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QUADRUPOLE NUCLEAR STATES AS A REPRESENTATIONS of a SU(5) GROUP

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Kvadrupol halları SU(5) cəbri vasitəsilə Skirma potensialında tədqiq edilir. Kollektiv hamiltonian keçid nüvələrinə ^{150,152}Nd, ^{150,152}Sm və ^{152}Gd tətbiq edilir. Qarşılıqlı təsir sabitləri əsas və həyəcanlanmış 0^+ hallarına $E2$ keçidlərinin ehtimalı, iki zərrəcikli keçidlərin spektroskopik faktorlarının müqayisəsinə görə təyin edilmişdir.

The quadrupole states are described by a SU(5) algebra with Skyrme potential. The collective Hamiltonian is applied to the transitional nuclei ^{150,152}Nd, ^{150,152}Sm and ^{152}Gd, where the constants are determined by fitting the experimental spectra, the relative $E2$ transition probabilities and the spectroscopic factors for the two-particle transitions to the ground and excited 0^+ states.

С помощью SU(5) алгебры описывается квадрупольные состояние в потенциале Скирма. Коллективный гамильтониан применяется к переходным ядрам: ^{150,152}Nd, ^{150,152}Sm и ^{152}Gd. Константы взаимодействия подбираются для основных и возбужденных 0^+ уровней по соответствующим экспериментальным данным: по $E2$ вероятностям, по спектроскопическим факторам двухнуклонных передач.

INTRODUCTION

For a long while, the systematic behavior of the first excited 0^+ states in some even-even transitional nuclei has attracted much attention, because those states often show too large anharmonicity to be considered as a member of the two-phonon triplet states in the phonon model. Generally speaking, the theory has been comparatively successful for deformed nuclei, but less for the spherical, or vibration nuclei. An important feature of the collective motion in nuclei is that, unlike other many-body systems, the excitation energies associated with vibrations and rotations are not very different from each other.

The purpose of this note is to point out that the group SU(6) might provide the appropriate framework for a unified description of the collective nuclear states. A.Arima and F.Iachello [1] showed that within this model both the vibrational and the rotational limit can be recovered.

The first case we have discussed here is only vibrational spectra in the framework of the subgroup SU(5). Exploiting the related symmetry group SU(5) we have been able to obtain simple analytic expressions for the eigenvalues of the boson Hamiltonian and for the intereband transition matrix elements as well as for side feeding from one band to the other. Back bending occurs naturally as the crossing of two bands and it can be predicted from the relative spacing of the low excited states.

The algebraic properties of the collective variable lead to a new quantum number N that implies in the boson representation the maximum number of phonons contained in the vibrational states. Because the boson-boson interaction in SU(5) invariant Hamiltonian splits the degeneracy of the multiplets, this limit describes an anharmonic vibrator. It should be noted, we describe finite dimensional systems in contrast with the geometrical description in which $N \to \infty$. It is worth noting that the knowledge of the invariance properties of the Hamiltonian provides directly a solution to the eigenvalue problem.

The transformation into the intrinsic frame of reference has been performed explicitly. Thus, the formulae for the potential energy the quadrupole moments are obtained as well as the spectroscopic factors for 0^+ state excitation in the two-nucleon transfer reactions. The proposed collective Hamiltonian is applied to the transitional nuclei Nd, Sm and Gd.

1. DERIVATION OF THE COLLECTIVE HAMILTONIAN

The symmetry structure of the nuclear many body system is in general very complex. However, since only few degrees of freedom play a dominant role in the description of the collective states, it is hoped that the Hamiltonian of the system when written in terms of these degrees of freedom has simple symmetry properties. We have suggested [1] that these symmetry properties are those of the six dimensional special unitary group SU(6) acting on a boson space.

To begin with, we claim that a number of positive-parity states can be generated in even-even nuclei as states of a system of N bosons having no intrinsic spin but able to occupy two levels, a ground-state level with angular momentum L = 0, and an excited state with angular momentum L =2. In the case in which the two levels are degenerate and there is no interaction between bosons, the five components of the excited L = 2 state, called d for convenience, and the single component of the ground L = 0 state, called s, span a six – dimensional vector space which provides the basis for the representations of the unitary group SU(6).

