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M² QUALIFY LASER BEAM PROPAGATION

ABDELHALIM BENCHEIKH, MOHAMED BOUAFIA
Applied Optics Laboratory, University Ferhat Abbas –Setif 19000 , Algeria

One of the most important properties of a laser resonator is the highly collimated or spatially coherent nature of the laser output beam. Laser beam diameter and quality factor M² are significant parameters in a wide range of laser applications. This is because the spatial beam quality determines how closely the beam can be focused or how well the beam propagates over long distances without significant dispersion. In the present paper we have used three different methods to qualify the spatial structure of a laser beam propagating in free space, the results are obtained and discussed, and we have found that the Wigner distribution function is a powerful tool which allows a global characterization of any kind of beam.

1. INTRODUCTION

The use of lenses and other optical elements is required by mainly laser applications so as to focus, modify, or shape the laser beam [1]. The adequate choice of the best optics for a particular laser application necessitates the knowledge of the basic properties of Gaussian beams. In most cases, the propagation of laser-beam can be approximated by assuming that the laser beam has an ideal Gaussian intensity profile, corresponding to the theoretical TEM_{00} mode.

In praxis, the real lasers output is not truly Gaussian. This discrepancy can be accommodated by using a quality factor M^2 (called the "M-square" factor). The quality factor M^2 has been defined to describe the deviation of the laser beam from a theoretical Gaussian one.

For a theoretical Gaussian beam $M^2 = 1$; but for a real laser beam $M^2 > 1$. In all cases, the M^2 factor, which varies significantly, affects the characteristics of a laser beam and cannot be neglected in optical designs [1-3].

Firstly, we will discuss the characteristics of a theoretical Gaussian beam $M^2 = 1$ and then we will show how these characteristics change as the beam deviates from its theoretical shape. In all cases, a circularly symmetric wave front is assumed, as would be the case for a helium neon laser. Diode laser beams are asymmetric and often astigmatic, which causes their transformation to be more complex. In order to gain an appreciation of the principles and limitations of Gaussian beam optics, it is necessary to understand the nature of the laser output beam. In TEM_{00} mode, the beam emitted from a laser is a perfect plane wave with a Gaussian transverse irradiance profile. The Gaussian shape is truncated at some diameter either by the internal dimensions of the laser or by some limiting aperture in the optical train. To specify and discuss propagation characteristics of a laser beam, we must give the theoretical aspect of laser beam propagation.

The aim of this paper consists on the use of a three techniques to evaluate the spatial structure of a laser beam; the first technique is based on the measurement of the width of the beam in different locations along the axis of propagation and then we plot the width as a function of the distance of propagation, from the curve we extract the spatial characteristics of the beam.

The second one is based on the measurement of the intensity moments in different locations along the axis of propagation, from these moments we extract the spatial characteristics of the beam, and we do that we have done with the fist method.

The last technique is based on the measurement of the Wigner distribution function; this last gives access to the whole spatial characteristics of the laser beam. And finally we discuss the results and we will finish with a conclusion.

2. THEORETICAL ASPECTS

2.1 Gaussian beam

Gaussian beams are the simplest and often the most desirable type of beam provided by a laser source. As we will see in this section, they are well characterized and the evolution is smooth and easily predicted. The amplitude function representing a Gaussian beam can be deduced from the boundary conditions of the optical resonator where the laser radiation is produced. The geometrical characteristics of the resonator determine the type of laser emission obtained. For stable resonators, neglecting a small loss of energy, the amplitude distribution is self-reproduced in every round trip of the laser through the resonator. Unstable resonators produce an amplitude distribution more complicated than in the stable case. Besides, the energy is leaking in large proportion for every round trip. For sake of simplicity we restrict this first analysis to those laser sources producing Gaussian beams. The curvature of the mirrors of the resonator and their axial distance determines the size and the location of the region showing the highest density of energy along the beam. The transversal characteristics of the resonator allow the existence of a set of amplitude distributions that are usually named as modes of the resonator. The Gaussian beam is the lowest degree mode, and therefore it is the most commonly obtained from stable optical resonators [2, 3].

