SHAHMERDAN SH. AMIROV

ELEMENTARY PHYSICS

BAKU-2012

MINISTRY of EDUCATION of AZERBAIJAN REPUBLICS

AZERBAIJAN MEDICAL UNIVERSITY

SHAHMERDAN SH. AMIROV

ELEMENTARY PHYSICS

(For Listeners of Preparatory Courses)

Approved by the decision of the Scientific Methodical Comission's "Physics" section of the Ministry of Education of Azerbaijan Republics
Dated: December 20, 2011 year
(minute N 5)

Reviewers :
Prof. D.Sc.(Phys.-Math) Sh. M. Nagiyev – Chief of laboratory of “ Nuclear researche**s”** at the Institute of Physics of NASA
Prof. D.Sc.(Phys.-Math) R.J. Gasimova - Chair of General Physics, Baku State University.
Associate prof.,Ph.D.(Phys.-Math) Sh.G.Bayramov- Chair of Biochemistry at Azerbaijan Medical University.
Head lecturer,Ph.D (Pedagojical sciences) V.A.Hajiyev-Chair of Medical Physics and Informatics at Azerbaijan Medical University.

“ Elementary Physics” Complied by **Sh. Sh. Amirov**, Ph.D. (Phys.-Math) Baku, 2012 , p.292

This book covers all the major subjects of elementary school physics .Formulas and Figures in the book are numerated individually for each chapter that makes easy its use by readers.

The manual is intended for listeners of preparatory courses. It will also prove useful to students and graduates of secondary schools, lyceums, colleges as well as teachers .

CONTENTS

Chapter 1. Kinematics

Chapter 2. Dynamics

Chapter 3. Statics

Chapter 4. Work, power and energy

Chapter 5. Mechanical oscillations

Chapter 6. Mechanical waves

Chapter 7. Mechanics of liquids

Chapter 8. Molecular Physics

Chapter 9 . Mutual conversion of liquids and gases

Chapter 10. Properties of liquids

Chapter 11 . Static electricity

Chapter 12. Direct electric current in metals

Chapter 13. Electric current in various media

Chapter 14 . Magnetism

Chapter 15. Electromagnetic induction

Chapter 16 . Electric oscillations and waves

Chapter 17. Reflection and refraction of light

Chapter 18. Mirrors and lenses

Chapter 19. Physical optics

Chapter 20. Relativistic mechanics

Chapter 21 . The Atomic spectra

Chapter 22 . The Atomic Nucleus

PREFACE

The complier of present book is an experienced teacher-physicist.

The text book is written in accordance with the full admission program to high schools on the subject of physics. The book covers all the major subjects of school physics. The full material is presented in 22 chapters. The book offers 580 numerated formulas and more than 155 Figures. Formulas and Figures in the book are numerated individually for each chapter that makes easy its use by readers. Each chapter is complied with presentation of important definitions and formulas.

My sincere thanks to the colleague of chair of Medical Physics and Informatics at Azerbaijan Medical University. Professors and teachers of this department read and commented upon various portions of the manuscript as we wrote them. Their suggestions were uniformly thoughtful and constructive, and this book has benefited greatly from them.

Special thanks goes to young painter ***Amirli Menzer*** *who took many drawings of manual.*

The book is intended for listeners of preparatory courses. It will also prove useful to students and graduates of secondary schools, lyceums, colleges as well as teachers .

Author wishes to express his gratitude to many physics students, teachers and science educators who have made suggestions for changes based on their use of the first edition of " Elementary Physics".

Author

CHAPTER 1

Kinematics

§ 1.1. Mechanical motion. Kinematic concepts

Mechanics is a branch of physics treating the simple forms of motion of bodies in space and time. It is divided into kinematics, dynamics and statics. **Kinematics** is study the pure motion without regard to its cause.

Mechanical motion of a body is the change in its position with time relative to other bodies. Mechanical motion has mainly two types; translational and rotatory. The simplest motion of a body is translational motion during which all points of a body move similarly, describing identical trajectories. The paths in a translational motion can be either straight lines or curves. Note, that in translational motion any straight line drawn in a body, remains parallel to itself (Fig.1.1a) . The motions of the needle in a sewing machine and of the hood in a motor car are translational.

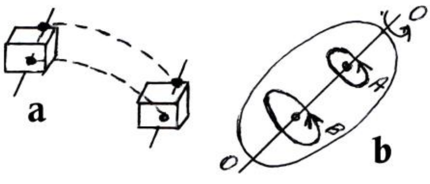


Fig.1.1

In rotary motion, all points of the body move in circles, whose centers lie on the same straight line (Fig.1.1b). This line is called the rotational axis. Note, rotational axis in the given reference frame may be either in move or at rest.

For example, an axis of rotor of generator at electric power station is at rest relative the Earth, while an axis of wheel of automobile is removing.

To solve some problems in mechanics it is necessary to consider the body as a mass point.

Point mass.. A body whose dimensions can be neglected under of the given problem is called a **point mass** .

For instance, Earth can be considered like a mass point during the revolving around the Sun.

The position of a body in space can be determined only with respect to other objects.

To describe the motion of body it is necessary to locate the positions of several points of it. This is done by various systems of coordinates. In rectangular coordinates of position of point in space is indicated by giving its distances x, y and z from three perpendicular lines called the x – axis, y – axis and z – axis respectively (Fig.1.2) A reference body to which coordinates are fixed and mutually synchronized clocks form so - called **reference frame**.

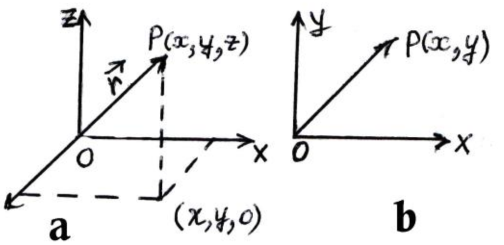


Fig.1.2

Path of motion. A certain line which is described by each point of the body during the motion is called the **path** of motion.. The path and graph of motion must be distinguished. So that, trajectory of body thrown vertically is linear; while graph of motion, i.e. dependence of vertical coordinate on the time is parabolic. The trajectory of motion may depend on the choice of the reference frame.

Depending on the view of trajectory the translational motion of a body may be **rectilinear** or **curvelinear.**

If the trajectory is a straight line the motion of the point is referred to as rectilinear. If the path is a curve, the motion is said to be curvelinear.

Distance. Distance is known as a scalar quantity equal the length of any part of trajectory. Distances passed by a body are added arithmetically.

Displacement. Suppose that a body has moved from point M to point N (Fig.1.3) The length of segment MN is the *distance* traveled. The vector $\vec{MN}$ directed from the initial position to the final one is called the *displacement* of the body.

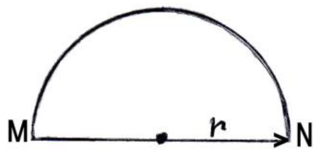


Fig.1.3

The motion may be either uniform or nonuniform due to character of the velocity. First consider the simplest form - uniform rectilinear motion.

§1. 2. Uniform rectilinear motion

A motion in which a body covers equal distances in equal intervals of time is known as **uniform motion**. The velocity in uniform motion is numerically equal the distance traveled per unit time.

$$\vec{\upsilon} = \frac{\vec{s}}{t} \quad . \qquad (1.1)$$

A distance traversed by a body in this motion is numerically equal to the area of polygon between velocity graph and x – axis.

Temporary dependences for (a) velocity, (b) displacement (c) and (d) coordinate and (e) distance traveled are illustrated in (Fig.1.4).

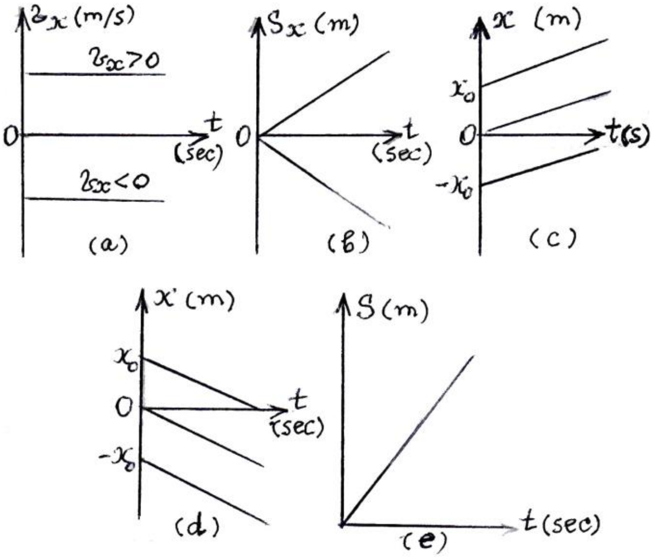


Fig.1.4

In Fig.1.4a upper and downer graphs correspond to the positive and negative directions of x – axis respectively.

A component of displacement along any axis is the difference of final and original coordinates. For example, x-component of displacement is given by

$$S_x = x - x_0 \; . \qquad (1.2)$$

Then for x-component of velocity we can write

$$\upsilon_x = \frac{S_x}{t} = \frac{x - x_0}{t} \, . \qquad (1.3)$$

The SI unit of velocity is m /s.

§1.3. Average velocity in rectilinear motion

Since the ratio of the distance to the corresponding time interval is different for various sections unlike the case of uniform motion, the nonuniform motion is characterized by average velocity. If the displacements s_1, , s_2,... .s_n and time intervals t_1, t_2,.... t_n are known the average velocity of entire motion is given by the formula:

$$\upsilon_{av.} = \frac{s_1 + s_2 + ... + s_n}{t_1 + t_2 + ... + t_n} \quad . \qquad (1.4)$$

Since each section of trajectory the motion can be considered as uniformly rectilinear with velocity υ equation (1.4) takes the form

$$\upsilon_{av.} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + ... + \upsilon_n t_n}{t_1 + t_2 + ... + t_n} \qquad (1.5)$$

In particular , if the distance covered consists of two sections, then

$$\upsilon_{av.} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2}{t_1 + t_2} \qquad (1.6)$$

Hence it follows, that if $t_1 = t_2 = t_{total} / 2$, then

$$\upsilon_{av.} = \frac{\upsilon_1 + \upsilon_2}{2} \quad . \qquad (1.7)$$

When the distances traveled by a body are equal, i.e. $s_1 = s_2 = s_{total} / 2$ then the average velocity is given by

$$\upsilon_{av.} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2} . \qquad (1.8)$$

§1.4. Acceleration in rectilinear motion

Acceleration is the rate at which velocity changes with time. The change in velocity of a body is the difference between its final υ_f and original velocities υ_0. Thus

$$\vec{a} = \frac{\vec{\upsilon}_f - \vec{\upsilon}_0}{t} . \qquad (1.9)$$

Magnitude of acceleration is equal to tangent of the angle between graph of velocity and axis of time. (Fig.1.5).

$$a = tg\beta$$

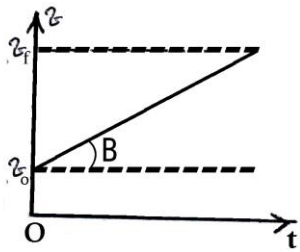


Fig.1.5

Thus, the slope of a speed–time graph represents acceleration.
In SI unit of acceleration is m/ sec ^{2} .

§ 1. 5. Kinematic equations

Consider the special case of a motion that a body is accelerated from rest., i.e. initial velocity $\upsilon_0 = 0$. For this motion the distance traveled at any given time can be given as

$$s = \frac{a\,t^2}{2} \quad \text{and} \quad s = \frac{\upsilon\, t}{2} , \qquad (1.10)$$

If the velocity of a body is increasing, the acceleration is positive ($a > 0$). If the velocity is decreasing, the acceleration is negative($a < 0$) .

