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ON SPONTANEOUS SYMMETRY BREAKING IN HOT QCD

SARADZHEV F.M.

Institute of Physics, National Academy of Sciences of Azerbaijan,
H.Javid pr. 33, 370143 Baku, E-mail: fuad_saradzhev@hotmail.com

We prove that nontrivial vacuum states which can arise in hot QCD are associated with the tachyonic regime of hadronic matter fluctuations. This allows us to improve the condition for such states to appear.

1. It is known that at phase transitions from hadronic to quark and gluon degrees of freedom nontrivial local vacuum states can appear in the hadronic phase [1]. These states are metastable and of particular interest since they have experimental signatures such as an enhanced production of η and η' mesons [2]. They can decay via CP violating processes such as $\eta \to \pi^0\pi^0$ and because of global parity odd asymmetries for charged pions. The decay rate of CP-odd metastable states was estimated in [3].

In [4] we used the mean-field approximation to develop the kinetic approach to the decay of the CP-odd phase in hot QCD and to derive a non-Markovian kinetic equation describing the production of η'- mesons. A different kinetic equation was derived for the production of tachyonic modes [5].

In the present Talk, we aim to show that in addition to these metastable states nontrivial vacua can appear according to the standard spontaneous symmetry breaking picture provided the hadronic matter fluctuations enter a tachyonic regime.

2. We start from the singlet Witten-DiVecchia-Veneziano effective Lagrangian density [6]

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + f^2\mu^2 \cos\left(\frac{\eta}{f}\right) - \frac{a_0}{2}\eta^2, \qquad (1)$$

where $f = \sqrt{\frac{3}{2}} f_\pi$ and $f_\pi = 92$ MeV is the semi-leptonic pion decay constant; $\mu^2 = \frac{1}{3}\left(m_\pi^{\ 2} + 2m_K^{\ 2}\right)$ is a parameter depending on $\pi-$ and K-meson masses. The parameter a_0 represents the topological susceptibility. For zero temperature $T = 0$, $a_0 = m_\eta^{\ 2} + m_{\eta'}^{\ 2} - 2m_K^{\ 2} \cong 0.726$ GeV^2, $\mu^2 \cong 0.171$ GeV^2 and $f_\pi \cong 93$ MeV. In response to non-zero temperature mesons change their effective masses, μ and a_0 becoming functions of T. The model is defined in a finite volume: $-L/2 \le x_i \le L/2, i = 1,2,3.$

The meson field $\eta(\vec{x},t)$ obeys the Klein-Gordon type equation

$$\left(\Upsilon + m_0^{\ 2}\right)\eta = J_{s,} \qquad (2)$$

where $m_0^{\ 2} \equiv a_0 + \mu^2$ and the current

$$J_s \equiv -f\mu^2\left[\sin\left(\frac{\eta}{f}\right) - \left(\frac{\eta}{f}\right)\right] \qquad (3)$$

is non-linear in η, i.e. contains orders η^3 and higher and is therefore completely determined by the self-interaction of the field η.

Following the mean-field approximation we decompose $\eta(\vec{x},t)$ into its space-homogeneous vacuum mean value $\phi(t) = \langle \eta(\vec{x},t)\rangle$ and fluctuations χ

$$\eta(\vec{x},t) = \phi(t) + \chi(\vec{x},t) \qquad (4)$$

with $\langle \chi(\vec{x},t)\rangle = 0$. The vacuum mean field is treated as a classical, self-interacting background field. It is defined with respect to the in-vacuum $|0\rangle$ as

$$\phi(t) \equiv \langle \eta(\vec{x},t)\rangle \equiv \frac{1}{L^3}\int d^3x \langle 0|\eta(\vec{x},t)|0\rangle, \qquad (5)$$

so in the limit $t \to -\infty$ $\phi(t) \to 0$, while quantum fluctuations take place at all times.

