FRACTAL CONCEPTIONS IN PHYSICAL AND NATURAL STRUCTURES

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On example of morphology investigation of surface structures of solid substances and breather skin is shown that fractal systems form the whole world of objects and phenomena which can be described by functions introduced by Veyershtrass and Van-der-Varden.

1. INTRODUCTION

Nowadays the percolation and fractal conceptions are successfully used in fundamental science and natural phenomena. Such approach has changed the description of disordered objects and processes of nature. Many problems in physics connected with behavior of complex systems have the deep analogies with either percolation phenomena or with fractal growth on different models [1-3]. The interest is caused by the transition to nano-objects and problems of nano-world that has led to new physical and geometric approaches. The perceptions fractal and fractal geometry for description of real objects and math abstractions have been introduced. This perceptions are widely used: almost all objects of real world are related to it [1,4].

The nature using the possibilities of fractal structure essentially effectively constructs the alive organisms including the human being, snakes and other alive organisms). The series of origin theoretical conceptions described in [5] is included in the base of investigation of fractal structure of alive organisms. The fractal can be the basis of structure-system organization, functioning and control of alive organisms and solid-state structures. The alive organism has the property of self-similarity on different levels of system hierarchy.

The fractal has the hierarchy and scale invariance. The self-similarity is its important property; its type isn't changed in any space scale. This means the safety of fractal similarity principle at different levels of structure consideration from atomic force microscope (AFM) up to observations from near space. The natural and anthropogenic fractals have clearly bordered scale interval in which they reveal their fractal structure [1-2]. In reality the any fractal has the some minimum and maximal length scale. The fractal object of each substance forms at definite physical conditions. The fractal structure of skin surface of alive objects has formed in the process of their origin.

The idea of infinite divisibility is applied for geometric, algebraic and stochastic fractals that cause the total self-similarity for math fractal objects being the idealized objects.

The math fractals can be applied to material objects only in the capacity of models as the approximation which is comfortable for calculations. If we consider the surface morphology of both the alive and inorganic origin in detail, then we can reveal their similarity.

Any mathematic methods applied for modeling of natural objects are relative approximation to reality. Finally we can lead to scales where the similarity of visible structure images can be revealed also as in nano-structures.

Many objects in nature can't be described with application of smooth curves, smooth surfaces which are character to usual geometric figures. The non-differentiable curves are suitable for description of such objects.

The theoretical and experimental description of fractal structures proving the unity of fractal alive and inorganic nature are the aim of the given work

2. THEORETICAL PART

In 1875 Dubua-Reymon informed the World about continuous non-differentiable function constructed by K.Veyershtrass [4].

Let's give an example, its function is defined by $f(x)=\sum_{n=0}^{\infty} a^n \cdot cos(b^n \pi x)$ series where $0<a<1$, ab is uneven natural number (moreover $ab > 1+\frac{3}{2}\pi$). This set is dominated by convergent progression $\sum_{n=0}^{\infty} a^n$, consequently it converges equally and its sum is continuous function from x everywhere. Veyershtrass showed by detail investigation that the finite derivative doesn't exist in any its point.

Let's give the more simple example of Van-der-Varden [6] constructed on the same idea only oscillating curves $y = cos\,\omega x$ are exchanged by oscillating kinked curves. Let's designate the magnitude of difference between x number and nearest number to it through $u_0(x)$. This function is linear one in $\left[\frac{S}{2}, \frac{S+1}{2}\right]$ interval where S is integral number; it is continuous one and has the period 1. The function plot presents itself the kinked curve, the separate elements of kinked curve have angular coefficient ± 1. For $\kappa=1,2\ldots$ let's define the functions $u_k(x)=\frac{u_o(4^k \cdot x)}{4^k}$, these functions will be linear ones in $\left[\frac{S}{2\cdot 4^k}, \frac{S+1}{2\cdot 4^k}\right]$ intervals, they are also continuous ones and have the period $\frac{1}{4^k}$. Its plot is also kinked curve but with small ripples. In all cases the angular coefficients of separate elements of kinked curve here are equal to ± 1.

Let's define the function *f(x)* for all real values of *x* by $f(x)=\sum_{k=0}^{\infty} u_k(x)$ equality. As it is obvious that $0 \le u_k(x) \le \frac{1}{2\cdot 4^k} (k=0,1,2...)$ then series is dominated by convergent progression $\sum_{0}^{\infty} \frac{1}{2\cdot 4^k}$ (as in the case of Veyershtrass function), the series converges equally and *f(x)* function is continuous one everywhere.