According to the group reduction [2]:

$$SU(6)\supset SU(5)\otimes U(1), \qquad (1.1)$$

the first case we will consider is that of the group SU(5) spanned by the 5 components of the d state alone. Only five labels are needed to classify the states. Three of them are the total boson number N, the total angular momentum L and its z – component M. The fourth is the seniority v. Instead of v one can introduce another quantum number n_β, which counts boson pairs coupled to zero angular momentum. n_β is related to v by

$$v = N - 2n_\beta . \qquad (1.2)$$

The representations of SU(5)contained in [N] are all symmetric representations

$$\lfloor n_\beta = 0\rfloor,\ \lfloor n_\beta = 1\rfloor,\ \lfloor n_\beta = 2\rfloor,\ \text{up to}\ \lfloor n_\beta = N\rfloor.$$

Finally one can introduce a fifth quantum number n_Δ, which counts boson triplets coupled to zero angular momentum. The total number N is partitioned by n_β and n_Δ as

$$N=2n_\beta+3n_\Delta+\lambda, \qquad (1.3)$$

and the possible values of the total angular momentum L are given in terms of λ by

$$L = \lambda, \lambda+1, \lambda+2, \ldots, 2\lambda-2, 2\lambda . \qquad (1.4)$$

The energy levels can be found by diagonalizing the model Hamiltonian

$$H = \varepsilon\sum_m d_m^+ d_m + \sum_\lambda q_\lambda \left[\left[d^+ d^+\right]^\lambda_\mu \left[dd\right]\right]^0_0 , \qquad (1.5)$$

with eigenvalues

$$E([N], n_\beta, v, n_\Delta, L, M) = \varepsilon n_\beta + \frac{1}{14}(3q_4+4q_2)N(N-1)+$$
$$\frac{1}{10}\left[q_0 - \left(\frac{1}{7}(3q_0+4q_2)\right) + \frac{7}{6}(q_0-q_2)\right](N-v)(N+v+3)+$$
$$\frac{1}{14}(q_0-q_2)\left[L(L+1)-6N\right], \qquad (1.6)$$

where $d^+(d)$ is the creation (annihilation) operator for a d – bosons and

$$q_\lambda = \left\langle d^2\,\lambda\,\mu \right| V \left| d^2\,\lambda\,\mu \right\rangle . \qquad (1.7)$$

For our considerations the algebraic properties of the binary operators dd, d^+d^+, d^+d are important. Obviously, in a given configuration space these operators from Lie algebra. In what follows our basic assumption is, that this Lie algebra there exists a subalgebra, which contains all operators for a construction of the collective variables as well as collective Hamiltonian and the collective states. In general, such algebra does not exist. However, for our aim it is satisfactory, if this subalgebra exists with respect to the subspace of the collective vibrational states only, in which we are interested.

It is important to notice that our collective Hamiltonian yields a finite energy matrix for a given value of N and a definite spin. This is a consequence of the symmetry properties of our collective operators. Hence one can really find exact solutions for the Hamiltonian (1.5).

Because the boson-boson interaction in (1.5) splits the degeneracy of the multiplets, this limit describes an anharmonic vibrator. It should be noted that here and in (1.6) $0 \le n_\beta \le N$, and thus we describes finite dimensional systems in contrast with the geometrical description in which $N \to \infty$. It is worth nothing that the knowledge of the invariance properties of the Hamiltonian provides directly a solution to the eigenvalue problem.

2. E2 – MATRIX ELEMENTS

Now the application of the collective Hamiltonian (1.5) to the transitional nuclei ^{150,152}Nd, ^{150,152}Sm and ^{152}Gd is presented. In general the qualitative structure of the energy spectrum obtained from our Hamiltonian is not significantly influenced by the other order terms. Hence the energy spectrum and the wave functions are determined by the constants q_0, q_2, q_4 and the maximal phonon number N. The numbers of valence nucleons in ^{150,152}Nd, $^{150,\,152}$Sm and ^{152}Gd are equal to 18,20,18, 20, 20 respectively. Hence for those nuclei, N must be less than 9, 10, 9, 10, 10, respectively. Besides, one has to take into account the fact that N must increase, or one approach the region of stable deformation, i.e. N must be larger in ^{152}Sm than in ^{150}Sm. For both cases ^{150}Sm and ^{152}Gd we took N =7, because their properties are similar. In ^{152}Sm we put N =9.

For the calculation of the E2 transitions the following expression of the electric quadrupole operator is used:

$$M(E2) = \frac{1}{7}(3q_4+4q_2)(d^+s + s^+d)^2_m +$$
$$+\frac{1}{70}(13q_0 - 10q_2 - 3q_4)(d^+d)^2_m \qquad (2.1)$$

This operator has a part (d^+s+s^+d) satisfying the selection rule $\Delta n_\beta = \pm 1$, and a part (d^+d) satisfying $\Delta n_\beta = 0$. In the SU(5) vibrational limit the first term gives rise to transitions form one n_β multiplet to another, while the second term gives rise to transitions within the same multiplet and to quadrupole moments. Of particular importance are those between members of the ground state band, for which one obtains

$$B(E2;I+2\to I)=\frac{1}{196}(3q_4+4q_2)^2(I+2)(2N-I)=$$
$$\frac{1}{4}\frac{(I+2)(2N-I)}{N}B(E2;2\to 0) \qquad (2.2)$$

