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BULK SPIN-WAVE REGIONS IN AN ANTIFERROMAGNETIC SUPERLATTICE

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We present theoretical studies of superlattice structures formed from alternating layers of two simple-cubic Heisenberg antiferromagnetic materials. The spin-wave regions for spin waves propagating in a general direction in the superlattice are derived by the Green function method. The results are illustrated numerically.

An exciting aspect of solid-state physics is the discovery or exploration of new classes of materials, whose physical properties may differ dramatically from textbook descriptions of simple solids. During the past decade, there has been considerable effort devoted to the synthesis and study of composite materials, and of superlattices formed from alternating layers of different materials [1-3]. In magnetic superlattices, elementary excitations such as spin waves are collective excitations of the structure as a whole, and as a consequence have properties distinctly different from the modes associated with any one constituent. Some qualitative features of superlattice are most easily explained for the simple – cubic structure in terms of modified single - film properties. The bulk spin-wave regions in simple-cubic Heizenberg antiferromagnetic material are derived in Ref.[4] The aim of this paper is to study by the Green function method [5] properties of an antiferrormagnetic superlattice with quantum Heisenberg spins at finite temperature and this theoretical studies are analogous to one from the Ref.[6],where ferromagnetic superlattice is considered. The Green function method gives opportunity to study the behavior of parameters characterising the superlattice. For example, magnetic sucseptibility, temperature dependence of magnetization, local density of magnon states and oth. may be studied by the Green function method.

We consider a simple cubic antiferromagnetic superlattice model in which the atomic planes of material 1 alternate with atomic planes of material 2 [2]. Each atomic plane is assumed to be the [001] planes. The materials are taken to be simple – cubic Heisenberg antiferromagnet, having exchange constant i_1 and i_2 and lattice constant **a.** Assuming exchange interactions may be both ferromagnetically and antiferromagnetically , it is possible two spin arrangements between constituents.

The Heisenberg Hamiltonian is used to describe for system:

$$H=\sum_{i,j} I_{ij}(\vec{S}_i\vec{S}_j)-\sum_i g\mu_B(H_0+H_i^{(A)})S_{i,a}^z-\sum_i g\mu_B(H_0-H_i^{(A)})S_{i,b}^z \qquad (1)$$

where H_0 is the internal field, which is assumed to be paralell to the spins along the **z** axis and $H_i^{(A)}$ **(i =1,2)** anisotropy field for **a** antiferromagnet with simple unaxial anisotropy along the z axis. We define a double – time Green function in real space $G_{ij}(t, t')=\langle\langle S_i^+(t); S_j^-(t')\rangle\rangle$. By magnetic symmetry, there are two sublattices corresponding to up and down spins for both the materials. Writing the equation of motion for Green function and employing the random-phase approximation one obtains a set of equations coupling four different types of Green function, namely two of type $G_{ij}(t, t')$ where **i** and j belong to the same sublattice, and two of the type F_{ij} (t, t') where i and j belong to different sublattices. Furthermore, to emphasize the layered structure we shall use the following the frequency and two-dimensional. Fourier transformation [2]

$$G_{i,j}(t,t') = \frac{1}{\pi^2}\int dk_{11} \exp[ik_{11}(r_i - r_j)]\frac{1}{2\pi}\int d\varpi G_{nn'}(\varpi,k_{11})\exp[-i\varpi(t-t')] \qquad (2)$$

where k_{11} is two-dimensional wave vector, ϖ is spin-wave frequency, n and n' indices of the layers to which r_i and r_j and belong, respectively. Assuming that n-th layer is of the material 1 and (n+1)-th layer is of the material 2, one obtains the following set of equations when the exchange interaction between constituents is antiferromagnetically

$$\begin{cases} (E-d_1-\frac{2}{3}-\frac{\varepsilon}{3})g_{nn'} - B_1 f_{nn'} - \frac{\varepsilon}{6}(f_{n+1,n'}+f_{n-1,n'}) = 2S_n\delta_{nn'}, \\ (E+d_1+\frac{2}{3}+\frac{\varepsilon}{3})f_{nn'} + B_1 g_{nn'} + \frac{\varepsilon}{6}(g_{n+1,n'}+g_{n-1,n'}) = 0, \\ (E-\alpha d_2-\frac{2\alpha}{3}-\frac{\varepsilon}{3})g_{n+1,n'} - B_2 f_{n+1,n'} - \frac{\varepsilon}{6}(f_{n+2,n'}+f_{n,n'}) = 2S_{n+1}\delta_{n+1,n'}, \\ (E+\alpha d_2+\frac{2\alpha}{3}+\frac{\varepsilon}{3})f_{n+1,n'} + B_2 g_{n+1,n'} + \frac{\varepsilon}{6}(g_{n+2,n'}+g_{n,n'}) = 0. \end{cases} \qquad (3)$$

