ABJ ANOMALIES AND EXTENDED SUPERSYMMETRIC SU(2)xU(1)xU´(1)-MODEL

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A possibility of the reduction of ABJ anomalies in the extended sypersymmetric SU(2)xU(1)xU´(1) – model has been studied. The hypercharges of the left and right isomuitiplets of the fermions have been calculated using conditions of cancellation of ABJ anomalies in the gauge theories.

It is well known that even after symmetric regularization a theory with fermions included does not imply the Word identities to be always valid since some one-loop diagrams in that case lead to the anomaly terms which interfere with recurrent restoration of these identities in high orders of perturbation theory. The above terms called ABJ anomalies were found by Adler [1, 2] and Bell and by Jackiws [3] in the current algebra studies.

For a group G with the fermionic sector consisting of the lefts Ψ_{Li} and rights Ψ_{Rj} isomultiplets the conditions of cancellation of ABJ anomalies has the form [4–7]:

$$\sum_i \mathrm{Tr}[(T_{ai}T_{bi}+T_{bi}T_{ai})T_{ci}]-\sum_j \mathrm{Tr}[(T_{aj}T_{bj}+T_{bj}T_{aj})T_{cj}]=0\,, \qquad (1)$$

where matrices T are the generators of the group, G.

Let us now consider the extended supersymmetric SU(2)xU(1)xU´(1) – model with the left fermionic doublets and the right fermionic singlets in the form $\Psi_L=\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L$, $\Psi_Q=\begin{pmatrix}u\\ d\end{pmatrix}_L$, $\Psi_u=u_L^c$, $\Psi_R=e_R^c$, $\Psi_D=d_L^c$ respectively [8,9]. The muitiplets hypercharge, Y, based on the known Gell-Mann – Nishijima formulae is determined as $Y=2\overline{Q}=Y_1+Y_2$. The Y_1 part of hypercharge is caused by the interaction with the first Maxwell flied and its superpartner while the other one, Y_2, by the interaction with the second Maxwell field and its superpartner. Hypercharges for each isomuitiplets of fermions are presented in table 1.

The matrices T in the expression (1) may be the generators of the group SU (2), and/or the group U (1), and U´(1).

Let us consider the following selections of T matrices:

1. The matrices T_{bi} and T_{ci}, are the generators of the group, SU (2) and T_{ai} equals Y_1 or Y_2. In such a case, from (1) we have

$$\sum_i Y_{1i}\mathrm{Tr}(T_{3i}^2)-\sum_j Y_{1j}\mathrm{Tr}(T_{3j}^2)=0\,, \qquad (2.1)$$

$$\sum_i Y_{2i}\mathrm{Tr}(T_{3i}^2)-\sum_j Y_{2j}\mathrm{Tr}(T_{3j}^2)=0\,. \qquad (2.2)$$

2. The matrices T_{bi} are the generator of the group SU (2), and

$$T_{ai}=Y_{1i}, T_{ci}=Y_{1i}\,;$$
$$T_{ai}=Y_{2i}, T_{ci}=Y_{2i}\,;$$
$$T_{ai}=Y_{1i}, T_{ci}=Y_{2i}\,;$$
$$T_{ai}=Y_{2i}, T_{ci}=Y_{1i}\,.$$

In the above case, from (1) we have

$$\sum_i Y_{1i}^2 Tr(T_{3i}) - \sum_j Y_{1j}^2 Tr(T_{3j}) = 0, \qquad (2.3)$$

$$\sum_i Y_{2i}^2 Tr(T_{3i}) - \sum_j Y_{2j}^2 Tr(T_{3j}) = 0, \qquad (2.4)$$

$$\sum_i {}_i Y_{1i} Y_{2i} Tr(T_{3i}) - \sum_j Y_{1j} Y_{2j} Tr(T_{3j}) = 0. \qquad (2.5)$$

3. The matrices T are the generators of U (1) or U′(1) groups, i.e.

$$T_{ai} = Y_{1i}, T_{bi} = Y_{1i}, T_{ci} = Y_{1i};$$
$$T_{ai} = Y_{1i}, T_{bi} = Y_{1i}, T_{ci} = Y_{2i};$$
$$T_{ai} = Y_{1i}, T_{bi} = Y_{2i}, T_{ci} = Y_{1i};$$
$$T_{ai} = Y_{2i}, T_{bi} = Y_{1i}, T_{ci} = Y_{1i};$$
$$T_{ai} = Y_{2i}, T_{bi} = Y_{1i}, T_{ci} = Y_{2i};$$
$$T_{ai} = Y_{2i}, T_{bi} = Y_{2i}, T_{ci} = Y_{1i};$$
$$T_{ai} = Y_{2i}, T_{bi} = Y_{2i}, T_{ci} = Y_{2i}.$$