The final velocity can be expressed by

$$\upsilon_f = \mathbf{at} \qquad \upsilon_f = \sqrt{\mathbf{2\,a\,s}} \qquad \text{or} \qquad \upsilon_f = \sqrt{\mathbf{2\,a\,(x - x_0)}} \quad (1.11)$$

From Eq. (1.9) for final velocity of the motion with initial velocity υ_0 we get a general expression

$$\upsilon_f = \upsilon_0 + \mathbf{at}\,. \qquad (1.12)$$

The distance traveled by an object starting with intially velocity is presented by

$$\mathbf{s} = \frac{\upsilon_0 + \upsilon_f}{\mathbf{2}}\,\mathbf{t}\;. \qquad (1.13)$$

Substituting (1.12) into (1.13) yields to

$$\mathbf{s} = \upsilon_0\,\mathbf{t} + \frac{\mathbf{a\,t^2}}{\mathbf{2}} \qquad \text{or} \quad \mathbf{x} = \mathbf{x_0} + \upsilon_0\mathbf{t} + \frac{\mathbf{at^2}}{\mathbf{2}}\,. \quad (1.14)$$

The final velocity can be calculated by the formulae too:

$$\upsilon_f = \sqrt{\upsilon_0^2 + \mathbf{2as}} \qquad \text{or} \qquad \upsilon_f = \sqrt{\upsilon_0^2 + \mathbf{2a(x - x_0)}}\,. \qquad (1.15)$$

Note, that if the motion is decelerated an acceleration in formulae (1.14) and (1.15) is negative($a < 0$) . At the end of decelerated motion the final velocity is zero.($\upsilon_f = 0$). Then for **stopping distance** we get

$$s = \frac{\upsilon_0^2}{2a} \qquad (1.16)$$

A distance traveled at n-th second (t =1 second) is given by the formula

$$\Delta S_n = \upsilon_0 + \frac{a}{2}(2n-1) \qquad (1.17)$$

Consider the graphs of motion with constant acceleration for particular cases:

1. Accelerated motion occurs in the positive direction of x-axis; $\upsilon_x \triangleright 0$; $a_x \triangleright 0$

In this case the kinematic equations for x-componenets of velocity, displacement and coordinates are given by

$$\upsilon_x = \upsilon_{x0} + a_x t\,, \qquad (1.18)$$

$$S_x = \upsilon_{x0} t + \frac{a_x}{2} t^2 \quad , \qquad (1.19)$$

$$x = x_0 + S_x = x_0 + \upsilon_{x0} t + \frac{a_x}{2} t^2 . \qquad (1.20)$$

Temporary dependencies of (a) velocity, (b) displacement, (c) coordinate and (d) a distance traveled are plotted in (Fig.1.6).

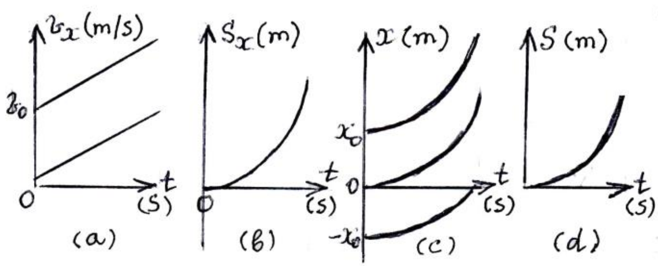


Fig.1.6

2. A decelerated motion in the positive direction of x-axis:

$\upsilon_x \triangleright 0; \quad a_x \triangleleft 0$

In this case kinematic equations are given by

$$\upsilon_x = \upsilon_{x0} - a_x t \quad , \tag{1.21}$$

$$S_x = \upsilon_{x0} t - \frac{a_x}{2} t^2 \ , \tag{1.22}$$

$$x = x_0 + S_x = x_0 + \upsilon_{x0} t - \frac{a_x}{2} t^2 \quad . \tag{1.23}$$

Temporary dependencies of (a) velocity, (b) displacement, (c) coordinate and (d) a distance traveled are presented in (Fig.1.7).

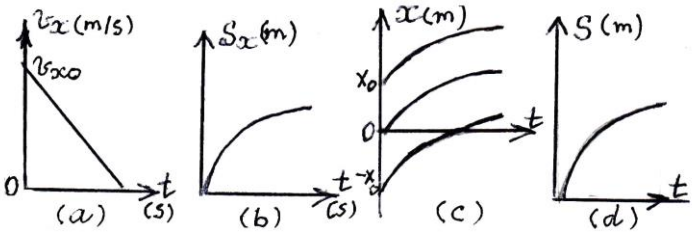


Fig.1.7

3. An accelerated motion in the negative direction of x-axis;

$\upsilon_x \triangleleft 0; \ a_x \triangleright 0$

Kinematic equations are given as

$$\upsilon_x = -\upsilon_{x0} + a_x t \quad , \tag{1.24}$$

$$S_x = -\upsilon_{x0} t + \frac{a_x}{2} t^2 \ , \tag{1.25}$$

$$x = x_0 + S_x = x_0 - \upsilon_{x0} t + \frac{a_x}{2} t^2 . \tag{1.26}$$

Temporary dependencies of (a) velocity, (b) displacement, (c) coordinate and (d) a distance traveled are presented in (Fig.1.8).

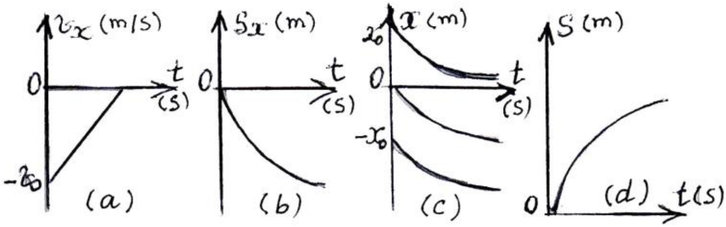


Fig.1.8

4. A decelerated motion in the negative direction of x-axis;
$\upsilon_x \lhd 0$; $a_x \lhd 0$

The kinematic equations are presented by

$$\upsilon_x = -\upsilon_{x0} - a_x t \ , \tag{1.27}$$

$$S_x = -\upsilon_{x0} t - \frac{a_x}{2} t^2 \ , \tag{1.28}$$

$$x = x_0 + S_x = x_0 - \upsilon_{x0} t - \frac{a_x}{2} t^2 \ . \tag{1.29}$$

Temporary dependencies of (a) velocity, (b) displacement, (c) coordinate and (d) a distance traveled are presented in (Fig.1.9).

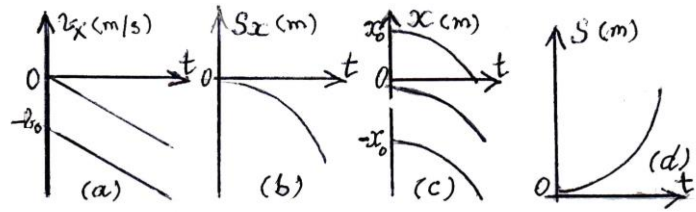


Fig.1.9

From figures above can be seen that a distance traveled is positive quantity for all types of motion with constant acceleration .

§ 1.6. Motion of a body thrown vertically downward. Free fall

This motion with initial velocity υ_0 is rectilinear accelerated motion.

At the instant of time ($t < t_{fall.}$) velocity and distance (height) are determined as

$$\upsilon = \upsilon_0 + gt \;, \quad h = \upsilon_0 t + \frac{gt^2}{2}. \qquad (1.30)$$

From (1.30) it follows that

$$\upsilon = \sqrt{\upsilon_0^2 + 2gh} \quad \text{and} \quad h = \frac{\upsilon^2 - \upsilon_0^2}{2g}. \qquad (1.31)$$

When $t = t_{fall.}$ substituting $h = H$ and $\upsilon = \upsilon_{final}$ we get:

$$\upsilon_{final.} = \upsilon_0 + gt_{fall.} \quad \text{and} \quad H = \upsilon_0 t_{fall.} + \frac{gt_{fall.}^2}{2} \qquad (1.32)$$

Hence we can find:

$$\upsilon_{final} = \sqrt{\upsilon_0^2 + 2gH} \quad \text{and} \quad t_{fall.} = \frac{\sqrt{\upsilon_0^2 + 2gH} - \upsilon_0}{g}. \qquad (1.33)$$

We can use the following relations as well:

$$\upsilon_{ave.} = \frac{\upsilon_0 + \upsilon_{final}}{g} \quad \text{and} \quad H = \frac{\upsilon_0 + \upsilon_{final}}{2} t \;. \qquad (1.34)$$

The height of the body at the n-th second (t=1 sec.) is

$$\Delta h_n = \upsilon_0 + \frac{g}{2}(2n - 1) \quad . \qquad (1.35)$$

Free fall. This motion is accelerated (with **g)** motion without initial velocity ($v_0 = 0$) At any instant of time t velocity and height are determined with the following formulas:

$$\upsilon_{final.} = gt \qquad \text{and} \qquad h = \frac{g\,t^2}{2} \quad . \qquad (1.36)$$

Hence it follows:

$$t = \sqrt{\frac{2\,h}{g}} \qquad \text{and} \qquad \upsilon_{final.} = \sqrt{2gh} \quad . \qquad (1.37)$$

For free fall motion the height at the n-th sec. is expressed as

$$\Delta h_n = h_n - h_{n-1} = \frac{g}{2}(2n-1) \quad . \qquad (1.38)$$

§ 1.7. Motion of a body thrown vertically upward

To describe this motion it is convenient to assume that the upward direction is positive. Since the g is directed downwards, the motion with a positive initial velocity will be uniformly decelerated with a negative acceleration $-$ g. The velocity of this motion at instant **t** is given by

$$\upsilon = \upsilon_0 - gt \quad . \qquad (1.\,39)$$

Since the final velocity υ at highest point becomes zero the time of ascent is determined by the relation

$$\mathbf{t} = \frac{\upsilon_0}{\mathbf{g}}.$$

The height at this instant is

$$h = \upsilon_0 t - \frac{gt^2}{2}. \qquad (1.40)$$

Substituting expression for time into (1.40) yields for maximum height to

$$\mathbf{h}_{\mathbf{max.}} = \frac{\upsilon_0^2}{\mathbf{2g}}. \qquad (1.41)$$

§1.8. Velocity in relative motion. Addition of velocities

If the velocities of two bodies with respect to Earth are $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ then the relative velocity of the first body with respect to the second is

$$\vec{\upsilon}_{12} = \vec{\upsilon}_1 - \vec{\upsilon}_2. \qquad (1.42)$$

The velocity of second body with respect to the first one is

$$\vec{\upsilon}_{21} = \vec{\upsilon}_2 - \vec{\upsilon}_1. \qquad (1.43)$$

The value of magnitude of relative velocity depends on the angle between vectors of velocities and can be calculated by the theorem of cosine.

$$\upsilon_{\mathrm{rel}} = \sqrt{\upsilon_1^2 + \upsilon_2^2 - 2\upsilon_1\upsilon_2\cos\alpha}. \qquad (1.44)$$

where α - is the angle between vectors of velocities.

If the $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ are parallel then

$$\vec{\upsilon}_{\text{rel.}} = \vec{\upsilon}_1 - \vec{\upsilon}_2 \ . \qquad (1.45)$$

When $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ are in opposite directions

$$\vec{\upsilon}_{\text{rel.}} = \vec{\upsilon}_1 + \vec{\upsilon}_2 \ . \qquad (1.46)$$

Now suppose that $\vec{\upsilon}_1$ is the relative velocity of the first body with respect to the Earth and $\vec{\upsilon}_2$ is the velocity of the second body with respect to the first body . Then the resultant velocity of the second body with respect to the Earth is

$$\vec{\upsilon}_{\text{res.}} = \vec{\upsilon}_1 - \vec{\upsilon}_2 \, .$$

Magnitude of the resultant velocity is

$$\upsilon_{res.} = \sqrt{\upsilon_1^2 + \upsilon_2^2 + 2\,\upsilon_1\,\upsilon_2 \cos\alpha} \ . \qquad (1.47)$$

In particular, if $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ are parallel, then

$$\upsilon_{\text{res.}} = \upsilon_1 + \upsilon_2 \ . \qquad (1.48)$$

If $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ are opposite directed vectors,

$$\upsilon_{\text{res.}} = \upsilon_1 - \upsilon_2 \ . \qquad (1.49)$$

§1.9. Curvelinear motion

Average velocity. Let us consider path 1-2, a portion of the path. Assume, that at the instant of time t the moving particle was at 1 and at the instant of time $t + \Delta t$ at 2.