Let's take any value of $x=x_0$. Calculating it with delicacy up to $\frac{1}{2\cdot 4^n}$ (where *n=0, 1, 2, ...*) on defect and excess we put it between numbers of $\frac{S_n}{2\cdot 4^n} \le x_0 < \frac{S_n+1}{2\cdot 4^n}$ type where S_n is integral number. The closed intervals $\Delta_n = \left[\frac{S_n}{2\cdot 4^n}, \frac{S_n+1}{2\cdot 4^n}\right]$ (*n=0, 1, 2, ...*) are put the one into another. In each of them there is such point x_n that its distance from x_0 point is equal to half of interval length: $|x_n - x_o| = \frac{1}{4^{n+1}}$, it is clear that $x_n \to x_o$ with increase of *n* variant.

Let's write $\frac{f(x_n)-f(x_o)}{x_n-x_o} = \sum_{k=o}^{\infty} \frac{u_k(x_n)-u_k(x_o)}{x_n-x_o}$.

At $k>n$ number $\frac{1}{4^{n+1}}$ is whole multiple of $\frac{1}{4^k}$ period of $u_k(x)$ function so $u_k(x_n)= u_k(x_o)$, the corresponding members of series transform into zero and can be excluded. If $k \le n$ then $u_k(x)$ function is linear one in Δk interval, it will be linear one in Δ_n interval including in it, moreover

$$\frac{u_k(x_n)-u_k(x_o)}{x_n-x_o} = \pm 1 \ (k=0, 1, ..., n).$$

Then finally we have $\frac{f(x_n)-f(x_o)}{x_n-x_o} = \sum_{k=0}^{n} (\pm 1)$, by other words this relation is equal to even integral number at uneven *n* and to uneven integral number at even *n*.

It is clear that at the difference relation not to any finite bound can't strive for, this means that our function at $x=x_o$ doesn't have the finite derivative.

Thus Van-der-Varden function is simpler than known Veyershtrass function. On the base of [6-7] it is established that Van-der-Varden function is non-differentiable in all points.

The previous history of these questions is the following. In second half of previous century the representatives of math school criticizing the analysis basics and firstly Veyershtrass and Piano constructed the continuous functions not having derivatives everywhere and curves everywhere tightly filled the square. The strange properties of these objects are connected with the fact that they are considered as one-dimension ones from modern point of view, whereas one can consider them as objects of more high and also fractional dimensionality on modern terminology by fractals.

"The fractal is the word introduced by Mandelbrot in order to combine the wide object class which plays the historical role in the deveopment of pure math of XIXc. under one heading. The roots of classic math are among right geometrical structures of Euclid and Newton translational dynamics. The modern math begins with Cantorian theory of multitudes and Piano curve filling the space. The historical revolution is caused by the opening of math structures not putting in limits of constructions of Euclid and Newton. These new structures were considered as "pathological ones, as some monster exhibition". The mathematicians created these "monsters" considered them as important evidences of the fact that world of pure math consists in it the unusual excess of possibilities which are far from limits of that simple structures that we can observe in nature.

3. FRACTALITY OF ALIVE AND INORGANIC NATURE

Firstly we consider the some known perceptions of that fact that chaos being the structureless one is able to create the order [1-2,8]. The biological organism consists in many structure-functional elements where informational component of each separate cell influences on both all rest subforms (textures, organs) as a whole [5].

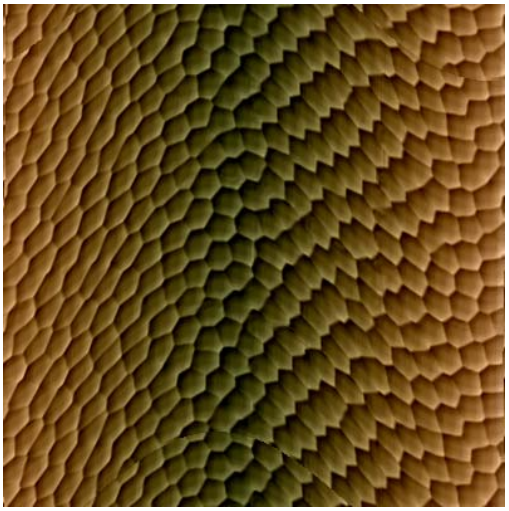

Fig.1. The fractal surface of snake skin.