Substituting Eq. (4) into Eq. (3) yields the following decomposition for the current

$$J_s = J_s^{(1)} + \bar{J}_s, \qquad (6)$$

where

$$J_s^{(1)} \equiv J_s^{(0)} + \mu^2\left[1-\cos\left(\frac{\phi}{f}\right)\right]\chi \qquad (7)$$

is the current in the first order in χ with the background field – fluctuations interaction term added, while the zero order of the current

$$J_s^{(0)} \equiv -f\mu^2\left[\sin\left(\frac{\phi}{f}\right)-\left(\frac{\phi}{f}\right)\right] \qquad (8)$$

represents only the self-interaction of the background field. The second current in the right-hand side of Eq. (6) includes terms of second and higher orders in χ

$$\begin{aligned}\bar{J}_s &= -f\mu^2\sin\left(\frac{\phi}{f}\right)\left[\cos\left(\frac{\chi}{f}\right)-1\right]-\\ &- f\mu^2\cos\left(\frac{\phi}{f}\right)\left[\sin\left(\frac{\chi}{f}\right)-\left(\frac{\chi}{f}\right)\right]\end{aligned}. \qquad (9)$$

Substituting Eq. (4) also into Eq. (2) and taking the mean value $\langle\ldots\rangle$ yields the vacuum mean field equation

$$\ddot{\phi} + a_0\phi + f\mu^2\sin\left(\frac{\phi}{f}\right) = \langle\bar{J}_s\rangle. \qquad (10)$$

Eq. (10) is a generalization of the vacuum mean field equation used in [5] for non-vanishing values of $\langle\bar{J}_s\rangle$ (see also [7]). In the Hartree-type approximation,

$$\langle\bar{J}_s\rangle = -f\mu^2\sin\left(\frac{\phi}{f}\right)\left\langle\cos\left(\frac{\chi}{f}\right)-1\right\rangle. \qquad (11)$$

The equation of motion for the quantum fluctuations reads

$$(\Upsilon + m_{eff}^{\ 2})\chi = J_s - \langle\bar{J}_s\rangle \qquad (12)$$

with

$$m_{eff}^{\ 2} \equiv a_0 + \mu^2\cos\left(\frac{\phi}{f}\right). \qquad (13)$$

For $a \equiv \left(a_0 / \mu^2\right) < 1,$ $m_{eff}^{\ 2}$ can be negative for some values of the background field indicating a tachyonic regime. Eqs.(10) and (12) are self-consistently coupled and include back-reactions. The vacuum mean field modifies the equation for fluctuations via a time dependent frequency, while the fluctuations themselves react back on the vacuum mean field via the source term $\langle\bar{J}_s\rangle$.

3. With the decomposition (4), we deduce from (1) the effective Lagrangian density governing the dynamics of fluctuations

$$\begin{aligned}&\mathcal{L}(\chi)\\ &= \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) + f^2\mu^2\cos\left(\frac{\phi}{f}\right)\cdot\left[\cos\left(\frac{\chi}{f}\right)-1\right]\\ &- f^2\mu^2\sin\left(\frac{\phi}{f}\right)\cdot\left[\sin\left(\frac{\chi}{f}\right)-\left(\frac{\chi}{f}\right)\right]\\ &-\frac{a_0}{2}\chi^2 - \langle\bar{J}_s\rangle\chi.\end{aligned} \qquad (14)$$

Expanding (14) in power series in χ, yields in the second order

$$\mathcal{L}^{(2)}(\chi) = \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \frac{1}{2}m_{eff}^{\ 2}\chi^2. \qquad (15)$$

For $a > 1,$ the second order effective potential of fluctuations

$$\mathrm{V}^{(2)}(\chi) = \frac{1}{2}m_{eff}^{\ 2}\chi^2 \qquad (16)$$

is χ^2-type potential with oscillating walls. During the time evolution of the background field, the potential (16) fluctuates around $\frac{1}{2}a_0\chi^2$ in tune with the time dependence of ϕ. For $a < 1,$ for some values of the background field the potential (16) becomes upside down without any stable, particle states.

Let us consider now the exact form of the effective potential ,

$$\begin{aligned}\bar{V}(\chi) &\equiv \frac{1}{f^2\mu^2}V(\chi) = \frac{a}{2}\left(\frac{\chi}{f}\right)^2 + \frac{1}{f\mu^2}\langle\bar{J}_s\rangle\left(\frac{\chi}{f}\right)\\ &-\cos\left(\frac{\phi}{f}\right)\cdot\left[\cos\left(\frac{\chi}{f}\right)-1\right]\\ &+\sin\left(\frac{\phi}{f}\right)\cdot\left[\sin\left(\frac{\chi}{f}\right)-\left(\frac{\chi}{f}\right)\right].\end{aligned} \qquad (17)$$

It also changes during the time evolution of ϕ. First of all, the term $\frac{1}{f\mu^2}\langle\bar{J}_s\rangle\left(\frac{\chi}{f}\right)$ shifts the minimum of χ^2-potential from $\chi = 0$ to $\chi = -\frac{\langle\bar{J}_s\rangle}{a_0}$,

$$\frac{a}{2}\left(\frac{\chi}{f}\right)^2 + \frac{1}{f\mu^2}\langle\bar{J}_s\rangle\left(\frac{\chi}{f}\right) = \frac{a}{2}\left(\frac{\chi}{f} + \frac{1}{a_0 f}\langle\bar{J}_s\rangle\right)^2 + \ldots \qquad (18)$$

the coordinate of the minimum oscillating in tune with the background field.