Let r_1 and r_2 are position vectors of a particle. From Fig.1.10. we see that

$$\vec{r}_2 = \vec{r}_1 + \Delta r \quad \text{or}$$
$$\vec{r}_2 - \vec{r}_1 = \Delta \vec{r},$$

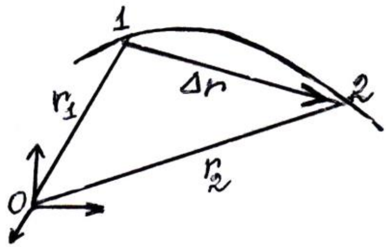


Fig.1.10

where Δr - is the difference of the position vectors.

The average velocity for the path AB is given by the relation

$$\upsilon_{av.} = \frac{\Delta \vec{r}}{\Delta t} \quad . \qquad (1.50)$$

Instantaneous velocity.

If we decrease the interval of time Δt, the point B will approach point A. These points finally merge and the direction of AB then coincides with the tangent to the curve at the point of merger. As Δt decreases, the ratio $\vec{AB}/\Delta t$ approaches a limit. The vector $\upsilon_{ins.}$, having the direction of the tangent to the curve at the given moment of motion and numerically equal to the limit of the ratio $AB/\Delta t$ as $\Delta t \to 0$, is called the instantaneous velocity.

$$\upsilon_{ins.} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \qquad (1.51)$$

§ 1.10. Uniform and nonuniform circular motions

A body moving on the circle with linear velocity constant in magnitude makes a uniform motion. In this motion the instantaneous speed is directed on the tangent of trajectory at any moment of time. In general the acceleration is defined as the rate of change of speed. (Fig.1.11).

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\upsilon}}{\Delta t}. \qquad (1.52)$$

Since acceleration $\vec{a}$ coincides in direction with $\Delta \vec{v}$ then it is directed towards the center of circle. This acceleration refers to *centripetal*. Magnitude of centripetal acceleration is given by

$$a_c = \frac{\upsilon^2}{r} \qquad (1.53)$$

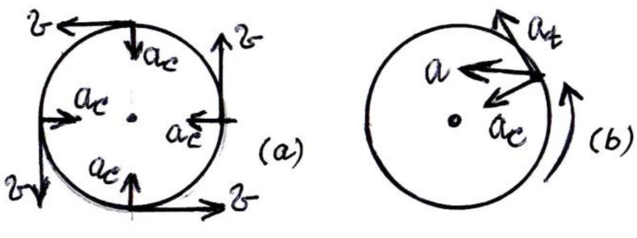


Fig.1.11

Angular velocity. The angular velocity of a point is the ratio of the angle of rotation (equal to 2π) of the position vector of the point to the time interval during which this rotation has occur.

$$\omega = \frac{2\pi}{T} \ \frac{\mathbf{rad}}{\mathbf{s}} . \qquad (1.54)$$

Having put into (1.54) the expression of frequency $f = 1/T$ we get

$$\boldsymbol{\omega = 2\pi} f \qquad (1.55)$$

relationship between the linear and angular velocities is

$$\upsilon = \omega r \ , \qquad (1.56)$$

where r is the radius of the circle.
Note, that angular velocity is defined through the angle of rotation φ by the equation

$$\omega = \frac{\Delta \varphi}{\Delta t} \ . \qquad (1.57)$$

Substituting (1.56) into (1.53) we get a relationship between the centripetal acceleration and angular velocity

$$a_C = \omega^2 \mathbf{r} \ . \qquad (1.58)$$

If the velocity of a body circular motion changes in magnitude it will have both normal and tangential accelerations. Normal and tangential components of acceleration are perpendicular to each other. The total acceleration is the vector sum of both accelerations (Look at Fig.1.11b):

$$\vec{a} = \vec{a}_c + \vec{a}_t .$$

The magnitude of total acceleration is equal to

$$a = \sqrt{a_c^2 + a_t^2} \quad . \qquad (1.59)$$

According to the equation (1.57) the angular acceleration is calculated as

$$\beta = \frac{\Delta\omega}{\Delta t} \quad . \qquad (1.60)$$

Like linear acceleration the angular acceleration is the rate of change in angular velocity.

§1. 11. Motion of a horizontally thrown body

This motion is the composition of two independent ones: horizontal and vertical. The motion of the x – projection is the motion with zero acceleration at a velocity υ_0. The motion of y projection is a free fall with an acceleration g under the action of the force of gravity with zero initial velocity.(Fig. 1.12) The υ_x component remains constant and equal to υ_0. But the υ_y component grows in proportion to time: $\upsilon_y = gt$

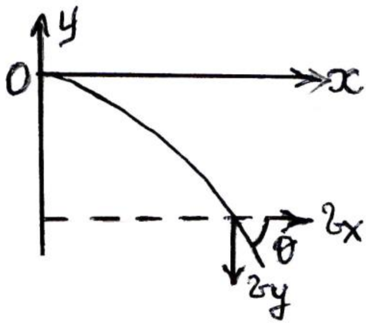


Fig.1.12

Then for coordinates x and y at instant t we have:

$$\mathbf{x = \upsilon_0 t} \qquad \text{and} \qquad y = \frac{gt^2}{2} . \qquad (1.61)$$

Solving (1.61) together for y yields to ***equation of path***

$$y = \frac{g}{2\upsilon_0} x^2 . \qquad (1.62)$$

From (1.62) it is seen, that path of free falling body with an initial horizontal velocity is a **parabola.**

Having put h= y in (1.61) gives

$$\mathbf{h = \frac{g\,t^2}{2}} . \qquad (1.63)$$

Having put **t** from (1.63) into the equation for ***x*** gives horizontal range **s**

$$\mathbf{s} = \upsilon_0 \sqrt{\frac{\mathbf{2h}}{\mathbf{g}}} . \qquad (1.64)$$

The directions of instantaneous velocity and tangent at the given point of path are coincide. From Fig 1.12

$$\mathrm{tg}\,\alpha = \frac{\mathbf{g\,t}}{\upsilon_0} . \qquad (1.65)$$

Magnitude of resultant velocity is given by

$$\upsilon = \sqrt{\upsilon_0^2 + \mathbf{g^2 t^2}} . \qquad (1.66)$$

§ 1.12. Projectile motion

Let us consider the motion of a body thrown upwards at an angle θ to the horizontal (Fig.1.13).

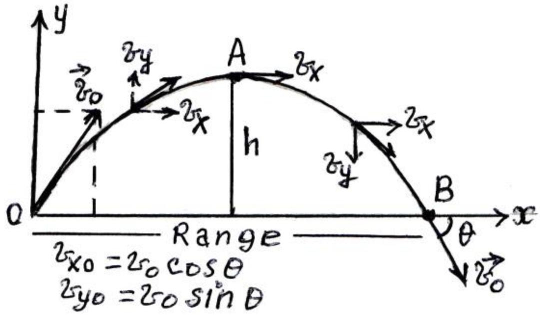


Fig.1.13

with an initial velocity υ_0. The x–component of the velocity is constant:

$$\upsilon_x = \upsilon_0 \cos\theta . \qquad (1.67)$$

The y – projection of velocity varies with time by the formula;

$$\upsilon_y = \upsilon_{0y} - g t , \qquad (1.68)$$

where $\upsilon_{0y} = \upsilon_0 \sin\theta$

From Eqs. (1.67) and (1.68) it is quite clear the described motion is composed of two independent motions;

a) the horizontal motion with constant velocity ;

b) the vertical motion with constant acceleration. At the highest point A of the trajectory the velocity has only horizontal component, while vertical component reduces with passage of time until the body reaches the highest point and becomes zero. Hence, equating the expression (1.68) to zero we get the instant $\mathbf{t_A}$ over which the body reaches the maximum height.

$$t_A = \upsilon_0 \sin\theta / g . \qquad (1.69)$$

The y coordinate varies with time as follows

$$y = (\upsilon_0 \sin\theta)t - \frac{gt^2}{2} \quad . \qquad (1.70)$$

From Eqs. (1. 69) and (1.70) for maximum height which the body reaches we get:

$$h_{max.} = \upsilon_0^2 \sin^2\theta / 2g \quad . \qquad (1.71)$$

From formula (1.71) it follows that the height increases with angle θ and reaches its maximum value equal to $\upsilon_0^2/2g$ at an angle θ = 90°

At any time instant t the velocity with magnitude υ is given by

$$\upsilon = \sqrt{(\upsilon_0 \cos\theta)^2 + (\upsilon_0 \sin\theta - gt)^2} \quad . \qquad (1.72)$$

If υ makes an angle β with the horizontal then

$$\beta = \mathrm{arctg}\frac{\upsilon_0 \sin\theta - gt}{\upsilon_0 \cos\theta} \quad . \qquad (1.73)$$

The flight time is equal to $2t_A$, thus

$$t_{fl.} = 2\,t_A = 2\,\upsilon_0 \sin\theta / g \quad . \qquad (1.74)$$

The range (R) is equal to the horizontal distance (x_{max}) traveled from starting point to the point where the body returns back to its origin:

$$R = x_{max} = \upsilon_{0x} t_{fl.} \quad \text{or} \quad R = \upsilon_0^2 \sin 2\theta / g . \qquad (1.75)$$

From equation (1.75) the range for a particular initial velocity is maximum when **sin 2θ is** maximum. The maximum value of **sin** 90^0 **=1** if **2θ=90°** **,** then **θ=45°** for a maximum range at a given initial velocity. For a projectile with a given initial velocity, the maximum range is attained with a projection angle of **45**°. For angles above and below **45**° the range is shorter and equal for equal differences from **45**°.

Comparison (1.71) and (1.75) gives

$$\frac{R}{h_{\max}} = \frac{4}{tg\theta}. \tag{1.76}$$

When projectile angle is then **45**° , $tg\theta = 1$. Hence

$$R = 4h_{\max} \tag{1.77}$$

Substituting (1.74) into (1.70) gives equation of motion, i.e., dependence of y – coordinate on the x one.

$$\mathbf{y = x\, tg}\, \theta - \frac{\mathbf{g}}{\mathbf{2}\, \upsilon_0^2 \,\mathbf{cos}\, \theta} \mathbf{x^2} \;. \tag{1.78}$$

Now consider energy relationships for a projectile.

At the starting point potential energy is zero, while kinetic energy is maximum: $E_p = 0;\ \mathrm{E_k} = E_{total} = \frac{m\upsilon_0^2}{2};\ \ \mathrm{p} = \mathrm{m}\upsilon_0$ where p- is the linear momentum

At the highest point the potential energy is maximum and kinetic energy is minimum

$$E_{p,\max} = mgh_{\max} = \frac{m\upsilon_0^2 \sin^2\theta}{2}\ , \tag{1.79}$$

$$E_{k,\min} = \frac{m\upsilon_{x0}^2}{2} = \frac{m\upsilon_0^2 \cos^2\theta}{2} \quad . \tag{1.80}$$

Consequently, total energy can be given by

$$E_{total} = E_{k,\max} = \frac{m\upsilon_0^2}{2} = \frac{p_0^2}{2m} = \frac{p_0\upsilon_0}{2} \quad . \tag{1.81}$$

From equations (1.79) and (1.80) we get

$$E_{k,\min} = E_{k,\max}\cos^2\theta \quad , \tag{1.82}$$

$$E_{p,\max} = E_{k,\max}\sin^2\theta \quad . \tag{1.83}$$

Comparison of last relationships yields to

$$\frac{E_{p,\max}}{E_{k,\min}} = tg^2\theta \, .$$

When projectile angle is $\theta = 45^0$, then $tg\theta = 1$, Hence

$$E_{p,\max} = E_{k,\min} = \frac{E_{total}}{2} = \frac{m\upsilon_0^2}{4} \tag{1.84}$$

CHAPTER 2

Dynamics.