The propagation of Gaussian beams through an optical system can be treated almost as simply as geometric optics. Because of the unique self-Fourier Transform characteristic of the Gaussian, we do not need an integral to describe the evolution of the intensity profile with distance. The transverse distribution intensity remains Gaussian at every point in the system; only the radius of the Gaussian and the radius of curvature of the wave front change [4].

The set of modes is characterized in every point along the propagation axis by two functions: $R(z)$ and $w(z)$. The first describes the radius in the transverse plane for which the amplitude of the field has decreased by a factor $1/e$ with respect to the amplitude value along the propagation axis, while the second parameter, with respect to the fundamental mode TEM_{00}, gives the radius of curvature of the wave front that intersects the propagation axis. The transversal intensity distribution of the laser beam (for the TEM_{00} only) has a Gaussian dependence, it is given by:

$$U(r,z) = A(z)\exp\left(-jk\frac{r^2}{2q(z)}\right) \quad (1)$$

And its radius $w(z)$ contracts to a minimum w_0 known as the waist of the beam. The two parameters $R(z)$ and $w(z)$ are determined by the waist size w_0 and by the distance z from the waist position, the complex ray of curvature between brackets is given as a function of the ray curvature of the wave front of the beam and of its width [1, 2, 3]:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)} \quad (2)$$

The longitudinal beam profile determined by the function $w(z)$ is a hyperbola with asymptotes forming an angle with the propagation axis.

The equations describing the beam radius $w(z)$ and the wave front radius of curvature $R(z)$ are:

$$w^2(z) = w_0^2\left(1+\left(\frac{z}{z_R}\right)^2\right) \quad (3)$$

$$R(z) = z\left(1+\left(\frac{z_R}{z}\right)^2\right) \quad (4)$$

The Rayleigh length and the divergence angle, respectively relate the beam waist to the wave length as follows:

$$z_R = \frac{\pi w_0^2}{\lambda} \quad ; \quad \theta = \frac{\lambda}{\pi w_0} \quad (5)$$

The beam size will increase, slowly at first, then faster, eventually increasing proportionally to z. The wave front radius of curvature, which was infinite at $z = 0$, will become finite and initially decrease with z. At some point it will reach a minimum value, and then increase with larger z, eventually becoming proportional to z.

2.2 Real aspect of laser beams and the intensity moments

In the most cases in practice the laser beam is not purely Gaussian because of the experimental limitations as; truncation, phase distortion, etc… (see fig.1)

The theory of Gaussian beam propagation is not sufficient to describe the evolution of the spatial characteristics of such beams as beam width and the divergence, the introduction of the intensity moments is very convenient in this case and the beam width and the divergence are given by (for the shake of simplicity we consider a circular beam with only one dimension x) [5]:

$$d(z) = 4\sqrt{\langle x^2 \rangle} \qquad \theta(z) = 4\sqrt{\langle \theta^2 \rangle} \quad (6)$$

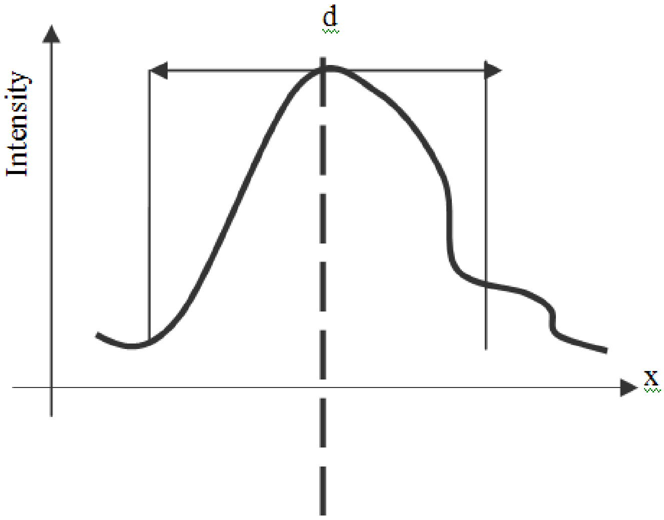

Fig.1: Intensity distribution of non Gaussian laser.