and when the exchange interaction between constituents is ferromagnetically

$$\begin{cases} (E-d_1-\frac{2}{3}+\frac{\varepsilon}{3})g_{nn'} - B_1 f_{nn'} - \frac{\varepsilon}{6}(g_{n+1,n'}+g_{n-1,n'}) = 2S_n\delta_{nn'}, \\ (E+d_1+\frac{2}{3}-\frac{\varepsilon}{3})f_{nn'} + B_1 g_{nn'} + \frac{\varepsilon}{6}(f_{n+1,n'}+f_{n-1,n'}) = 0, \\ (E-\alpha d_2-\frac{2\alpha}{3}+\frac{\varepsilon}{3})g_{n+1,n'} - B_2 f_{n+1,n'} - \frac{\varepsilon}{6}(g_{n+2,n'}+g_{n,n'}) = 2S_{n+1}\delta_{n,n'}, \\ (E+\alpha d_2+\frac{2\alpha}{3}-\frac{\varepsilon}{3})f_{n+1,n'} + B_2 g_{n+1,n'} + \frac{\varepsilon}{6}(f_{n+2,n'}+f_{n,n'}) = 0. \end{cases} \qquad (4)$$

where $E = (\varpi - g\,\mu_B H_0)/6I_1S$, $g=G(\varpi, k_{11}) \cdot 6I_1S$ and the expression of other terms appearing in the set of equations (3) and (4) are given in the Appendix.

The system is also periodic in the z direction, which lattice constant is d=2a. According to Bloch"s theorem we introduces the following plane waves [2]

$$g_{n+2,n'} = g_{n,n'} \exp(ik_z d),$$
$$f_{n+2,n'} = f_{n,n'} \exp(ik_z d) \qquad (5)$$

The set of equations (3) and (4) may be rewritten under following matrix form

$$Mu = s \quad \text{and} \quad Nu = s \qquad (6)$$

where

$$M = \begin{vmatrix} E-A_1 & -B_1 & 0 & -T^* \\ B_1 & E+A_1 & T^* & 0 \\ 0 & -T & E-A_2 & -B_2 \\ T & 0 & B_2 & E+A_2 \end{vmatrix} \qquad u = \begin{vmatrix} g_{n,n'} \\ f_{n,n'} \\ g_{n+1,n'} \\ f_{n+1,n'} \end{vmatrix}$$

$$N=\begin{vmatrix} E-A_1 & -B_1 & -T^* & 0 \\ B_1 & E+A_1 & 0 & T^* \\ -T & 0 & E-A_2 & -B_2 \\ 0 & T & B_2 & E+A_2 \end{vmatrix} \qquad s=\begin{vmatrix} 2S_n\delta_{nn'} \\ 0 \\ 2S_{n+1}\delta_{n+1,n'} \\ 0 \end{vmatrix} \qquad (6a)$$

where $T=\frac{\varepsilon}{6}$ (1 + exp (ik_zd)) and T^* is the complex conjugate of T. The Green functions for the four-sublattice model are obtained by solving the equations (6). We can also give combined expressions of the Green functions

$$g_{nn}(E)=\sum_{\ell}\frac{C_1(E_{k\ell})}{E-E_{k\ell}}, \qquad f_{nn}(E)=\sum_{\ell}\frac{D_1(E_{k\ell})}{E-E_{k\ell}},$$

$$g_{n+1,n+1}(E)=\sum_{\ell}\frac{C_2(E_{k\ell})}{E-E_{k\ell}}, \qquad f_{n+1,n+1}(E)=\sum_{\ell}\frac{D_2(E_{k\ell})}{E-E_{k\ell}}, \qquad (7)$$