§2.1. Newton′s laws of motion. The concept of force

Dynamics is a branch of mechanics which study the motion of bodies with regard to the reasons of motion. An object at rest will not move unless the forces acting on it are no longer in equilibrium. An object in motion will not slow down, speed up, or change its direction unless a force acts upon it. All changes in the motion of objects are due to forces. The five forces known to scientists are the gravitational, electric, magnetic, nuclear and weak interaction forces.

Newton's First Law. Newton's first law of motion implies that there is no fundamental difference between an object at rest and one that is moving with uniform velocity. This law of motion states; **an object continues its state of rest or uniform motion along a straight line,unless it is compelled upon by an unbalanced external force impressed upon it**. This law means that it is the natural tendency of an object to resist a change in its motion. This tendency is called **inertia** of a body. Mass of matter in any body is the quantitative measure of inertia.Mass is measured in kilograms (kg)

Newton's Second Law. Consider a sitation when there is either a single applied force or two or more applied forces whose vector sum is not zero. The second law of motion states: " **The effect of an applied force on a body is to cause it to accelerate**

in the direction of he force. The acceleration is direct proportional to the force and is inversely proportional to the mass of the body". Newton′s second law can be written in the form

$$\vec{F} = m\vec{a} \quad , \qquad (2.1)$$

where $\vec{F}$ is the net force applied on an object, *a* is the resultant acceleration of an object and *m* is the inertial mass of the object.

The SI unit of force is newton (N). A force of one newton will accelerate a 1 – kg mass at the rate of $1 m/s^2$. $1N = kgms^{-2}$

Newton's Third Law. When any body exerts by force on the second body the second body also exerts by equal force in the opposite direction on the first body.

Third law of motion is also known as the law of action and reaction.According to this law a single force can never exit. Action of any body on another is always accompained by a reaction of the second on the first. Third law of motion states: **To every action (force) there is always an equal and opposite reaction**

$$F_1 = -F_2 \text{ or } \quad m_1a_1 = -m_2a_2 \ . \ , \qquad (2.2)$$

where F_1 is the force acting on the first body, F_2 –the force acting on the second body, m_1 and m_2-the masses of the first and second bodies, respectively. Fig.2.1a shows that F_1-force exerted by the table on hand and F_2-force exerted by hand on table.

Walking is a good example of Newton′s third law of motion.As we attemt to push the earth away from us, the net force exerted on us is the reaction of the ground acting vertically upward as a result of our exertion on the ground vertically downward minus our weight. If the net force is positive we will be able to jump.The reaction of the Earth on our feet propels us forward (Fig.2.1b).

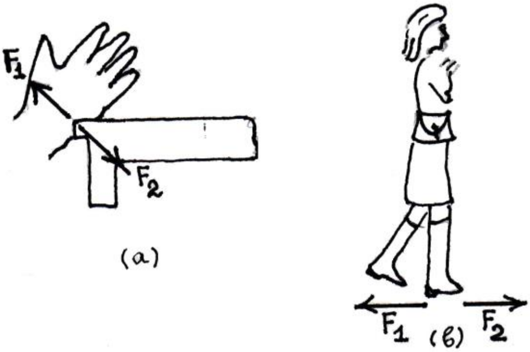


Fig.2.1

The third law of motion leads to the law of conservation of momentum.

§ 2.2. Linear momentum. Impulse. Law of conservation of linear momentum

Momentum of the mass point is a vector quantity, equal to the product of the mass and its velocity.

$$p = m\upsilon . \qquad (2.3)$$

Direction of momentum is the same with the direction of velocity. A momentun of the system composed of n mass point is equal to the sum of momenta of all masses.

Momentum is measured in $kgms^{-1}$.

If the applied forces are constant then

$$F = m\,\frac{\upsilon - \upsilon_0}{t} \qquad \text{or} \qquad Ft = p - p_0 .$$

The quantity **Ft** is referred to **impulse.** Thus a change in momentum is equal to the impulse. Here F is the vector sum of internal and external forces. The sum of internal forces is equal to zero in accordance with the Newton s third law. Thus change in momentum per unit time is equal to the resultant external force.

$$\frac{\Delta \mathbf{p}}{\mathbf{t}} = \mathbf{F}_{\mathbf{ext.}} \quad . \qquad (2.4)$$

Eq. (2.4) is known as principal equation of dynamics.

If the system is isolated, i.e. the system is acted by no exterior force, then Δp= 0 or p=const. This is law of conservation of momentum..

§ 2. 3. Elastic collision of balls

First, let us consider perfectly elastic collesion. We assume that two balls move along the same straight line (along the line connecting their centres). A ball of mass m_1 moving with a velocity v_1 called the incident approaches a second ball of mass m_2, moving with a velocity v_2 called the target.(Fig.2.2a) After the collision let the velocities of the two balls be υ'_1 and v'_2.

From the law of conservation of momentum it follows that

$$m_1\upsilon_1 + m_2\upsilon_2 = m_1\upsilon_1' + m_2\upsilon_2' \qquad (2.5)$$

During elastic collisions the total energy is also conserved:

$$\frac{m_1\upsilon_1^2}{2} + \frac{m_2\upsilon_2^2}{2} = \frac{m_1\upsilon_1'^2}{2} + \frac{m_2\upsilon_2'^2}{2} \quad . \qquad (2.6)$$

After rearranging Eqs. (2.5) and (2.6) may be written as (2.7) and (2.8) respectively

$$m_1\left(\upsilon_1 - \upsilon_1''\right) = -m_2\left(\upsilon_2 - \upsilon_2'\right) \;, \qquad (2.7)$$

$$m_1\left(\upsilon_1^2 - \upsilon_1'^2\right) = -m_2\left(\upsilon_2^2 - \upsilon_2'^2\right) \;. \qquad (2.8)$$

Dividing Eq. (2. 8) by (2. 7) gives:

$$\upsilon_1 - \upsilon_2 = \upsilon'_2 - \upsilon'_1 \qquad (2.9)$$

On the other hand when $m_1 = m_2$ from (2. 5) we get:

$$\upsilon_1 + \upsilon_2 = \upsilon_1' + v'_2 \qquad (2.10)$$

The sum of the last equations gives $\upsilon'_2 = \upsilon_1$ while their difference gives $\upsilon'_1 = \upsilon_2$. Thus the balls interchange their velocities, the second ball acquires the velocity which the first ball had before the collision. If the second ball initially was at rest (υ_2=0) then υ'_1 =0 i.e. the incident ball which was moving with velocity v_1 comes to rest while the target ball that was at rest begins to move with a velocity v_1. Thus the incident ball transfers all its kinetic energy to the target ball.

Relative velocity of the balls with respect to each other before collision and after collision has the same magnitude but it is reversed after collision. Solving (2.7) and (2.8) for υ'_1 and υ'_2 gives

$$\upsilon'_1 = \frac{(m_1 - m_2)\upsilon_1}{m_1 + m_2} + 2m_2\upsilon_2$$

$$\upsilon'_2 = \frac{(m_2 - m_1)\upsilon_2 + 2m_1\upsilon_1}{m_1 + m_2} \qquad (2.11)$$

Let us examine these formulas:

Suppose that the second ball initially is at rest ($\upsilon_2 = 0$). Then expressions (2. 11) will have a simplified form:

$$\upsilon'_1 = \frac{m_1 - m_2}{m_1 + m_2}\upsilon_1$$

(2.12)

$$\upsilon'_2 = \frac{2m_1}{m_1 + m_2}\upsilon_1$$

Consider some interesting cases:

a) $\upsilon_2 = 0$, $m_1 = m_2$ In this case we get $\upsilon_1' = 0$ and $\upsilon_1 = \upsilon_2'$,i.e., when one of the balls is at rest the velocity of moving one is transferred to another.

b) $\upsilon_2 = 0$, Suppose $m_1 >> m_2$ In this case from Eq.(2.12) we get $\upsilon_1' = \upsilon_1$ and $\upsilon_2' = 2\upsilon_1$. That is the incident ball keeps on moving without losing much energy while the target ball moves with a velocity double that of the incident ball.

c) $\upsilon_2 = 0$ Suppose $m_1 << m_2$ From Eq.(2.6) we see that $\upsilon_1' = \upsilon_1$ while $\upsilon_2' = 0$ This means that small incident ball just bounces off in the opposite direction while the heavy target remains almost motionless.

§ 2. 4. Inelastic collision of balls

In the case of inelasticity, after collision the two bodies move together with a certain velocity υ .(Fig.2.2b).

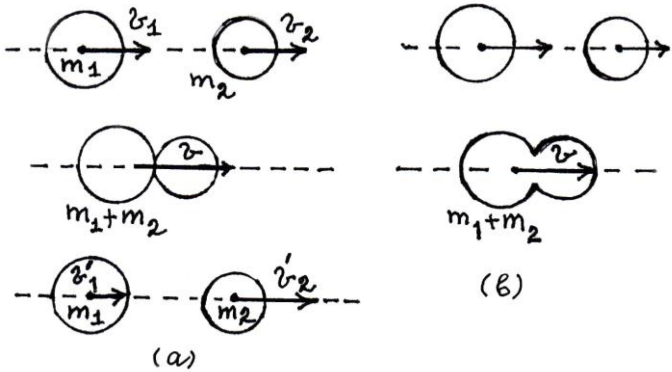


Fig.2.2

Suppose that a body of mass m $_1$ has a velosity υ_1 and a body of mass m_2 has a velosity υ_2 before collision.

According to the law of conservation of momentum momentum;

$$m_1\upsilon_1 + m_2\upsilon_2 = (m_1 + m_2)\upsilon$$

Hence

$$\upsilon = \frac{m_1\upsilon_1 + m_2\upsilon_2}{m_1 + m_2} \qquad (2.13)$$

Note, here energy does not conserved and the change in kinetic energy is determined by the formula;

$$\Delta E = -\frac{m_1 m_2}{2(m_1 + m_2)}(\upsilon_1 - \upsilon_2)^2 < 0 \qquad (2.14)$$

Thus, from (2.14) we see that a decrease in mechanical energy causes the increase in internal energy of a system of two bodies.

§ 2. 5. Jet propulsion

Consider an action of.a rocket in result of conbustion of fuel. The gases heated up high temperature are thrown out from the of rocket's tail with speed υ. A rocket and gases thrown out by its engine are interracting. In accordance with the conservation of momentum in the absence of external forces the vector sum of momenta of interacting bodies remains constant. Untill starting into operation momenta of both rocket and fuel are equal to zero. Hence, the vector sum of momentums of rocket and gases expired must be equal zero when engines are going into operation as well.

$$M\vec{V} + m\vec{\upsilon} = \mathbf{0}$$

where M – is the mass of rocket, V – is the rocket s velocity , m –is the mass of thrown out gases, υ – is the speed of the expiration of gases.

Hence we get

$$M\vec{V} = -\ m\vec{\upsilon}$$

This formula is used for the calculation the rocket's speed when a change in its mass **M** is negligible.

The gases being pulled out from tail of rocket act on it with a certain force , called **jet force of draft.** In order to find it we use the basic equation of dynamics (2. 4). We shall divide both sides by t. If taking into account that $M\frac{\Delta \mathbf{V}}{\Delta \mathbf{t}} = \mathbf{F}$ represents the **jet force of draft and** $\mu = \frac{\Delta \mathbf{m}}{\Delta \mathbf{t}}$ is the **fuel charge per unit time .**

Hence it follows

$$\mathbf{F} = -\mu \mathbf{v} \qquad (2.\ 15)$$

Thus**,** jet force of draft is directly proportional to the fuel charge per second and speed of gases; it is directed in opposite direction to the gas expiration.

The **stock of fuel** is calculated by the formula

$$M_{fuel} = \mu t \qquad (2.16)$$

where **t–** is the time.

§ 2. 6. Kepler′s laws

Kepler's first law: The orbits of the planets are ellipses one focus of which is located on the Sun (Fig.2.3 a).