Here the second moments of the intensity distribution $I(x,y,z)$ at the location z are given by:

$$\langle x^2 \rangle(z) = \frac{\iint (x-\langle x\rangle)^2 I(x,y,z)\,dxdy}{\iint I(x,y,z)\,dxdy} \quad (7)$$

$$\langle \theta^2(z) \rangle = \frac{\iint \langle \theta^2 \rangle \bar{I}(x,y,z)dxdy}{\iint \bar{I}(x,y,z)dxdy} \quad (8)$$

Here $\langle x \rangle$ is the first moment of the intensity distribution giving coordinates of the beam centre:

$$\langle x(z) \rangle = \frac{\iint \langle x \rangle I(x,y,z)dxdy}{\iint I(x,y,z)dxdy} \quad (9)$$

$$\langle \theta(z) \rangle = \frac{\iint \langle \theta \rangle \bar{I}(x,y,z)dxdy}{\iint \bar{I}(x,y,z)dxdy} \quad (10)$$

$$\langle x\theta(z) \rangle = \frac{\iint \langle x\theta \rangle I(x,y,z)dxdy}{\iint I(x,y,z)dxdy} \quad (11)$$

The invariant parameter and the quality factor M^2

We can introduce a matrix which describes the different spatial parameters of a laser beam (parameters of propagation), this matrix is called in the literature a beam matrix M, and it is given with the physical signification of each element of the matrix by:

$$M=\begin{pmatrix}\langle x^2\rangle & \langle x\theta\rangle\\ \langle x\theta\rangle & \langle\theta^2\rangle\end{pmatrix}=\begin{pmatrix}Width & Curvature\\ Curvature & Divergence\end{pmatrix} \quad (12)$$

The propagation of a given laser beam along the optical systems described by ABCD matrix, is given by [3, 5]:

$$\begin{pmatrix}\langle x_2^2\rangle & \langle x_2\theta_2\rangle\\ \langle x_2\theta_2\rangle & \langle\theta_2^2\rangle\end{pmatrix}=\begin{pmatrix}A & B\\ C & D\end{pmatrix}\begin{pmatrix}\langle x_1^2\rangle & \langle x_1\theta_1\rangle\\ \langle x_1\theta_1\rangle & \langle\theta_1^2\rangle\end{pmatrix}\begin{pmatrix}A & B\\ C & D\end{pmatrix}^T \quad (13)$$

We can write [5]:

$$\langle x_2^2\rangle\langle\theta_2^2\rangle-\langle x_2\theta_2\rangle^2=\langle x_1^2\rangle\langle\theta_1^2\rangle-\langle x_1\theta_1\rangle^2 \quad (14)$$

Where, the sub notations 1 and 2 are for the input and out beam.

Finally we can show that the quantity $\langle x^2\rangle\langle\theta^2\rangle-\langle x\theta\rangle$ is invariant with propagation with lossless optical systems. This invariant is given by [3.5]:

$$\langle x^2\rangle\langle\theta^2\rangle-\langle x\theta\rangle^2=\left(\frac{\lambda}{4\pi}M^2\right)^2 \quad (15)$$

In the previous paragraph we have found that for the Gaussian beams there is a parameter which remains invariant along the propagation through *ABCD* optical systems. If we rearrange the equation (15) we find the famous quality factor known as M² factor [3, 5-9], given by:

$$M^2=\frac{4\pi}{\lambda}\sqrt{\langle x^2\rangle\langle\theta^2\rangle-\langle x\theta\rangle} \quad (16)$$

This parameter gives a compact characterization of any kind of laser beams; it describes the deviation of the real laser beam from the ideal model which is the Gaussian one. The value of M² is 1 for the ideal case (Gaussian beam), and it become greater than 1 when the intensity distribution deviates from the Gaussian model.

2.3. Wigner distribution function

A partially-coherent light beam is described better by its second order functions. One of these functions is the cross spectral density function that, in the two dimensional case, can be written as [6, 7, 10, 11,12]:

$$\Gamma(x,s,z)=\left\langle E\left(x+{}^{s}\!/_{2},z\right)E^*\left(x-{}^{s}\!/_{2},z\right)\right\rangle \quad (17)$$

Where; $E(x,z)$ is the distribution of the electric field along the x axis at a given distance of propagation z, * means the complex conjugate and $\langle\ \rangle$ stands for an ensemble average.

