Multi-quark equations in Nambu—Jona-Lasinio model

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Introduction

A number of perspective physical applications of Nambu – Jona-Lasinio (NJL) model is connected with multi-quark functions (for example: meson decays, pion-pion scattering, baryons, pentaquarks etc.). These multi-quark functions arise in higher orders of the mean-field expansion (MFE) for NJL model. In present work we review some results of investigation of higher orders of MFE for NJL model and list some possible physical application of the multi-quark equations. To formulate MFE we have used an iteration scheme of solution of Schwinger-Dyson equation with fermion bilocal source, which has been developed in works [1-3]

The leading approximation and the first order of MFE maintain equations for the quark propagator and the two-particle function and also the first-order correction to the quark propagator. A consideration of these equations is the usual field of investigations of NJL model [4,5]. In the second order of MFE the equations for four-particle and three-particle functions arise, and in the third order the equations for six-particle and five-particle functions arise. (Note, that the construction of the five-particle functions gives us a possibility to investigate the pentaquark states in NJL model.)

Mean-field expansion for Namdu—Jona-Lasinio model

The Lagrangian has the following form:

$$L = \bar{\psi} i \hat{\partial} \psi + \frac{g}{2}\left[(\bar{\psi}\psi)^2 + \left(\bar{\psi} i \gamma_5 \tau^a \psi\right)^2\right].$$

The Lagrangian is invariant under transformations of chiral group $SU(2)_V \times SU(2)_A$, which correspond to u-d quark sector.

The unique connected Green function of the leading approximation is the quark propagator $S^{(0)}$. Other connected Green functions appear in the following iteration steps. The quark propagator in the chiral limit is

$$S^{(0)}(p) = (m - \not{p})^{-1}$$

where m is the dynamical quark mass, which is a solution of gap equation

$$1 = -\frac{8ign_c}{(2\pi)^4} \int \frac{dp}{m^2 - p^2}.$$

The iteration-scheme equations give us the equation for first-order two-particle function:

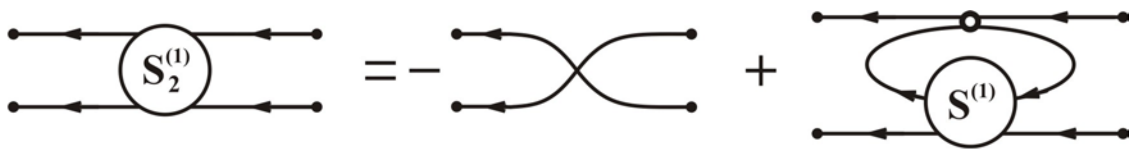

Fig.1 Two-quark functions equation

and the first-order correction to quark propagator:

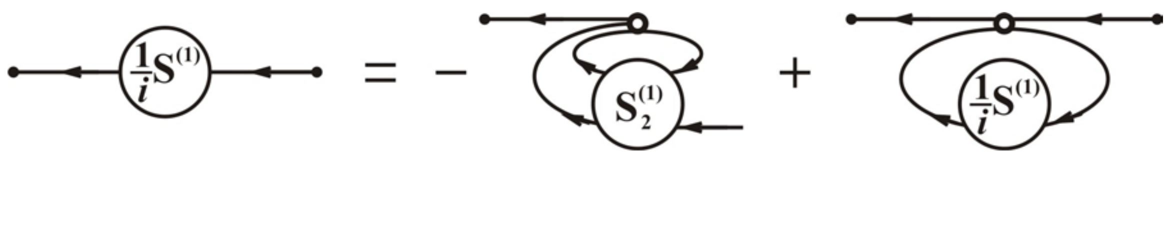

Fig.2 The equation of quark propagator

Here the graphical notation are used:

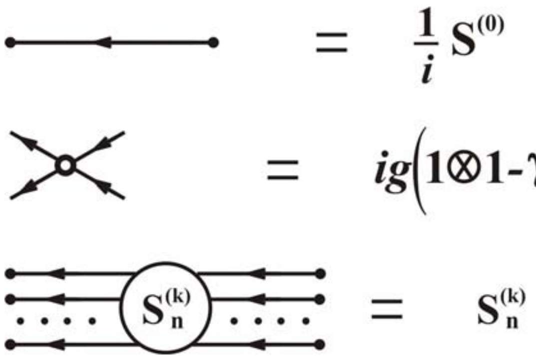

Fig. 3 The graphical notation

To describe the solution of the first-order equation for two-particle function and for future purposes we introduce the composite meson propagators by following way:

a) Let us define scalar-scalar function

$$S_\sigma(x-x') \equiv \mathrm{tr}\left[S_2^{(1)}\begin{pmatrix} x & x \\ x' & x' \end{pmatrix}\right] \sim \langle \overline{\psi}\psi(x)\overline{\psi}\psi(x')\rangle$$