Kepler's second law: A planet moves so that the imaginary line between it and the Sun sweeps out equal areas in equal time intervals.

In Fig.2.3b two areas A and B are equal. In the equal intervals of time the planet can go from point 1 to 2 or from 3 to 4.

Third law: A square of ratio of periods need to make a revolution around the Sun and ratio of average distances from the Sun are equal for any two planets.

This result can be obtained by equating the centripetal force and the force of attraction between the Sun for any planet . For pair of arbitrary planets we can write:

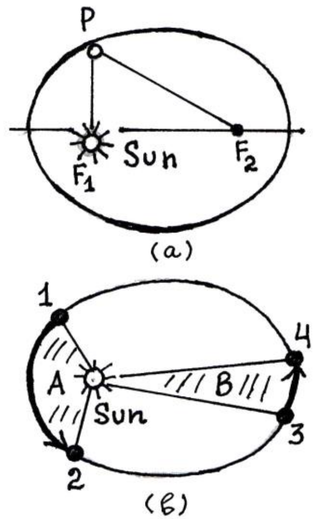


Fig..2.3

$$G\frac{Mm_1}{r_1^2} = \frac{m_1\upsilon_1^2}{r_1} \qquad \text{for the first planet}$$

(2.17)

$$G\frac{Mm_2}{r_2^2} = \frac{m_2\upsilon_2^2}{r_2} \quad \text{for the second planet}$$

Substituting $\upsilon = 2\pi$ r/T(where T is the period of revolution) in these formulas, finally gives

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \qquad (2.18)$$

§ 2. 7. The Law of Universal Gravitation

The force of gravity on the Earth's surface gives an acceleration g=9.80 m/sec^2 to all bodies. How to calculate the

centripetal acceleration of Moon?. The Moon moves around the Earth on the circular orbit. Radius of orbits is appoximately equal 385000 km,and period of revolution T is 27.3 day or 2358720 sec. Then the speed of this motion is

$$\upsilon = 2\pi\, r/T = 1.02x10^{3}\, m/\text{sec.}$$

Consequently, $a_{c.}= \upsilon^2/ r = 2.73x10^{-3}$ ms^{-2} or $a_c \approx 2.78x10^{-4}$ g which is the same as $a_c \approx (1/3600)g$.

Thus, the acceleration of Moon directed toward the Earth is 1/3600 part of acceleration on the surface of the Earth.A distance between Moon and Earth is 385000km.,which is greater as 60 times than the radius of Earth, equal to 6380 km. This means that Moon is far from the center of Earth by 60 time in comparison with the body on the Earth. Newton had a conclusion that gravitational force decreases as the inverse square of the distance from the Earth′s center.Kepler's first law supported this conclusion, since a force of this kind can only lead to a circular or elliptical motion. The weight mg of the body,which is the force of gravity uponit,is always proportional to its mass m. Newton s third law of motion requaires that,if the earth attracts body, the body also attracts the earth. If the earth s attraction for an apple depends upon apple′ s mass, then the reaction force exerted by the apple on the earth depends upon the earth s mass.Hence the graviational force between two bodies is proportional to both of their masses.Thus, Newton summarized the above conclusions in a single form: **Every body in the universe attracts every other body with a force proportional to both of their masses and inversely proportional to the square of the distance between them.**

Force of attraction between two objects is directed along the line connecting those objects (Fig.2.4a).

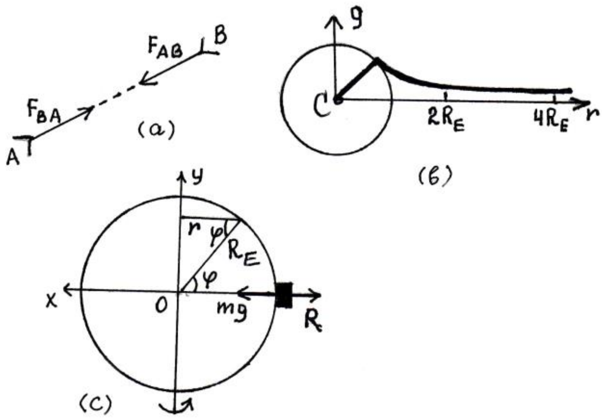


Fig.2.4

Newton′ s law of gravity has the following equation form:

$$F=G\frac{m_1m_2}{r^2}. \qquad (2.19)$$

Here **r** – is the distance between the bodies and G is the constant of nature being the same number everywhere in the universe. G=6.670 $\times10^{-11}$ Nm^2 kg^{-2}.

According to (2.17) a body of mass m at the surface of the Earth is attracted by it with a force

$$P=GmM_{Earth}/R^2_{Earth} \quad , \qquad (2.20)$$

where M_e is the mass of the Earth and R_E is its radius. But, P=mg. Substituting it into (2. 18) gives

$$M_E=gR^2/G \qquad (2.21)$$

According to (2. 19) the free fall acceleration is

$$g=GM_E/R^2_E \qquad (2.22)$$

Or

$$g= CR_E \qquad (2.23)$$

where $C=\frac{4\pi\rho}{3}G$ and ρ – is the Earth′s density. Hence it follows that g is directly proportional to the radius of the Earth up to the surface of latter.

If a body with mass m will be at the height h above Earth's surface, then from (2. 21) we get

$$g_h=GM_E / (R+h)^2 \qquad (2.24)$$

The dependences (2. 23) and (2. 24) are demonstrated in Fig.2.4b

§ 2.8. Variation of acceleration due to gravity with latitude

The body on the Earth ′s surface lying at latitude φ moves with angular velocity ω.Now consider, how is the acceleration due to gravity depends on position of the body on the Earth ′s surface.

As can be seen from Fig.2.4c the resultant of two forces gives the body a centripetal acceleration $a=\omega^2 R$. Then

$$mg - N = m\omega^2 R \qquad (2.25)$$

Where R– is the Earth′s radius, N–is the support s reactional force, ω – is the Earth′ s angular velocity. If the support is taken and the body is given the free fall possibility, then it will move under action of force, determined with acceleration at equator: $F_{eq}=mg_{eq}$. Then the equation (2. 25) takes the form:

$$mg - m\omega^2 R = mg_{eq.} \quad or \quad g_{eq} = g - \omega^2 R$$

where g– is the l acceleration of free fall without regard to the Earth′ s rotation around its axis.

Similarly for another parallel circles (corresponding to various angles φ) we get

$$g_\varphi = g - \omega^2 r \qquad (2.26)$$

Since in Fig .2.4c $r = R\cos\varphi$, then

$$g_\varphi = g - \omega^2 R\cos\varphi \quad \text{or} \quad g_\varphi = g - (4\pi^2/T^2)\cos\varphi . \qquad (2.27)$$

§ 2. 9. Variation of *g* with altitude and depth below the Earth's surface

From Eq. (2. 22) is seen

$$g_h = g\frac{R_E^2}{(R_E + h)^2} = g\left(1 + \frac{h}{R_E}\right)^{-2}$$

Assuming h<< R_E and ignoring the higher powers of h/R we obtain

$$g_h = g - \frac{2gh}{R_E} \qquad (2.28)$$

Taking the Earth to be a uniform sphere, an observer at depth x below the surface is inside a sphere of radius R and thickness *x*, so that the attraction due to this part of the Earth′s mass is zero. The Earth's effective mass is sphere of radius (R− *x*). While at depth x, the value of the acceleration due to garvity g_x is

$$g_x = \frac{GM_E}{(R_E - x)^2} = g\left(1 - \frac{x}{R_E}\right) \qquad (2.29)$$

where $g = \frac{4}{3}\pi R_E \rho G$ and ρ - is the density of the Earth.

From Eq.(2. 29) can be seen that at the centre of the Earth ($x=R_E$) $g_x=0$. Hence as we go below the surf ace of the earth, the value of g decreases.

§ 2.10. Cavendish′s experiment

The value of the gravitational constant G is found experimentally by Cavendish. A light rod is suspended from its centre by means of fine quarts fibre as shown in (Fig.2.5).
Two identical balls each of mass ***m*** are suspended from the end of the thread. When heavy balls each of mass M are brought near them on opposite sides of the small balls, they exert forces of attraction on the suspended balls. Each of these forces has

magnitude $G\frac{Mm}{r^2}$. where r is the distance between the centres of the balls M and *m*. These forces produced torque which causes the fibre to twist. The twist produced in the fibre is proportional to the magnitude of the torque that is,

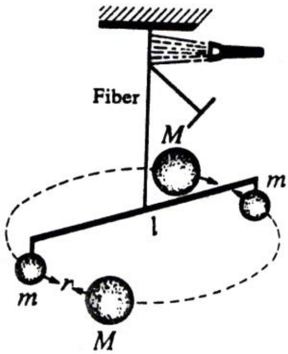


Fig.2.25

$$T=c\theta \qquad (2.30)$$

where c is the torsion constant of the fibre which can be calculated by applying a known torque and θ – is the angle through which the fibre is twisted.

If *l* – is the length of the rod then the magnitude of the torque due to gravitation is given by

$$T = G\frac{M\mathbf{m}}{\mathbf{r}^2}l \qquad (2.31)$$

Equating (2.30) and (2.31) yields to

$$G = \frac{c\ \theta\mathbf{r}^2}{Mml} \qquad (2.32)$$

and θ is measured by observing the deflection of beam of light reflected from the small mirror attached to the fibre. The lamp and scale arrangement is used to measure the angle of twist accurately.

All the quantities in the above equation except G, are known. The value of G found from this experiment is $G=6.673\text{x}10^{-11}\text{Nm}^2\text{kg}^{-2}$.

§2.11. The weight of an object

The force with which a body experiences the force of gravity acting on a support or a suspender is called the *weight* of an object.

The **weight** may be distinguished from the force of **gravity** (attraction) and is the force acting on the connection. Naturally weight is the force of elasticity.The weight may be equal to the force of **gravity,** when

– the body will be at rest relatively to the Earth,

– the body will be in the inertial reference frames.

If the lift moves upward or downward with constant acceleration the weight observed in it will differ from that the force of gravity. Let R be the force of reaction and P be the weight of the body. Consider the motion of lift for the following cases:

1. Suppose, that the vectors of velocity v and acceleration a are directed downward (opposite direction of z axis (Fig.2.6a,).

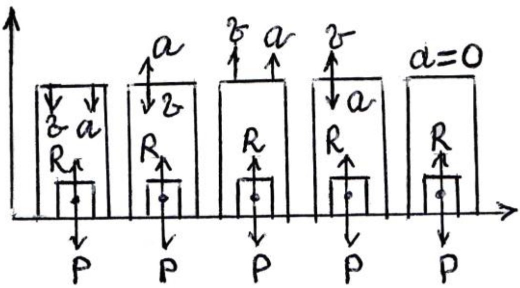


Fig.2.6

Then

$$R-P=-ma \quad \text{or} \quad R=m\,(g-a) \qquad (2.33)$$

2.Direction of velocity is downward, but acceleration is directed upwards (Fig.2.6b,) ;

$$R-P=ma \quad \text{or} \quad R=m\,(g+a) \qquad (2.34)$$

3. Both velocity and acceleration are directed upward

(Fig.2.6c,) :

$$R–P=ma \quad or \quad R=m (g+a) \qquad (2.35)$$

4.Direction of υ is upward but acceleration is directed downward (Fig.2.6d) :

$$R–P = –ma \quad or \quad R= m(g–a) \qquad (2.36)$$

Weight of an object P equals in magnitude and opposite in direction to the force of reaction R .

5. Weight of an object upon its motion over convex and concave paths (Fig.2.7) are given by the formulas respectively;

$$P = m(g - \frac{\upsilon^2}{r}) \quad , \quad P = m(g + \frac{\upsilon^2}{r})$$

Thus, if the body moves in non-inertial reference frames the weight differs from the force of gravity. An apparent weight is less

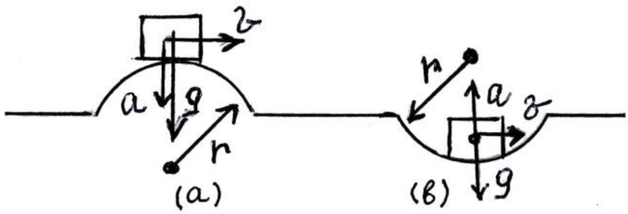


Fig.2.7

than the gravitational force on the body.When a=g then P is equal to zero. It is called the state of **weightlessness.** In this case motion is the free fall.The above described second and third cases are known as **overload**. Since the free falling acceleration varies at different points of the Earth′s surface the weight of the same body will be various at different points of the Earth′s surface.