The Wigner distribution is defined as the Fourier transform of the cross spectral density as follows [6, 7, 12].

$$W(x,u,z)=\int\Gamma(x,s,z)\exp(-2i\pi us)ds \quad (18)$$

The use of the Wigner distribution in optics has been deeply studied and it seems to be very well adapted to the analysis of partially coherent beam because it contains information about the spatial irradiance distribution and its angular spectrum at the same time, so it allows to give the local spatial frequencies at any location.

We define both the width, divergence and the curvature as functions of the second order moments of the Wigner distribution as [6-7, 10-12]:

$$\langle x^2\rangle_W(z)=\frac{\iint(x-\langle x\rangle)^2W(x,u,z)dxdu}{\iint W(x,u,z)dxdu} \quad (19)$$

$$\langle\theta^2\rangle_W(z)=\frac{\iint(u-\langle u\rangle)^2W(x,u,z)dxdu}{\iint W(x,u,z)dxdu} \quad (20)$$

$$\langle x\theta\rangle_W(z)=\frac{\iint\langle xu\rangle W(x,u,z)dxdu}{\iint W(x,u,z)dxdu} \quad (21)$$

3. EXPERIMENTAL RESULTS

The first step of the experiment is to take different caustics which correspond to the different propagating distances z by using the setup presented in the figure.2

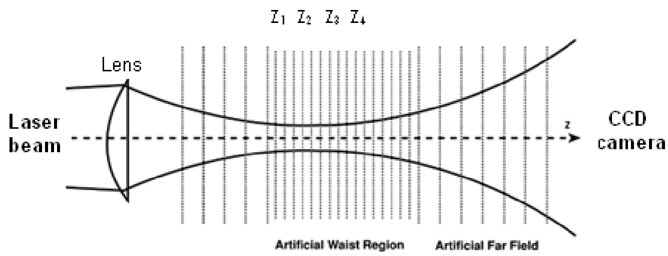

Fig.2. The record of the different caustics

We have a lens which focuses the laser beam, and then we take different images (caustics) in different locations along z axis of propagation. The images taken by a CCD camera are presented bellow:

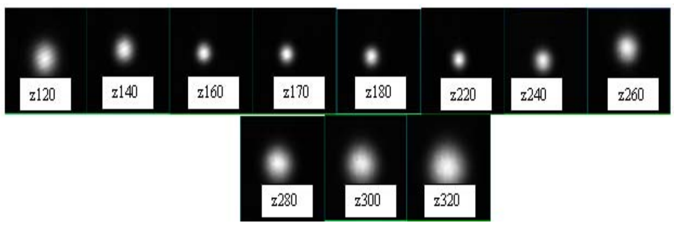

Fig.3. Caustics spots of laser beam in different z positions in mm.

With the three methods we use the same caustics (images) to characterize the spatial structure of the laser beam.

***The first method*:**

The evolution of the width of laser beam along free space is represented in figure 4. From this curve we extract the beam waist and the divergence, and then the M² factor.

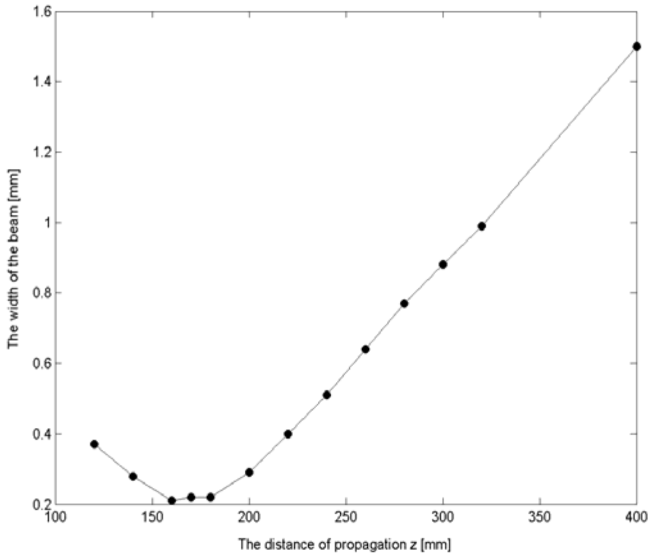

Fig.4. The evolution of laser beam width along free space. $d_0 = 2w_0 = 0.4\,mm$. The quality factor is found $M^2 = 1.77$

***The second method "Moment method"*:**

With the same manner as the first method, but in this one we measure the intensity second order moments at different distances of propagation z, we have plotted the curve by using 'Beam analyzer' software. The curve is given by the figure 5.