From the equation for two-particle function we obtain (in momentum space)

$$S_\sigma(p) = \frac{1}{ig}\left(1 - i\Delta_\sigma(p)\right).$$

Here we define the following function, which we call $\sigma-$ meson propagator:

$$\Delta_\sigma(p) = \frac{Z_\sigma(p)}{4m^2 - p^2}$$

where $Z_\sigma(p) = \dfrac{I(4m^2)}{I_0(p^2)}$ and $I_0(p) = \int d\tilde{q}\,\dfrac{1}{\left[m^2-(p-q)^2\right]\left(m^2-q^2\right)}$.

b) Pseudoscalar-pseudoscalar function is defined as

$$S_\pi^{ab}(x-x') \equiv \mathrm{tr}\left[S_2^{(1)}\begin{pmatrix} x & x \\ x' & x' \end{pmatrix} \gamma_5 \frac{\tau^a}{2} \gamma_5 \frac{\tau^b}{2} \right] \sim \left\langle \overline{\psi} \gamma_5 \frac{\tau^a}{2} \psi(x) \overline{\psi} \gamma_5 \frac{\tau^b}{2} \psi(x') \right\rangle .$$

From the equation for two-particle function we obtain (in momentum space)

$$S_\pi^{ab}(p) = -\frac{1}{ig}\left(\delta^{ab} - i\Delta_\pi^{ab}(p)\right) .$$

Here we define the pion propagator

$$\Delta_\pi^{ab}(p) = -\frac{\delta^{ab} Z(p)}{p^2},$$

where

$$Z_\sigma(p) = \frac{I(0)}{I_0(p^2)}.$$

Second-order equations

Second-order generating functional is

$$G^{(2)} = \left\{ \frac{1}{4!}\mathrm{Tr}\left(S_4^{(2)} * \eta^4\right) + \frac{1}{3!}\mathrm{Tr}\left(S_3^{(2)} * \eta^3\right) + \frac{1}{2}\mathrm{Tr}\left(S_2^{(2)} * \eta^2\right) + \mathrm{Tr}\left(S^{(2)} * \eta\right) \right\} G^{(0)}.$$

The equations for four-quark and three-quark functions are

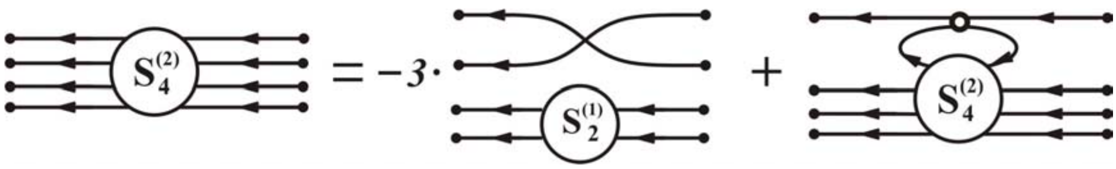

Fig.4 Four quark functions equation

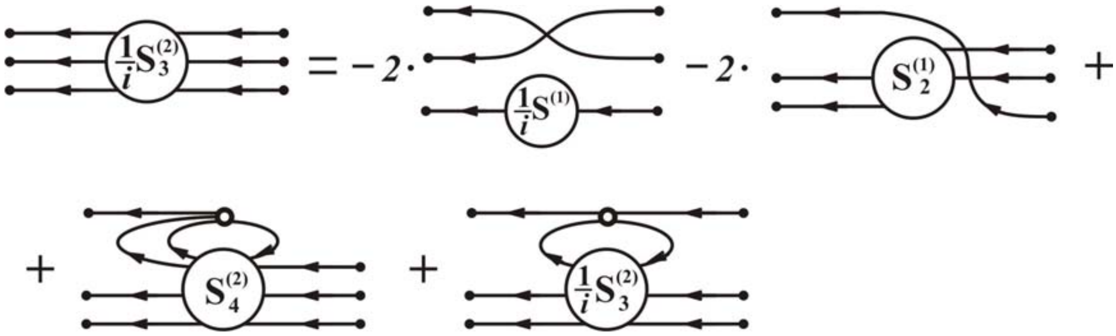

Fig.5 Three quark functions equation

The equation for the four-quark function has a simple exact solution which is the product of firs-order two-quark functions:

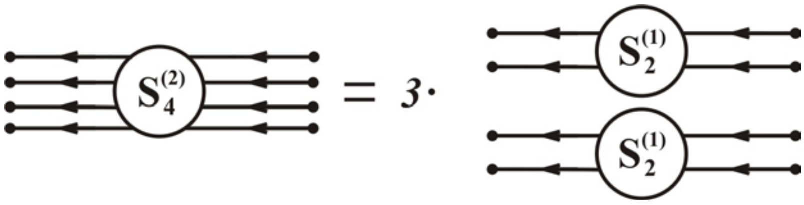

Fig.6 The solution of the four quark functions equation

Vertex $\sigma\pi\pi$

The existence of above exact solution for the four-quark function gives us a possibility to obtain a closed equation for the three-quark function. As a first step in an investigation of this rather complicated equation we shall solve a problem of definition of $\sigma\pi\pi$ -- vertex with composite sigma-meson and pions.