When a body is in horizontally accelerated motion its weight is expressed by

$$P = m\left(g^2 + a^2\right)^{1/2} \qquad (2.37)$$

If the accelerated motion is directed at an angle φ relatively the g then weight is expressed as

$$P = m\left(g^2 + a^2 \pm 2ga\cos\varphi\right)^{1/2} \qquad (2.38)$$

§ 2.12. Cosmic velocities

A body starting from any point with definite velocity revolves over circular trajectory around the Earth. The velocity of such motion is called **first cosmic velocity** and is directed horizontally. A cosmic rocket moving over circular orbit nearly the Earth s surface is acted by two forces: centripetal and force of gravity. According the Newton s second law

$$\mathbf{G}\frac{\mathbf{m\,M_E}}{\mathbf{R_E^2}} = \frac{\mathbf{m}\ \upsilon^2}{\mathbf{R_E}}$$

Hence for velocity we get

$$\upsilon_1 = \sqrt{\mathbf{G}\frac{\mathbf{M_E}}{\mathbf{R_E}}} = \sqrt{\mathbf{g\,R_E}} = \mathbf{7.93x10^3}\,\frac{\mathbf{m}}{\mathbf{s}} \approx \mathbf{8}\ \frac{\mathbf{km}}{\mathbf{s}} \qquad (2.39)$$

The velocity with which a rocket may leave the gravitational field of Earth and become the **artificial planet** is known as the **second cosmic velocity** υ_2. Expression for the second cosmic velocity we get in accordance with the law of conservation of energy. In this case a distance between artificial planet and Earth can be accepted equal to infinite . Thus

$$\frac{m\upsilon^2}{2} = G\frac{m\,M_E}{R_E}$$

Hence,

$$\upsilon_2 = \sqrt{2}\,\upsilon_1 = 11.2\ \frac{km}{s} \qquad (2.40)$$

In this case trajectory is the parabola.

If the velocity has the range $\upsilon_1 < \upsilon < \upsilon_2$ the trajectory represents an ellipse which nearly focus is located at the centre of Earth.

When velocity is greater than the second cosmic velocity $(\upsilon > \upsilon_2)$ the trajectory becomes hyperbola

At least, if the $\upsilon < \upsilon_1$, then trajectory is the ellipse, farther focus which coincides with the centre of the Earth.

§ 2.13. Elasticity. Force of elasticity

Elasticity is defined as that property of matter by virtue of which it tends to return to its original shape or size after any change of shape or size. A force can deform the body, i.e. displace the components of its parts relative other. Then arrising the oppositional force equal in magnitude to deforming one is called a force of elasticity. These forces are due to interaction between particles and have nature of electricity.

Strain. A strain is given by the equation:

$$\varepsilon = \frac{\textbf{Change of length}}{\textbf{Original length}} = \frac{\Delta l}{l_0} \qquad (2.41)$$

Stress is defined as the internal force per unit of cross – sectional area on which the force acts. It tends to resist the external load. For the specimen under consideration stress is expressed as

$$\sigma = \frac{\mathbf{F}}{\mathbf{S}} \qquad (2.42)$$

Stress is measured in (N/m^2) or in Pa.

Elastic limit. The maximum stress a substance will bear and yet return to its original shape or size when the load is removed is known as its elastic limit. In Fig.2.8 OA represents the proportionality limit of the specimen. The section AB is residence deformation. Magnitude of stress corresponding to point B is the elastic limit. Section CD indicate fluidity. Under stress closed to point K a specimen can be broken.

In general, any change in shape or size is called a deformation. The following types of deformation are known: extension, compression, shear, bending and torsion. However, any deformation can ultimately be reduced to two simple types: extension and shear. Consider deformation of extension . The change in length is expressed

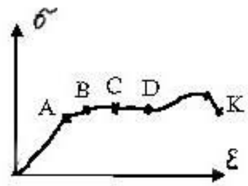


Fig.2.8

as

$$\Delta l = \frac{1}{E}\frac{Fl_0}{S} \qquad (2.43)$$

where **S** the cross-sectional area of the specimen, **E** is the Young′s modulus and l_0 is the original length. Fig.2.8

Expression (2.43) can be rewritten as

$$F = k\,\Delta l_0 \qquad (2.44)$$

where

$$k = \frac{E\,S}{l_0}$$

Further substituting (2.43) and (2.44) into (2. 42) we get

$$\sigma = E\,\varepsilon \qquad (2.\ 45)$$

Last expression is the Hooke′ s law, whicih states: **Up to the elastic limit, stresses are proportional to their corresponding strains.**

Mechanical stress produces also the transverse compression. Cross- sectional area of a body changes due to its longitudinal changing. This sort of deformation is characterised by Poussin s coefficient χ

$$\frac{\Delta\,\mathbf{d}}{\mathbf{d}} = \chi\,\frac{\Delta\,\mathbf{l}}{\mathbf{l}} \qquad (2.\ 46)$$

Deformation of shear is described by the equation

$$\tau = G\,\gamma \qquad (2.\ 47)$$

where τ – the tangent stress, G – modulus of shear and γ – shear deformation . The relation among these quantitues and Young′ s modulus can be put as

$$G = \frac{E}{2\,(1+\chi)} \qquad (2.\ 48)$$

§ 2.14. Frictional forces

A forces appearing in bodies contacting in direct are called frictional forces. Frictional forces are dividing in following kinds: 1.external,2.internal,3.static, 4.sliding, 5.rolling

1. First consider the **static friction**. Suppose, that the body is not at rest on the surface of horizontal plane . If we slightly increase the external force F , then the body still does not move. This means, that static friction has increased together with F as well. If we increase F still further the body will ultimately start to move along the plane. It means, that force of static friction has its maximum and this maximum is less than the applied external force F. Thus, static friction is equal and opposite to the external force. Static friction depends on the force of reaction, with which bodies in contact press against each other. If denote the coefficient of static friction by μ_0 we can write;

$$\mathbf{F}_{s.f.} = \mu_0 \mathbf{N} \qquad (2.49)$$

where N– is the force of reaction.

Coefficient of static friction depends on the nature of material.

2. Now let consider the **sliding friction**. If the two bodies in contact are in motion relative to each other then the arising force of friction is called sliding. The external force needed for sliding of the body on the plane is less than the force required to move the body on the plane. For the case of sliding friction we have:

$$\mathbf{F}_{sl.\,f.} = \mu_1 \mathbf{N} \qquad (2.50)$$

where μ_1 is the coefficient of sliding friction. Coefficient μ_1 depends on the nature and quality of surfaces in contact and the velocity of motion.

3. To demonstrate the role of **rolling fricion** let us take the cylinder on the horizontal plane. It is not difficult to see that the external force required to shift the cylinder from the state of rest and to move is much smaller than that required for sliding this cylinder without rolling. Thus, rolling friction is much smaller than sliding frict ion.: Force of rolling friction is expressed as

$$\mathbf{F}_{R.f.} = \mu_2 \frac{N}{R} \qquad (2.51)$$

where R is the radius of the rolling body. Finally, note that relation among these coefficients of friction is as following:

$$\mu_2 < \mu_1 < \mu_0$$

§ 2.15. Motion over an inclined plane

The plane is called inclined, which forms a certain angle with the horizontal. To describe this motion we must decomposite the force of gravity in two components F_1 and F_2 as in Fig.2.9.

$$F_1 = mg\sin\alpha; \quad F_2 = mg\cos\alpha \qquad (2.52)$$

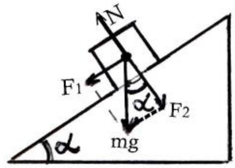


Fig.2.9

where α – is the angle of repose. Since $F_2 = N$ (the force of plane′s surface reaction) the frictional force

$$F_{frict.} = \mu R = \mu mg\cos\alpha \qquad (2.53)$$

If the friction is absent then body left to itself would slide down the plane.In order to hold the body some other force should be applied to it.

If a frictional force $F_{frict.}$, there is only then moving to become of body in states of rest or move depends on the ratio between forces F_1 (directed downwards) and $F_{frict.}$: there are some cases:

a) If $F_1 < F_{frict.}$, the body is at rest

b) If $F_1 = F_{frict.}$ the body is in uniform motion.

c) If $F_1 > F_{frict.}$ the body is in uniform accelerated motion.

$$a=g(\sin\alpha - \mu\cos\alpha) \qquad (2.54)$$

Consider the following cases:

1. Force of tension is equal to zero.

a) Accelerated downward motion.

$$a=g(\sin\alpha - \mu\cos\alpha) \qquad (2.55)$$

b) Decelerated downward.motion

$$a=g(\mu\cos\alpha - \sin\alpha) \qquad (2.56)$$

2. Force of tension differs from zero:

a) Accelerated upward motion

$$ma=T - mg\sin\alpha - \mu mg\cos\alpha \qquad (2.57)$$

b) Accelerated downward motion

$$ma= mg\sin\alpha - \mu mg\cos\alpha - T \qquad (2.58)$$

c) Decelerated upward motion

$$ma= mg\sin\alpha + \mu mg\cos\alpha - T \qquad (2.59)$$

d) Decelerated downward motion

$$ma= T - mg\sin\alpha - \mu mg\cos\alpha \qquad (2.60)$$

For the inclined plane the following relations are true.

$$\mathrm{tg}\alpha=\frac{h}{S}, \quad \sin\alpha=\frac{h}{l} \quad \text{and} \quad \cos\alpha=\frac{s}{l} \qquad (2.61)$$

where *S*-is the base of inclined plane, *h*-its height, and *l*-is the length.

CHAPTER 3

Statics

§3.1. Rest and equilibrium

Statics deals with the problems concerning the motion of bodies, from point of view of equilibrium conditions. To solve some of problems on engineering constructions such as bridges, inclined planes, we must know equilibrium conditions for forces.

First consider the difference between **rest** and **equilibrium**; A body is said to be in state of rest when its **velocity** is zero

Rest; $\upsilon = 0$ for translational motion,
(3. 1)
$\omega = 0$ for rotational motion.

A body is said to be in a state of equilibrium when its **acceleration** is zero

Equilibrium; $a = 0$ for translational motion,
(3. 2)
$\beta = 0$ for rotational motion

A body may be at rest without being in equilibrium or it may be in equilibrium without being at rest.

3.2. First condition of equilibrium

We consider equilibrium condition for a body without a fixed axis.

a) The sum of all forces upward must be equal to the sum of the forces downward.

Writing this fact by an equation for five forces F_1, F_2, F_3, F_4, and F_5 applied on the body as shown in Fig. 3.1

$$F_{1y} + F_{2y} + F_{3y} = F_{4y} + F_{5y}.$$

Transposing gives

$$F_{1y} + F_{2y} + F_{3y} - F_{4y} - F_{5y} = 0.$$

Hence the equation may be written in the briefer form;

$$\sum F_y = 0 . \qquad (3.3)$$

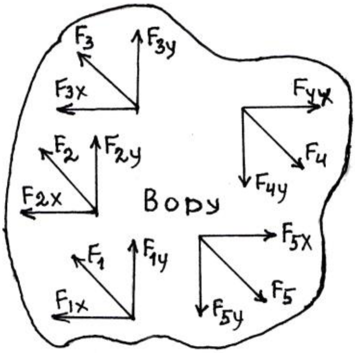


Fig.3.1

b) The sum of the forces to right must equal the sum of the forces to left

$$F_{4x} + F_{5x} = F_{1x} + F_{2x} + F_{3x} .$$

Transposing

$$F_{4x} + F_{5x} - F_{1x} - F_{2x} - F_{3x} = 0 \quad \text{or}$$

$$\sum F_x = 0 \qquad (3.4)$$

In the case of motion in space we would take a third, or z-axis perpendicular to the X – and Y –axes; it would then be necessary also that

$$\sum F_z = 0 \qquad (3.5)$$

Thus, conditions (3. 3), (3.4) and (3.5) may be extended to any number of forces acting on a body.