The poles of the Green functions occur at energies

$$E_k=\pm\sqrt{0.5[-p\pm\sqrt{p^2-4Q}]}. \qquad (8)$$

The expressions of the terms appearing (6-8) are given in the Appendix. Equations [7-8] are the main results of this paper. It can be verified from equation (8) that when both media are identical, $I_1=I_2=I$ the equations (8) reduces to the well-known expression of bulk-spin wave dispersion equation for antiferromagnetic constituents [7]. In Fig.1,2,3 the results numerically illustrated for a particular choice of parameters. Fig.2 shows the spin-wave regions for the superlattice as a function of the quantity q, while Fig.1 shows those for the components 1 and 2 [4]. In Fig.3 the spin-wave regions are shown as a function of ε for q=0 and q=0.5. All these figures correspond to $-1\le\cos k_zd\le 1$.

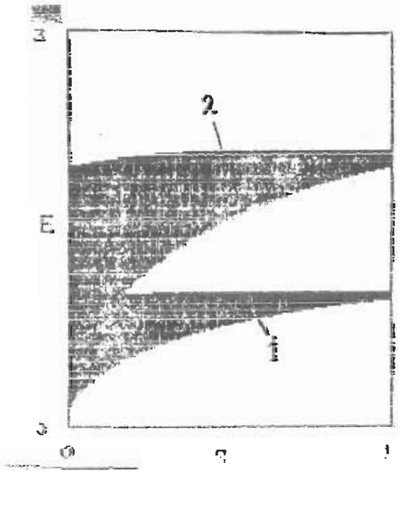


Fig.1

The bulk spin-ware regions for the components 1 and 2 as a function of transverse components of wavevectors (d_1=0,001, d_2=0,002, α=2).

The analysis of the results shows that the width of the bulk-spin wave regions in the antiferromagnetic superlattice is depended on transverse components of wave vectors and exchange interaction. When the exchange interaction between constituents is antiferromagnetically the bulk-spin wave regions are narrower than the bulk-spin wave regions when that is ferromagnetically.

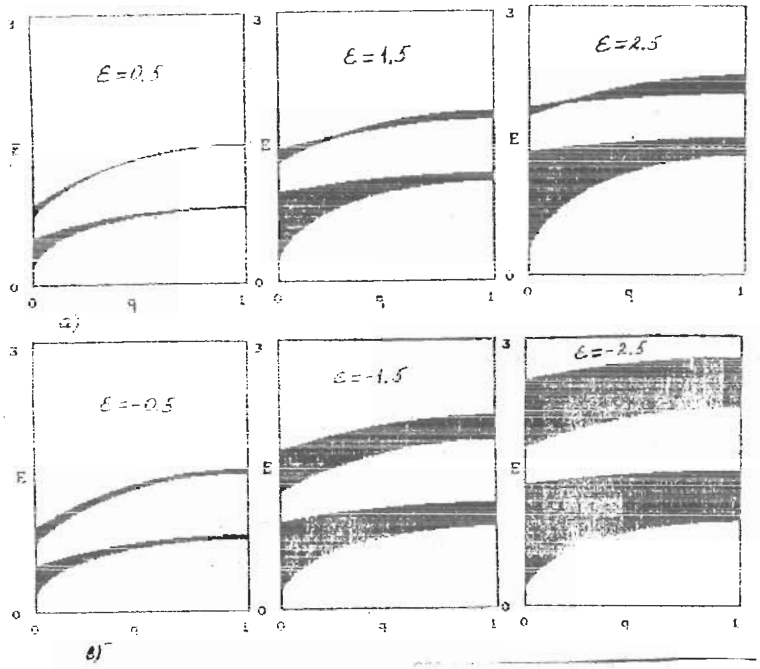
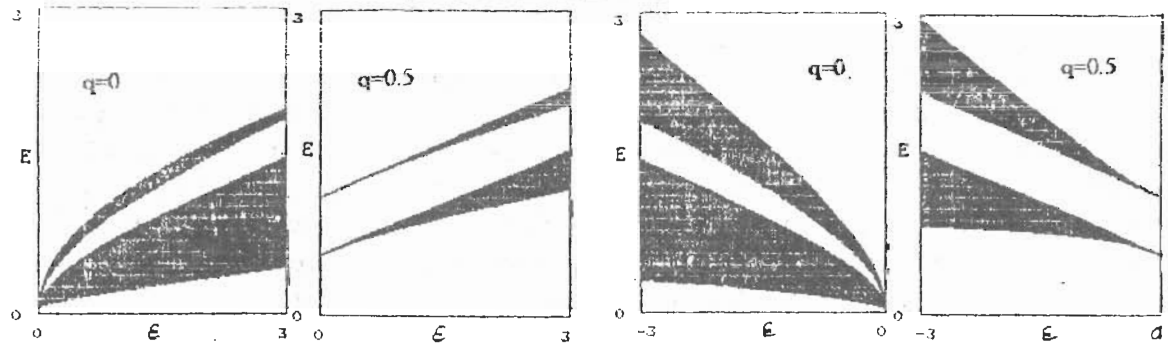


