundaram, and A.C.R.Cossard,* Phys.Rev.B.48, 4524, 1993.
[16] *R. C. Miller, D. A. Kleinman, and A. C. Gossard,* Phys. Rev. B 29, R7085, 1984.
[17] *T.G. Ismailov,G.G. Jabrailova*. News of Baku University 3,125, 2008.

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SONLU QALINLIQLI BOŞ KVANT SİLİNDRİNDƏ FARADEY EFFEKTİ

Sonlu qalınlıqlı boş kvant silindrində zonalararası Faradey fırlanması (FF) nəzəri olaraq tədqiq edilmişdir.Silindrin qalınlığının müxtəlif qiymətlərində FF düşən işığın enerjisinin funksiyası kimi hesablanmışdır.Göstərilmişdir ki, FF əyrisində rezonans pikləri görünür.Seçmə qaydaları tapılmışdır.

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ЭФФЕКТ ФАРАДЕЯ В ПОЛОМ КВАНТОВОМ ЦИЛИНДРЕ КОНЕЧНОЙ ТОЛЩИНЫ

Теоретически исследовано межзонное Фарадеевское вращение (ФВ) в полом квантовом цилиндре конечной толщины. Изучено ФВ в зависимости от энергии падающего света для различных значений толщины цилиндра. Показано что на кривой ФВ появляются резонансные пики. Получены правила отбора.

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