In addition, in the tachyonic regime the effective potential exhibits the spontaneous symmetry breaking.

Let us compare the form of (17) for two different values of the background field, $\phi = 2\pi$ and $\phi = \pi$. For both values, $\langle \bar{J}_s \rangle = 0$ in the Hartree-type approximation. For $\phi = 2\pi$, the effective potential takes the form

$$\bar{V}(\chi) = \frac{a}{2}\left(\frac{\chi}{f}\right)^2 - \cos\left(\frac{\chi}{f}\right) + 1. \qquad (19)$$

It is positive for all values of χ and its minimum is at $\chi = 0$.

For $\phi = \pi$, the effective potential becomes

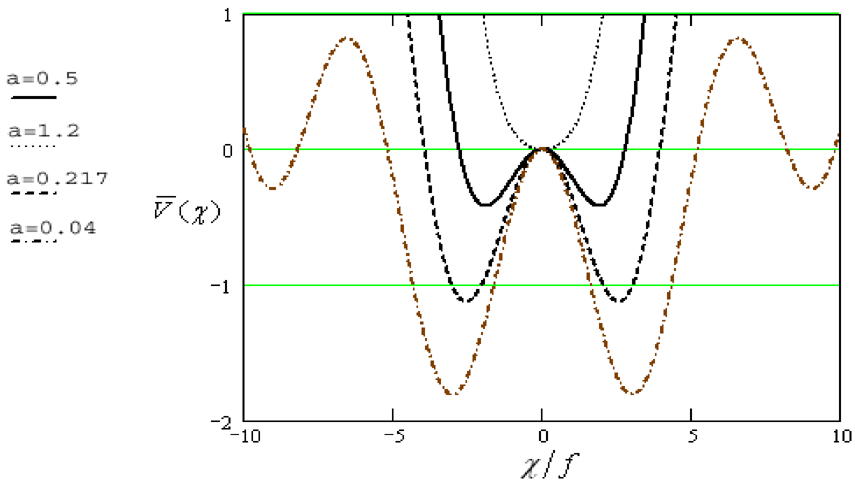
$$\bar{V}(\chi) = \frac{a}{2}\left(\frac{\chi}{f}\right)^2 + \cos\left(\frac{\chi}{f}\right) - 1. \qquad (20)$$

It is minimized for

$$a\left(\frac{\chi}{f}\right) = \sin\left(\frac{\chi}{f}\right). \qquad (21)$$

For $a \geq 1$, Eq. (21) has only trivial solution $\chi = 0$. However, for $a < 1$ nontrivial solutions appear.

The shape of the potential $\bar{V}(\chi)$ for different values of a is plotted below. The nontrivial local minima appear for $a < 1$. For $a > 1$, the spontaneous symmetry breaking does not occur. The special value $a_{sp} = 0.217$ was found in [1]. For $a < a_{sp}$, the number of nontrivial local minima is increasing with decreasing values of a. The nontrivial minima are of different energy; the ones of higher energy are metastable and can decay by a tunneling.



4. The tachyonic regime can be characterized as a regime of spontaneous symmetry breaking. Although $a_{sp} = 0.217$ is specified in [1] as a special value defining the first local minima, we have shown that nontrivial minima appear even for $a_{sp} < a < 1$.

Whether the system evolves in the tachyonic or non-tachyonic regime is fixed by the value of the background field. During its time evolution, energy is transferred from ϕ to χ. As a result, ϕ is damped, while the number of particles in quantum fluctuations increases. If, for example, $\phi(t = 0) = 2\pi$, then the quantum fluctuations first evolve in the standard, non-tachyonic regime. As soon as $\phi(t)$ reaches π, the tachyonic regime starts (for $a < 1$), and the intensive production of tachyonic modes results in a rapid damping of ϕ.

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