Fig.2
The bulk spin-wave regions in the superlattice as a function of transverse components of the wavevectors for different va-lues of ε (d_1=0,001, d_2=0,002, α=2).
a). spin arrangements between constituents are antiferromagnetically,
b). spin arrangements are ferromagnetically



a) b)

Fig3.
The bulk spin-wave regions in the superlattice as a function of exchange interaction between constituents (d_1=0,001, d_2=0,002, α=2) a). ε>0, when spin arrangements between constituents are antiferromagnetically, b). ε<0, when those are ferromagnetically.

APPENDIX

The terms appearing in the equations [3-8] are

$$A_1 = d_1 + \frac{2}{3} \pm \frac{\varepsilon}{3},$$

$$A_2 = \alpha d_2 + \frac{2\alpha}{3} \pm \frac{\varepsilon}{3},$$

$$B_1 = \frac{2}{3}(1-q),$$

$$B_2 = \frac{2\alpha}{3}(1-q),$$

$$p = -A_1^2 - A_2^2 + B_1^2 + B_2^2 \pm 2TT^*,$$

$$Q = (A_1A_2 + B_1B_2 - TT^*)^2 - (B_2A_1 + B_1A_2)^2.$$

where

$$d_i = \frac{g\mu_B H_i^{(A)}}{6J_iS} \quad (i=1,2);\ q = 1 - \frac{1}{2}(\cos k_x a + \cos k_y a)$$

$$\varepsilon = \frac{J}{J_1} \qquad \alpha = \frac{J_2}{J_1}.$$

Plus sign in the expressions and ε>0 are taken when exchange interaction is antiferromagnetically, minus sign and ε<0 are taken when that is ferromagnetically.

$$C_i(E_{k\ell}) = \frac{2S\{\, (E_{k\ell}+A_i)(E^2_{k\ell}-A_{i'}^2) + B_{i'}^2(E_{k\ell}+A_{i'}) \pm TT^*(E_{k\ell}\pm A_{i'})\,\}}{\prod_{\ell'}(E_{k\ell}-E_{k\ell'})},$$

$$\ell \neq \ell' = \overline{1,4}$$

$$D_i(E_{k\ell}) = \frac{-2S\{\, B_i(E^2_{k\ell}-A^2_{i'}) - B_{i'}TT^* + B_iB_{i'}^2\,\}}{\prod_{\ell'}(E_{k\ell}-E_{k\ell'})},$$

$$\ell \neq \ell' = \overline{1,4} \qquad i \neq i' = 1,2.$$

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ANTİFERROMAQNİT İFRAT QƏFƏSDƏ SPİN DALGA ZONASI

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İki sadə kubik Heyzenberq antiferromaqnitdən təşkil olunmuş ifrat qəfəsə baxılır. Qrin funksiyası metodundan istifadə edərək ifrat qəfəsin oxu boyunca yayılan spin dalğaları üçün həcm spin-dalğa zonaları müəyyən edilmişdir. Nəticə kəmiyyətcə təsvir olunmuşdur.

ОБЛАСТЬ ОБЪЕМНЫХ СПИНОВЫХ ВОЛН В АНТИФЕРРОМАГНИТНЫХ СВЕРХРЕШЕТКАХ

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Рассмотрена сверхрешетка состоящая из чередующихся слоев простых кубических Гейзенберговских антиферромагнетиков. Методом функции Грина определена область спиновых волн распространяющихся вдоль оси свехрешетки. Результаты представлены численно.