Consequently, from (1) we have

$$\sum_i Tr(Y_{1i}^2 Y_{2i}) - \sum_j Tr(Y_{1j}^2 Y_{2j}) = 0, \qquad (2.6)$$

$$\sum_i Tr(Y_{1i} Y_{2i}^2) - \sum_j Tr(Y_{1j} Y_{2j}^2) = 0, \qquad (2.7)$$

$$\sum_i Tr(Y_{1i}^3) - \sum_j Tr(Y_{1j}^3) = 0, \qquad (2.8)$$

$$\sum_i Tr(Y_{2i}^3) - \sum_j Tr(Y_{2j}^3) = 0. \qquad (2.9)$$

The conditions, (2.1) - (2.9), can be written in the form

$$\sum_i Tr(T_{3i}^2 Q_i) - \sum_j Tr(T_{3j}^2 Q_j) = 0, \qquad (3.1)$$

$$\sum_i Y_{1i} Tr(T_{3i}^2) - \sum_j Y_{1j} Tr(T_{3j}^2) = 0, \qquad (3.2)$$

$$\sum_i Tr(Y_{1i}^3) - \sum_j Tr(Y_{1j}^3) = 0, \qquad (3.3)$$

$$\sum_i Y_{1i}^2 \, TrQ_i - \sum_j Y_{1j}^2 \, TrQ_j = 0, \qquad (3.4)$$

$$\sum_i Y_{1i} Tr(Q_i^2) - \sum_j Y_{1j} Tr(Q_j^2) = 0. \qquad (3.5)$$

where sum over i define sum by the left components of isomultiplets, and sum over j define sum by the right components of isomultiplets.

From (3.1) – (3.5) we can get the following equations:

$$y_L + 3y_{QL} = 0,$$
$$2y_L^3 + 6y_{QL} - y_R^3 - 3y_{uR}^3 - 3y_{dR}^3 = 0,$$
$$y^2 - y_{QL}^2 + y_R^3 - y_R^2 - 2y_{uR}^2 + y_{dR}^2 = 0,$$
$$3y_L + 5y_{QL} - 3y_R - 4y_{uR} - y_{dR} = 0.$$

By solving the above equations, we have

$$y_R = 2{,}64 y_{QL}, y_{uR} = -2{,}74 y_{QL}, y_{dR} = -0{,}76 y_{QL} \qquad \text{in the region } y_R \geq 1{,}45 y_{QL},$$
$$y_R = -13{,}31 y_{QL}, y_{uR} = 14{,}91 y_{QL}, y_{dR} = -11{,}69 y_{QL} \qquad \text{in the region } y_R \leq -13{,}48 y_{QL}.$$

Note that the obtained hypercharge values for isomultiplets allow for the conclusion that the proposed supersymmetric SU (2)xU(1)xU′(1) - model [8] is free from ABJ – anomalies and leads to the significant reduction of the number of the model parameters. The last result also makes the proposed model favorable from the point of view of the comparison with experimental database.

References

[1] S. L. Adler. Phys.Rev., 177, p.2426, 1960.
[2] S. L. Adler. "Lectures on elementary particles and quantum field theory". Proc. 1970 Brandies Summer Institute (ed. S. Deser et al.) MIT Press, Cambridge, Massachusetts.
[3] I. S. Bell, R. Jackiv. Nuovo Cimento, 60A, p.315, 1972.
[4] C. Bouchiat, J. Iliopoulos, Ph. Meyer. Phys. Lett., 38B, p.519, 1972.
[5] H. Georgi, S. L. Glashow. Phys. Rev., D9, p.416, 1974.
[6] D. Gross, R. Jackiv. Phys. Rev., D6, p.477, 1972.
[7] Т.-П. Ченг, Л.-Ф. Ли. «Калибровочные теории в физике элементарных частиц», Москва, «Мир», 1987.
[8] Ф. Т. Халил-заде. Препринт № 315, ИФАН Азерб.ССР, Баку, 1989.
[9] Ф. Т. Халил-заде. «Fizika», том V, № 1, стр.20, 1999.

Table 1

FERMIONS	Y	Y_1	Y_2
Ψ_L	-1	y_L	$-1-y_L$
Ψ_L	2	y_R	$2-y_R$
Ψ_Q	$y_Q = \frac{1}{3}$	y_{QL}	$y_Q - y_{QL}$
Ψ_u	$y_u = -\frac{4}{3}$	y_{uR}	$y_u - y_{uR}$
Ψ_D	$y_d = \frac{2}{3}$	y_{dR}	$y_d - y_{dR}$