Let us introduce a function:

$$W^{ab}_{\sigma\pi\pi}(xx'x'') \equiv \mathrm{tr}\left[S_3^{(2)} \begin{pmatrix} x & x \\ x' & x' \\ x'' & x'' \end{pmatrix} \gamma_5 \frac{\tau^a}{2} \gamma_5 \frac{\tau^b}{2} \right] \sim \left\langle \bar{\psi}\psi(x) \bar{\psi}\gamma_5 \frac{\tau^a}{2} \psi(x') \bar{\psi}\gamma_5 \frac{\tau^b}{2} \psi(x'') \right\rangle$$

and define:

a) scalar vertex

$$V_{\sigma}(xx'x'')\equiv \mathrm{tr}\left(S^{(0)}(x-x')S_2^{(1)}\begin{pmatrix} x' & x \\ x'' & x'' \end{pmatrix}\right) = 2in_c\int dx_1 v_S(xx'x_1)\Delta_{\sigma}(x_1-x''),$$

(here $v_s(xx'x'') = \mathrm{tr}(S_0(x-x')S_0(x'-x'')S_0(x''-x))$, is the triangle diagram).

b) pseudoscalar vertex

$$V_{\pi}^{ab}(xx'x'')\equiv \mathrm{tr}\left(S^{(0)}(x-x')\gamma_5\frac{\tau^a}{2}S_2^{(1)}\begin{pmatrix} x' & x \\ x'' & x'' \end{pmatrix}\gamma_5\frac{\tau^b}{2}\right) = 2in_c\int dx_1 v_P(xx'x_1)\Delta_{\pi}^{ab}(x_1-x''),$$

(here $v_P(xx'x'') = \mathrm{tr}\left(S^{(0)}(x-x')\gamma_5\ S^{(0)}(x-x'')\gamma_5 S^{(0)}(x''-x)\right)$).

With these definitions we obtain from the second-order equation for $W_{\sigma\pi\pi}^{ab}$, which can be easily solved in the momentum space.

The connected part of $W_{\sigma\pi\pi}^{ab}$ is an amplitude of decay $\sigma \to 2\pi$.

It has a following form:

$$\left[W_{\sigma\pi\pi}^{ab}(pp'p'')\right]^{con} = \frac{2n_c}{i}\Delta_{\sigma}(p)\left[v_P(pp'p'')+v_P(pp''p')\right]\Delta_{\pi}^{aa_1}(p')\Delta_{\pi}^{a_1b}(p''),$$

or, graphically:

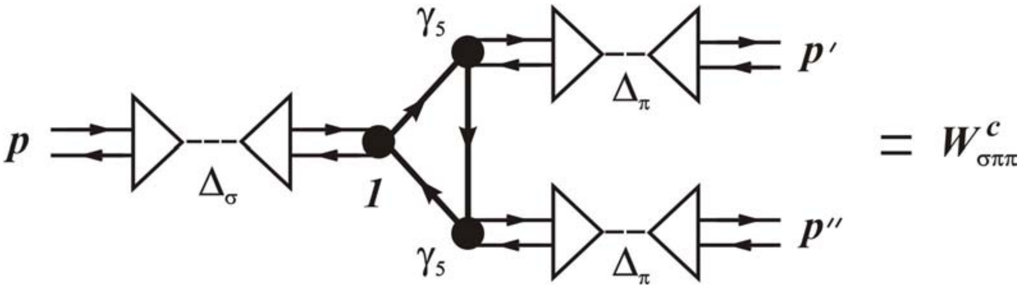

Fig.7 The connected part of $\sigma\pi\pi$.