§3.3. Equilibrium of a body due to action of three forces

Lami's theorem; When a body is held in equilibrium by three forces, each force is proportional to the sine of the angle between the other two forces:

$$F_1 : F_2 : F_3 = \sin\varphi_1 : \sin\varphi_2 : \sin\varphi_3 . \quad (3.6)$$

If a body is in equilibrium under the action of three forces

1. The forces must be lie in a plane.
2. The forces must meet in a point. The force polygon must be a closed triangle.

§3.4. Torque or moment of force

The product of the magnitude of force F and the distance from the axis to the line of action of the force is known as the **torque** or **moment of force**. The remembered distance is called the arm of force. Denoting the moment of force by M, we can write

$$\vec{M} = \vec{F}l \qquad (3.7)$$

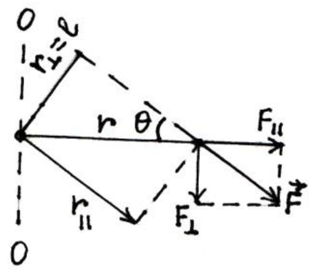


Fig.3.2

It can be seen from Fig. 3.2 that ***l* = *r* sin *θ*** and r is the position vector. Then we can write

$$M = rF_{\perp} \quad ; \qquad M = r_{\perp}F \quad ; \qquad M = \frac{F_{\perp} l}{\sin\theta} = \frac{F_{\perp} r_{\perp}}{\sin\theta} \qquad (3.8)$$

where $F_{\perp} = F\sin\theta$ is the component of F perpendicular to r and $r_{\perp} = r\sin\theta$ is the component of r perpendicular to F. If angle θ is equal to zero or π, then no perpendicular component of F exists i.e., $F_{\perp} = F\sin\theta = 0$, so the line of action of force F passes through the origin and the moment arm $r_{\perp}$ about the origin is zero. In this case equations (3. 8) implie that the torque M is zero.

§3.5. Second condition of equilibrium. A body with a fixed axis

We consider equilibrium condition for a body with a fixed axis. If a body does not rotate or rotates with a constant angular velocity. the body is said to be in rotational equilibrium. When a torque is applied to a body about an axis, it produces a rotation in it. So the second condition of equilibrium is stated as follows: A body will be in rotational equilibrium only if the sum of all the external torques acting on the body about any arbitrary axis is zero.

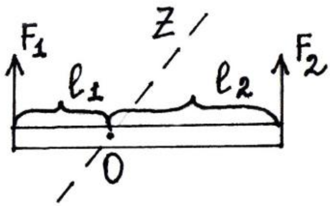


Fig.3.3

$$\vec{M}_1 + \vec{M}_2 + ... + \vec{M}_n = 0 \qquad (3.9)$$

Consider two forces F_1 and F_2 applied to a body as shown in Fig.3.3.The force F_1 produces counter clockwise torque $M_1 = F_1 l_1$ while F_2 produces anticlockwise torque $M_2 = -F_2 l_2$ about an axis through O along Z axis. For this body to be in rotational equilibrium

$$M_1 + M_2 = F_1 l_1 - F_2 l_2 = 0 \;; \quad F_1 l_1 = F_2 l_2 \qquad (3.10)$$

This formula expresses the **lever rule.**
Thus, for the equilibrium of a body with a fixed axis it is necessary that the product of the magnitude of the force and the distance between the axis and the line of action of force be the same for the two forces.

§3.6. Addition of forces along the same straight line

If two forces act along the same line, then resultant force is equal in magnitude to the sum or difference(in dependence of their directions) of the forces F_1 and F_2 are directed as a larger force.

$$\mathbf{F_R = F_1 + F_2} \quad \text{or} \quad \mathbf{F_R = F_1 - F_2} \qquad (3.11)$$

§3.7. Forces with the same and different points of application

If the two forces were applied to the same point, their resultant force is defined with the rule of parallelogram. The resultant force is equal to the diagonal of parallelogram , which sides are the given forces. From Fig.3.4a it follows that

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\gamma}\ , \qquad (3.12)$$

where $\gamma = \alpha + \beta$ – the angle between the forces F_1 and F_2.

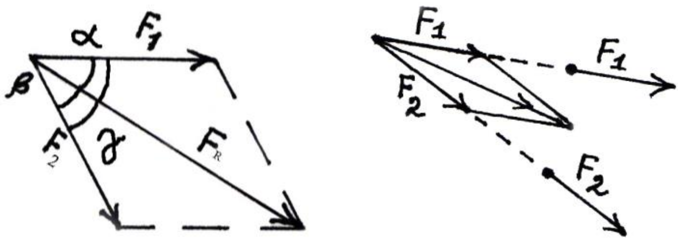


Fig.3.4

If the forces are perpendicular, i.e. $\gamma = 90°$, then

$$F_R = \sqrt{F_1^2 + F_2^2} \quad \text{(Pythagorean theorem) .} \qquad (3.13)$$

Direction of F_R is defined by the relations:

$$\sin\alpha = \frac{F_2}{F_R}\sin\gamma \qquad \text{and} \qquad \sin\beta = \frac{F_1}{F_R}\sin\gamma \quad . \qquad (3.14)$$

In order to find the resultant of two forces shown as in Fig.3.4b we must continue lines of action until they will meet at the same point. Then we may use the parallelogram rule.

§3.8. Addition of parallel and anti-parallel forces. Force couple

The resultant force F_R is equal in magnitude to the sum of the forces F_1 and F_2 if we neglect the mass of the lever. From equilibrium condition for the lever we get in Fig.3.5

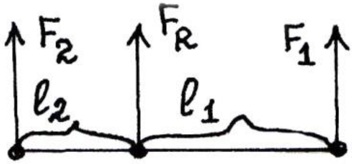


Fig.3.5

$$\frac{F_1}{F_2} = \frac{l_2}{l_1} \quad . \qquad (3.15)$$

Thus, the resultant o two parallel forces is equal to the sum of the forces, has the same direction and is applied at a point dividing the distance between the points of application of forces in inverse proportion to the applied forces.

The resultant of two anti-parallel forces is equal to the

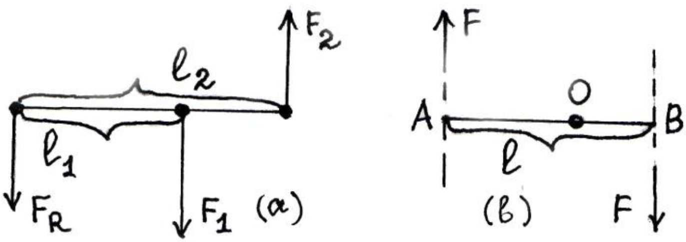


Fig.3.6

vector sum of the forces, have the same direction as the larger force and is applied at a point dividing the distance between the points of application of forces in inverse proportion to the forces of which it is composed (Fig.3.6a)

$$\mathbf{\frac{F_1}{F_2} = \frac{l_2}{l_1}}$$

Two equal and anti-parallel forces not laying on the same straight line is called the **force couple**. When the couple act on a free body then it will produce a rotation about the axis, passing through point, which is known as the centre of gravity. From the Fig. 3.6b the resultant moment M of couple is

$$M = Fl ,$$

where distance $l =$ AO + OB, O is the point, from which an axix passes.

§3.9. Decomposition of forces

In order to de-composite the force it is needed to know the direction or magnitude of components. From Fig. 3.7 it can be seen that

$$F_1 = F\frac{\sin\alpha}{\sin(\alpha+\beta)} \qquad F_2 = F\frac{\sin\beta}{\sin(\alpha+\beta)} . \qquad (3.16)$$

In case, when the given force is to be decomposited in two perpendicular components, i.e. $\alpha + \beta = 90°$ and $\sin\beta = \cos\alpha$, from equations (3. 16) we get:

$$F = F\sin\alpha = F\cos\beta \quad \text{and}$$
$$F = F\cos\alpha = F\sin\beta \quad . \qquad (3.17)$$

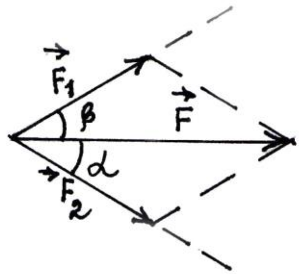


Fig.3.7

§3.10. Center of mass

Consider a particle having mass m. Its weight is acting towards the centre of the Earth. An extended object may be considered as composed of a large number of such small particles.(Fig.3.8a) Each constituent particle of the extended object is acted upon by a force directed toward the centre of the earth. Since the size of the extended object is very small as compared to that of the earth, the acceleration due to gravity may be taken as uniform over it. Therefore each particle of the object

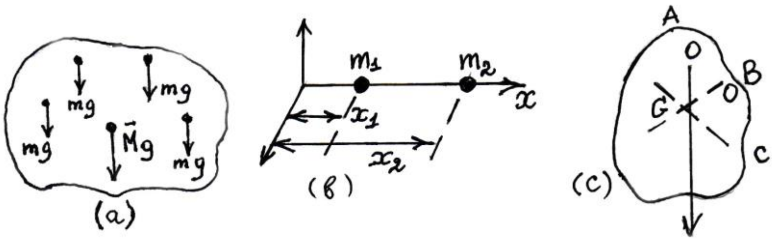


Fig.3.8

experiences the same force and these forces on the various particles are parallel to each other and are in the same direction. Hence their resultant is given by their algebraic sum., is called the weight. The point through which the resultant Mg passes is called the centre of gravity.

Since *g* is nearly uniform over the extended object, it can be concluded from W=Mg that the masses of the constituent particles can be replaced by the total mass of the object at its **center of gravity** in a uniform gravitational field. This point is also known as the centre of mass. It must be remembered that if the gravitational field is not uniform over the extended object or system of point masses, the centre of gravity and centre of mass will not coincide. The centre of gravity is called the point about which the sum of moments of the forces of gravity acting on all the particles of the body is equal to zero. The expression for the position vector **R** of the centre of gravity has the form:

$$R = \frac{m_1 r_1 + m_2 r_2 + \cdots + m_n r_n}{m_1 + m_2 + \cdots + m_n} \qquad (3.18)$$

where m_1, m_2 ...m_n are the masses. and r_1, r_2 ..... r_n are the position vectors of the particles. When the particles of the body are considered to be distributed along a straight line, for example x – axis we get the following expression for the position of the centre of gravity.

$$X=\frac{m_1x_1+m_2x_2+\cdots+m_nx_n}{m_1+m_2+\cdots+m_n} \quad . \qquad (3.19)$$

Here x_1 , x_2 ... x_n are the coordinates of the particles .

In particular, in case of two dimensions for the system of two bodies (Fig.3.8b) we can write:

$$x=\frac{m_1x_1+m_2x_2}{m_1+m_2}\ ; \qquad y=\frac{m_1y_1+m_2y_2}{m_1+m_2}\ . \qquad (3.20)$$

In these formulas masses are used instead of weights since the acceleration of the gravitational force cancels out. Then we can conclude that the point found has an objective significance that does not depend on the gravitational conditions of weightlessness. Thus we speak of the centre o inertia or centre of mass of a body instead of its centre of gravity.

An accurate method for determination the centre of mass of a flat plate is to suspend it in turn from points A, B, and C as shown in Fig.3.8c so that it hangs freely in each case. If a plumb line is hung from the same support and lines are drawn to indicate the vertical position of the thread, then the point of intersection of the lines gives the centre of mass of the plate. The positions of the centre of mass of some familiar objects are given below:

.