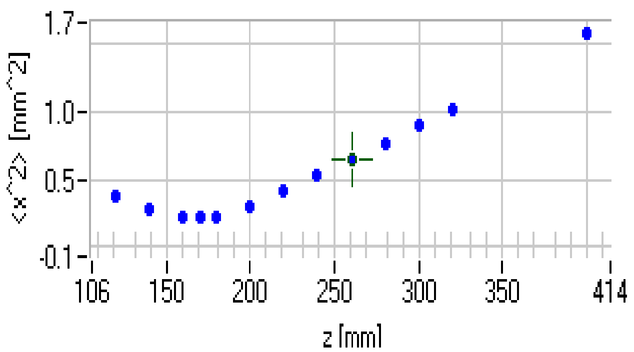

Fig.5. The evolution of the second order moment of laser beam along free space.

From the curve we extract the spatial characteristics as follows: the waist: $d_0 = 2w_0 = 4\sqrt{\langle x^2 \rangle} = 0.43\,mm$, and the divergence $\theta = 4\sqrt{\langle \theta^2 \rangle} = 2.83\times10^{-6}\,rad$. The quality factor is given by $M^2 = \frac{BPP}{(\lambda/4\pi)} = 1.77$

***The third method "Wigner distribution"*:**

For this method, we construct the Wigner distribution function from the different caustics, the reconstruction is based on the mathematical concept of Radon transform and filtered back projection theorem, in the present experiment we have reconstructed the Wigner distribution by using the 'Beam analyzer' software and the result is given bellow in the figure 6.

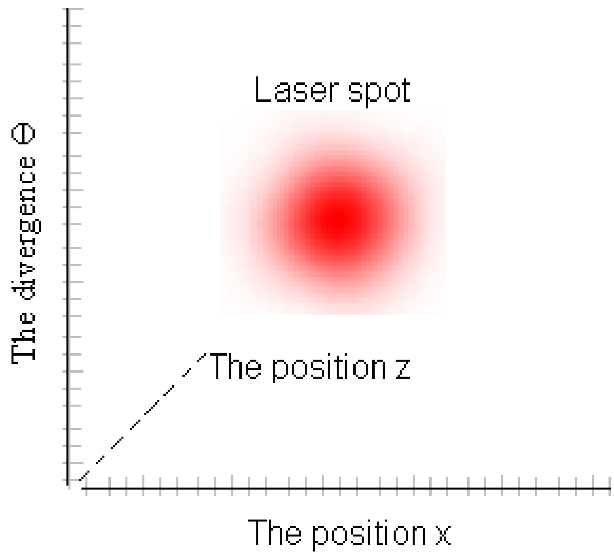

Fig.6. Wigner distribution of reconstructed caustics.

The software beam analyzer allows calculating the different moments (first, second and mixed orders) of the Wigner distribution function. It gives the results in a form of a (4*4) matrix.

The beam matrix is given by:

$$S=\begin{pmatrix} \langle x^2 \rangle & \langle xu \rangle \\ \langle xu \rangle & \langle u^2 \rangle \end{pmatrix} = \begin{pmatrix} \langle 2.84\times10^{-3} \rangle & \langle -4.76\times10^{-6} \rangle \\ \langle -4.76\times10^{-6} \rangle & \langle 2.83\times10^{-6} \rangle \end{pmatrix}$$

From this matrix we extract the waist $d_0 = 2w_0 = 4\sqrt{\langle x^2 \rangle} = 0.42\,mm$, and the divergence $\theta = 2.83\times10^{-6}\,rad$. Finally we get the value of the quality factor as $M^2 = \frac{BPP}{(\lambda/4\pi)} = 1{,}78$.

4. CONCLUSION

In this work three methods are used to characterize the laser beam quality. The results obtained from experiments show that the use of the Wigner distribution gives more information about the characteristics of laser such as coherence. The use of the Wigner distribution is more important in partially light characterization.

5. ACKNOWLEDGMENTS

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