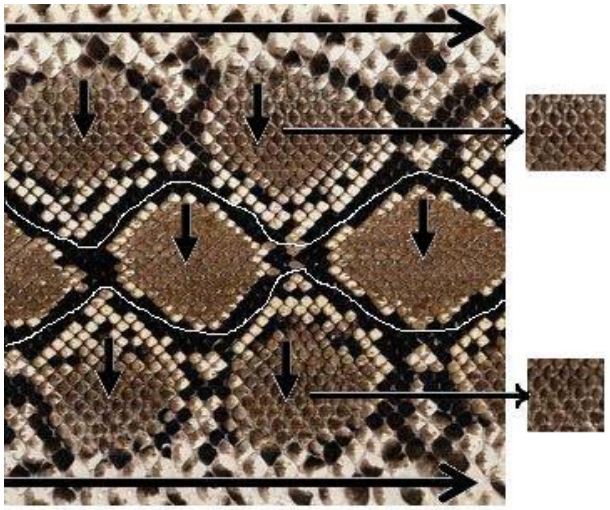

Fig.2. The fragments of snake skin with percolation morphology.

The elements systematizing the informational interconnections existing in all organs, organism systems, including the skin of alive organisms should interest us. Let's consider this on example of snake skin. The photo of snake skin surface is given on fig.1. The snake body is covered by dry parallel flakes (flakes of skin origin) which partially cover each other (fig.1). Many snakes living on earth surface have the flakes on front abdominal wall which are bigger in many times than on top surface. The separate series of wide flakes which play the important role in mobility of wood and earth snakes.

It is well known that snake during crawling has the wave along its whole body. This wave can be started on stripes which are along back from heat up to tail. We will discuss this question once more during snake design. For example, Caucasian viper has the wide black zigzag stripe on the edge. The sand Echis has the light transversal stripes. Ethis vocalizates the character dry rustling with help of small ribbed flakes on body's side. The fancy regular flake dispositions length and breadth of the body change in the dependence on its type.

The following factors have the essential meaning at snake definition by external features:

- the form and dimension of flakes;
- the quality of horny scutes and flakes covering the body;
- the space distribution of longitudinal zigzag stripes;
- the zigzag stripes transversally surrounding the body the series.

The above mentioned factors reveal the fractality features of patterns on snake body surface that is proved by photos given on fig.1 and 2. Firstly our attention should be paid to:

- the zigzag position of flakes from head up to tail differing from position transversally to their body;
- these longitudinal zigzags are continuous (they don't break) (see fig.2)
- the color of longitudinal stripes differs from transversal ones;
- the flake position character has the regular form (see fig.2);
- the separate islands consisting in flakes have their own dimensions (see fig.1 and 2).

If we cut the small elements from them then they will have the form of fractal objects, they are emphasized on fig.2 and mentioned by vertical hands.

The longitudinal zigzag stripes are mentioned on fig.2. One can suppose that the foundations of these stripes on snake body play the role of elements on information bond. Probably analyzing the informational-exchange process the brain sends the "wave" on passing channels (the channels are mentioned by serpentine line and horizontal direct lines on fig.2) along whole body that gives the jump to snake wave-like motion. We can say that flaky-fractal structure of snake appears in evolution process for formation of its optimal dynamics.

Note that fractality features reveal in all natural phenomena and physical objects [8-12]. The observable structures and physical phenomena are comfortable analyzed from fractal presentations of point of view. There are many such works in reference. We give only the series of monographs [1-2, 8, 11], however, the whole directions connected with study of fractal structure of solid bodies, their fractal dimensionality and properties have already formed. The geometric objects which are lines of surface body of strongly dissect form can be related to the number of such fractals and they demonstrate the repeatability in wide range of scales [3-8,10]. We can say about them from only fractal comparative point of view on the surface of alive organisms and crystalline bodies. The use of fractal conceptions in physics has led to perception of new group of properties of disordered systems and given the additional information about them. The establishment of morphological bond between absolutely different physical objects is our task (the snake skin surface and crystal surface). The photos of snake skin are given above (fig.1 and 2).

Nowadays we consider the photos of fractal interlayer surface of layered crystals. The electron-microscopic images of Bi_2Te_3 <impurity> taken on atomic force microscope (AFM) of cleaved surface (0001) are given on fig.3. As it is seen from photo these images are analogous by form and look like the patterns on snake skin (fig.1). The difference consists in scale. On fig.3 we see consider the nano-fragments the dimensions of which are in limits from 10 up to 50nm having the conical form near foundation with pointed tops.