From the structure of (2.2) it also follows that in the SU(5) limit there are no $\Delta n_\beta = 2$ transitions and yet the quadrupole moment of the first excited 2^+ state can be different from zero because of the $\Delta n_\beta = 0$ term. Thus the observed large quadrupole moments may be compatible with the observed retardation of the $\Delta n_\beta = 2$ transitions. The theoretical and experimental ratios of the B (E2) values for $^{150,\,152}$Nd, ^{150,152}Sm and ^{152}Gd are shown in table 1. The agreement between this theory and experimental results is good. The experimental data are taken from [3].

3. SPECTROSCOPIC FACTORS FOR TWO-NUCLEON TRANSFER REACTIONS

We have studied the spectroscopic factors for the two-particle transitions to the ground and excited 0^+ states in the model with pairing and quadrupole forces [4]. The transitions operators was taken to be of the form, for e.g. (p,t) reaction

$$\Gamma(p,t)=\sum_{\nu} b_{\tilde{\nu}} b_{\nu}\,, \qquad (3.1)$$

where b_ν is the annihilation operator of the nucleon in the state $|\nu\rangle$. Then the spectroscopic for the transition to the ground state is

$$S_0 = (\Delta/G)^2 \qquad (3.2)$$

where G is the strength parameter of the pairing force, Δ - energy gap.

The corresponding formula is too cumbersome to be given here. Therefore we present only the approximate expressions for two cases

1) $\omega \approx 2\Delta$ (pairing vibration)

$$S/S_0 \approx v^2(N-5)^2\,. \qquad (3.3)$$

2) $0<\omega<2\Delta$ (pairing + quadrupole force)

$$S/S_0 \approx \frac{c_2^2}{4}(N-n_\beta+\nu)^2(N-n_\Delta+2)^2\,. \qquad (3.4)$$

One can see from eq. (3.4) the strong dependence of the ratio S/S_0 on the distribution of the single-particle moments. Therefore the low energy 0^+ states are expected to be populated strongly in nuclei having large single-particle quadrupole moments of the same sign in the vicinity of the chemical potential [4].

The theoretical and experimental ratios of the S/S_0 values for $^{150,\,152}$Nd, ^{150,152}Sm and ^{152}Gd are shown in table 1. The experimental data are taken from [5].

4. SUMMARY AND CONCLUSIONS

It is proposed to describe the collective vibrational degree of freedom by an algebra, which is formed by five components. This algebra proves to be the algebra of the group SU(5). This description stresses the symmetry rather than the geometry of the intrinsic system. The symmetry properties of the model Hamiltonian have played an important role in the development of other branches of physics and it is hoped that they may as well elucidate the description of the collective nuclear states.

Table1. Theoretical and experimental ratios of the B(E2) and S/S_0 values for nuclei ^{150,152}Nd, $^{150,\,152}$Sm and ^{152}Gd

	$\frac{B(E2;I+2\to 2)}{B(E2;2\to 0)}$		$S/S_0(t,p)$		$S/S_0(p,t)$	
	Theor	Exp.	Theor	Exp	Theor	Exp
^{150}Nd	4.52					
I)*		5±	2.76	1.20	3.66	-
II)*		0.3	1.09		1.44	
^{152}Nd	5.05	-				
I)*			1.16	0.70	1.22	-
II)*			0.68		0.72	
^{150}Sm	18	13±2				
I)*			1.32	0.08	1,35	-
II)*			0.93		0.49	
^{152}Sm	5.66	6±				
I)*		0.4	2.63	0.74	5.50	0.28
II)*			1.38		2.63	
^{152}Gd	46	52±5				
I)*			2.13	-	3.35	0.13
II)*			1.40		1.40	

I)* with pairing force; II)* with pairing + quadrupole forces .

In the resultant collective Hamiltonian the information about single particle energies and matrix elements of the interaction involves few constants. Hence the general properties of the collective Hamiltonian can be easily discussed in terms of these constants. The algebraic properties of collective variable lead to a quantum member N, which implies in the boson representation the maximum number of phonons contained in the collective states. The proposed collective Hamiltonian is applied to the transitional nuclei ^{150,152}Nd, ^{150,152}Sm and ^{152}Gd, where the constants are determined by fitting the experimental spectra, the relative $E2$ transition probabilities and the spectroscopic factors for the two-particle transitions to the ground and excited 0^+ states. The agreement between the experimental data and the theoretical description is good.

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