We preset here first results of calculation of triangle diagram

$$v_P(pp'p'')=\int d\tilde{q}\,tr\left[S^{(0)}(p+q)\gamma_5 S^{(0)}(p''+q)\gamma_5 S^{(0)}(q)\right].$$

Here p – 4-momentum of meson, and p', p'' – 4-momenta of pions:

$$v_P = 4m\left[I_0(p)+(pp'')I_3(p,p'')\right],$$

where the integral I_0 above written integral, and

$$I_3(p,p'')=\int\frac{d\tilde{q}}{\left(m^2-(p+q)^2\right)\left(m^2-q^2\right)\left(m^2-(p''+q)^2\right)}$$

is convergent integral, which on mass shell

($p^2 = 4m^2$, $p'^2 = p''^2 = 0$, $(pp') = (p'p'') = 2m^2$) it has following value

$$I_3^{ms}=\frac{1}{256m^2}.$$

Third-order equations

The third-order generating functional is

$$G^{(3)}[\eta]=\left\{\frac{1}{6!}\mathrm{Tr}\left(S_6^{(3)}*\eta^6\right)+\frac{1}{5!}\mathrm{Tr}\left(S_5^{(3)}*\eta^5\right)+\frac{1}{4!}\mathrm{Tr}\left(S_4^{(3)}*\eta^4\right)+\right.$$

$$\left.+\frac{1}{3!}\mathrm{Tr}\left(S_3^{(3)}*\eta^3\right)+\frac{1}{2}\mathrm{Tr}\left(S_2^{(3)}*\eta^2\right)+\mathrm{Tr}\left(S^{(3)}*\eta\right)\right\}G^{(0)}.$$

The equations for six-quark functions and five-quark function are

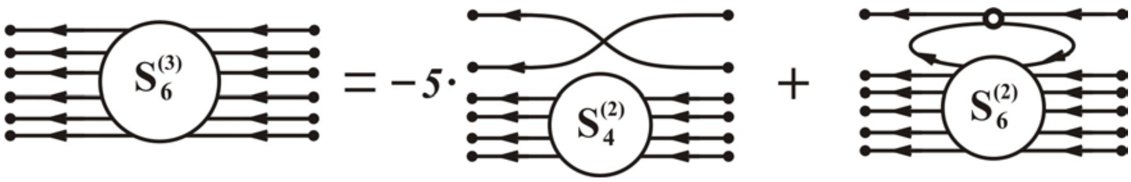

Fig. 8 Six-quark functions equation

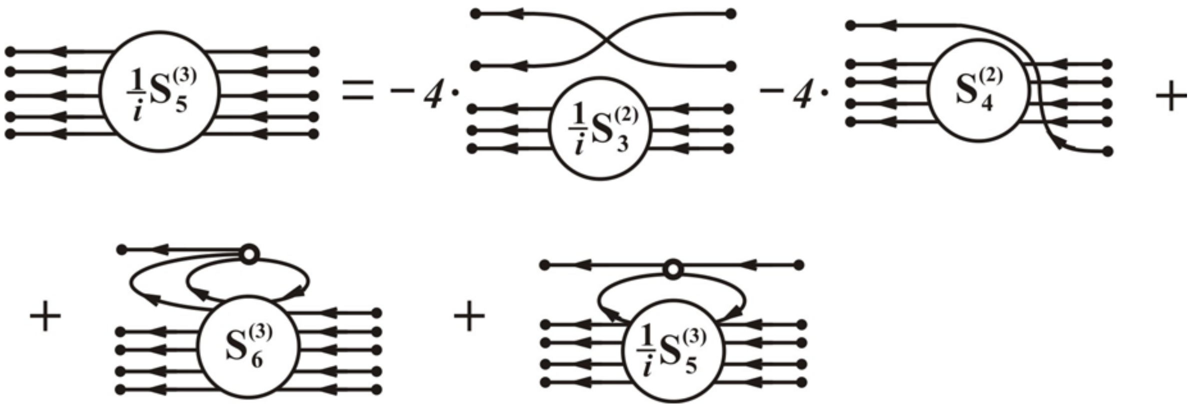

Fig. 9 Five-quark functions equation

The equations for the six-quark function and for the five-quark function in our iteration scheme are new, and the equations for four-quark function $S_4^{(3)}$, three-quark function $S_3^{(3)}$, two-quark function $S_2^{(3)}$ and quark propagator $S^{(3)}$ have the same form as the second-order equations exept of the inhomogeneous term, which contains the six-quark function and the five-quark function.

Conclusion

In conclusion we list some possible physical application of the multi-quark equations.

1. Vector meson decays (in generalized NJL model).
2. Nucleons as the bound states in three-quark function:

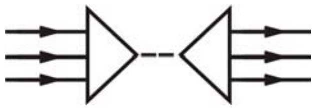

Fig. 10 Nucleons as the bound states

3. Pentaquarks as the bound states in five-quark function:

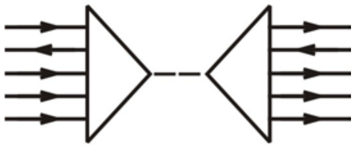

Fig. 11 Pentaquarks as the bound states

4. $\pi\pi -$scattering in four-quark function of third order.

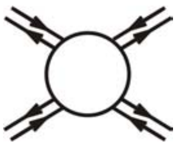

Fig. 12 $\pi\pi -$scattering

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