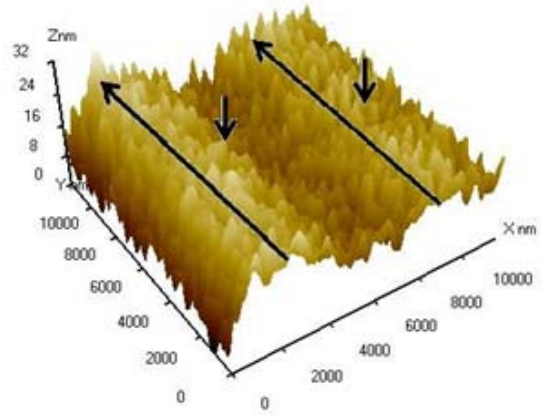

Fig.3. The fractal surface of Bi_2Te_3 <impurity>.

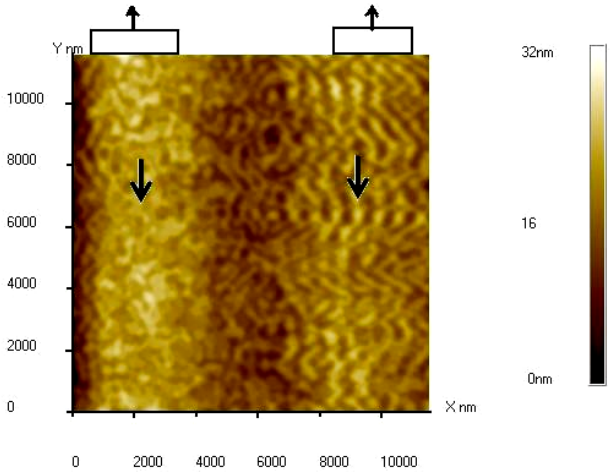

Fig.4. The percolation structure of (0001) Bi_2Te_3 <impurity> surface.

Note that formation of fractal systems in macro- (fig.1 and 2) and in nano-systems (fig.3) is connected with revealing of dissipative self-organization in unbalance conditions.

The photo of solid-state percolation cluster presented on fig.4 is very similar with such skin pattern given on fig.2.

The fractal coverings (aggregates, clusters, nano-particles) form in the result of merge of solid particles carrying out in alive and inorganic bodies in different mediums.

Here we compare the morphology of fractal aggregates forming on snake surface from the moment of their origin (they genetically put into fractal "constructions" of snakes as a whole) with fractal systems forming in Van-der-Waals of layered semiconductors (bismuth telluride). Such nano-particle can be observed not only in layered crystals and they can be obtained by many methods on their surface [12]. The growth mechanisms of these formations are similar for free fractal systems for bound fractals on the surface.

Note that forming fractal aggregates have the form of small "pyramides" directed perpendicularly to (0001) Bi_2Te_3 surface. Here we also have the fractal relief as on snake surface that leads to essential increase of surface general square. The consequences of this fact lead to the point that snake fractal surface (having the natural character) is the best receiver of different radiations that requires the special investigations. As we have mentioned the stripes from flakes from head up to tail (see fig.2) can serve as percolation channels. However the study of chemical composition of stripes so-called percolation structures is required. Besides, they can consist in materials present themselves the mixture of dielectrics, metals and other organic substances. The dynamics tasks of snake motion, questions of information spreading on surface of snake construction can be studied with help of percolation theory [1-2,8].

CONCLUSIONS

The attention on functions which aren't smooth or regular providing the essentially better presentation of many natural phenomena is paid. The fractal perception is used at analysis of morphology of snake flake surface and morphology of interlayer crystal surface by telluride bismuth type. In both cases the structures look like to fractals of surrounding us nature: the images of structure of crystal surface obtained with help of electron microscope and surface snake patterns effectively constructed by the nature. The physical body of alive organisms (snakes and etc) is fractal one; the principle of unique simple one controlling the different the complex one is put in snake (human being) genome, this fact is well known. The unique fractality of alive organisms and solid crystals becomes the fact.

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FİZİKİ VƏ TƏBİİ STRUKTURLARDA FRAKTAL KONSEPSİYASI

Bərk cisimlərin və canlıların səthlərində yaranan morfoloji fraktal strukturlarını Veyerştrass və Van-der-Varden funksiyaları ilə təsvir etmək mümkündür.

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ФРАКТАЛЬНЫЕ КОНЦЕПЦИИ В ФИЗИЧЕСКИХ И ПРИРОДНЫХ СТРУКТУРАХ

На примере исследования морфологии поверхностных структур твердых тел и кожи живых существ показано, что фрактальные системы образуют целый мир объектов и явлений, которые можно описывать функциями, введенными Вейерштрассом и Ван-дер-Варденом.

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