Name of object.	Position of centre of mass.
1. Uniform rod	Centre of rod
2. Circular plate	Centre of plate
3. Plate (rectangular	Intersection of their diagonals or parallelogram in shape) intersection of the lines joining the midpoints of opposite sides
4. Triangular plate	Intersection of medians
5. Rectangular block	Intersection of diagonals

6. Sphere	Centre of sphere
7. Cylinder	Midpoint of the axis

§3.11. Types of equilibrium

Equilibrium of a body is called **stable** if after a small displacement of a body from original position are appeared forces or moments of forces returning the body to its original position. In the stable equilibrium the potential energy of a body is a minimum, and for any overtoaching motion its center of mass raises upward(Fig 3.9 a)

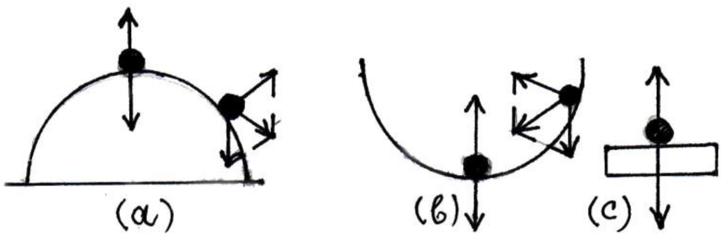


Fig.3.9

If the forces or moments of forces which return the body after very small shock to its original position are not appeared then the body will no longer stay nearly the original position and this equilibrium is called **unstable.** Its potential energy is a maximum, for any motion its centre of mass falls down (Fig.3.9 b)

If the body is at rest on a horizontal support a displacement of the body does not disturb its equilibrium at all, since the force exerted by the plane on the body and the force of gravity balance each other in any position of the body. Such an equilibrium is known as **neutral**, or **indifferent** equilibrium. Its potential energy is the same in all positions, and the height of its centre of mass remains unchanged (Fig.3.9 c).

Criterion of stability. A body will be in a condition of stable equilibrium provided the line of action of the resultant of the forces upon it falls within its base, and any motion tends to raise its center of mass.

§3.12. Equilibrium of a body on the inclined plane

From Fig. 3.10 it follows, that

$$\mathbf{P_t} = \mathbf{P}\ \frac{\mathbf{h}}{\mathbf{l}} = \mathbf{P \sin}\,\alpha \qquad \text{and}$$

$$\mathbf{P_N} = \mathbf{P}\ \frac{\mathbf{s}}{\mathbf{l}}\ \mathbf{\cos}\,\alpha\ , \qquad (3.20)$$

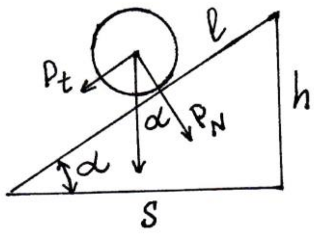


Fig.3.10

where P – the force of gravity, P_N – the perpendicular (normal) component of P to the inclined plane, s – the length of base of plane, l – the length of inclined plane, h – is the height of inclined plane and α – angle of repose of plane.

§3.13. Pulleys

Usually two types of pulleys are distinguished: **fixed** and **moveable** pulleys. The fixed pulley allows to change only the direction of the applied force as is shown in Fig. 3.11a.

An acceleration of system of loads can be calculated as

$$a=\frac{m_2-m_1}{m_2+m_1}g \qquad (3.21)$$

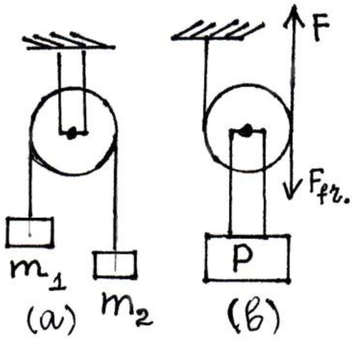


Fig.3.11

where m_1 and m_2 represent the sum of mass weighted on the left and right hand sides of pulley respectively, g – is the acceleration due to gravity.

The moveable pulley gives a gain in force. When a moveable pulley is at rest or in uniform rotation the sum of all the applied forces and the sum of all the torques is equal to zero. From Fig.3.11b it follows P=2F and hence

$$F=\frac{\mathbf{P}}{\mathbf{2}}\,, \qquad (3.22)$$

If the system is acted by a certain friction then equation (3.22) is rewritten as

$$F-F_{fric.}=\frac{\mathbf{P}}{\mathbf{2}}\,. \qquad (3.23)$$

CHAPTER 4

Work, power and energy

§4.1. Work

When work is done, the force that does the work is the net force. If the force is applied to an object at an angle with the direction of motion, the net force is the component of the force that acts in the direction of motion (Fig.4.1)

Work is done when an object moves some distance due to an applied force. General expression for the work done is given as

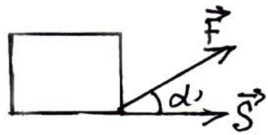


Fig.4.1

$$W=Fs\cos\alpha \tag{4.1}$$

Here α – is the angle between vectors of displcement and force. The product **s** × **cos** α is the projection of displcement in the direction of the force. Denoting this projection by s_F , we get another expression for the work

$$W= Fs_F \tag{4.2}$$

When directions of force and displacement coincide, work is found by the product of magnitudes of the force F and displacement s

$$W = Fs \qquad (4.3)$$

If the force and displacement are at right angle ($\alpha = 90°$) to each other, the work done is zero **(** W= 0 **).**

The work of gravitational force is calculated by

$$W_{grav.} = GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \quad , \qquad (4.4)$$

where r_1 and r_2 are position vectors and charcterise the initial and final positions of moving object.

If the distance between points increases the work is negative ($W < 0$, $r_2 > r_1$), and when move toward each other is positive. ($W > 0$, $r_2 < r_1$) .

If the trajectory of the point mass is closed ($r_1 = r_2$) then the work done is zero.

When a mass m is removed from the Earth′s surface up to height h smaller than the Earth ′s radius the work is defined by

$$W = -mgh \quad . \qquad (4.5)$$

The work done by the frictional force is calculated by

$$W = \mu RS \quad , \qquad (4.6)$$

where R – the force of reaction and μ is the coefficient of friction. When a body moves upwards ower the inclined plane work done is opposed to the gravitational and frictional forces. In this case the work done is given by the formula:

$$W = mgs\,(\sin\alpha + \mu\cos\alpha) \quad , \qquad (4.7)$$

where s is the length of the inclined plane.

When a constant torque M acts through an angle φ in rotational motion the work done is

$$W = M\varphi \quad . \qquad (4.8)$$

The work done by an elasic spring equals to

$$W = \frac{k x_2^2}{2} - \frac{k x_1^2}{2} \quad . \qquad (4.9)$$

The ratio of useful work to the total work is called the **coefficient of useful work** (**efficiency**)

$$\eta = \frac{W_{useful}}{W_{total}} \quad . \qquad (4.10)$$

The SI unit of work is the work done by the force of one newton over the displacement of one meter. This unit is known as a joule J . (1 J = 1 N x 1 m)

§4.2. Power

The power is a physical quantity, equal the ratio of work done by the time interval

$$P = \frac{W}{t} \quad . \qquad (4.11)$$

When power is constant the work is calculated as the area of rectangle between the power graph and the time axis (Fig.4.2).

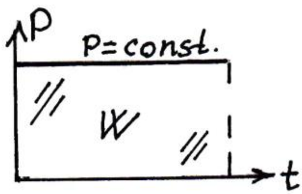


Fig.4.2

Having put (4.3) into (4.11) and taking into account υ=s/t for power gives

$$P = F\upsilon \quad . \qquad (4.12)$$

If the directions of velocity and the force coincide, the power is equal to the force multipiled by the velocity of the point of application of the force. In particular, for the power, developed by the motor of machine we have

$$P = 2\pi f M \qquad (4.13)$$

where f - is the frequency, M=Fr – the torque, and r – is the radius of pulley.

Power is measured in watts (Wt). A watt is one joule per second: 1 Wt = 1 J / (1 sec.)

§4.3. Potential and kinetic energies

Energy is a physical quantity characterising the ability of a body to do work. When a body changes its state the energy changes as well.

Energy is measured in the units of work, viz. in joules. In mechanics are considered two type of energy: *potential* and *kinetic.*

Potential energy is determined only by the position of the body. A body lifted above the Earth′s surface has a potential energy, equal to the product of the force of gravity and the height to which it has been lifted. For the height very smaller as compared to the Earth′s radius the potential energy is expressed as

$$E_P = m\,g\,h\,. \qquad (4.14)$$

When the height h can be compared with the Earth ′s radus R_E the acceleration due to gravity g depends on the distance. Then dependence of mg versus the h will have the form as illustrated in Fig.4.3a When g is constant the E_P is equal in magnitude to the area of polygon shown in Fig.4.3b.

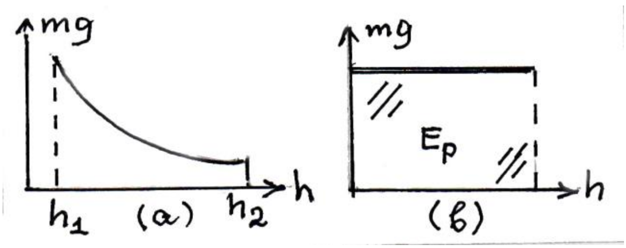


Fig.4.3

Potential energy of elastic spring is determined by the formula

$$E_P = k\frac{\mathbf{x}^2}{\mathbf{2}}\,, \qquad (4.15)$$

where x – is the change of length of the specimen and k is the coefficient of elasticity or spring constant.

Kinetic energy. Any moving body has a certain ability to do work, i.e. a certain energy due to its motion, which is called a kinetic energy. Consider as an exsample of an object of mass m resting on a frictionless surface. A constant force F acts on it through a displacement **s** . The force will accelerate the object in accordance with Newton’s second law of motion *F= ma*. If we multiply both sides of this equation by s , the left side of the equation represents work done on the mass.

$$Fs = mas \qquad (4.16)$$

The speed of an object starting from rest is given by $\upsilon^2 = 2as$ (see 1.19) This expression can be rearranged to read $as = \upsilon^2 / 2$ Substituting into the equation Fs = mas, we get

$$Fs = \frac{m\upsilon^2}{2} \qquad (4.17)$$

The expression (4.17) relates the work done on the mass to its resulting speed. The right hand side of the equation states the amount of work mass m moving with velocity υ can do as it is brought to rest. The energy the object has because of its velocity equals the work that was done to give the mass its velocity. Thus, the quantity $m\upsilon^2 / 2$ is called the kinetic energy E_K of the object.

$$E_K = \frac{m\upsilon^2}{2} . \qquad (4.18)$$

Fig. 4.4a shows the dependence of E_K on υ^2. Here $tg\ \theta = m/2$. If the object starts moving with a certain initial velocity then the work performed is defined by the difference of kinetic energies

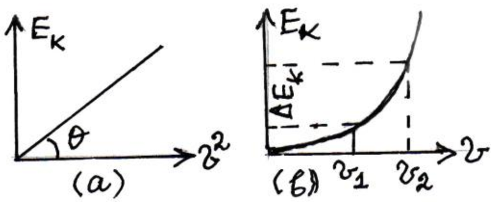


Fig.4.4

$$W = E_2 - E_1 \qquad (4.19)$$

The dependence of E_K on υ is demonstrated in Fig. 4.4b as a parabola.

§4.4. Law of conservation of mechanical energy

The total mechanical energy of the body is the sum of its kinetic and potential energies. The law of conservation of energy states that the total energy of a system cannot change, unless work is done on the system. Within an isolated system energy can change from one form to another, but the total amount of energy always remains the same:

$$E_{total} = E_{k,\text{max.}} + E_{p,\text{min.}} \tag{4.20}$$

$$E_{total} = E_{k,\text{min.}} + E_{p,\text{max.}} \tag{4.21}$$

Energy can never be lost by a system. Let us consider the free falling motion of a body. In Fig.4.5a are given time dependencies of kinetic an potential energies. The dependencies of these energies versus the height are shown in Fig.4.5b. It is seen that in both cases a decreasing in potential energy causes an equal increase in kinetic energy.

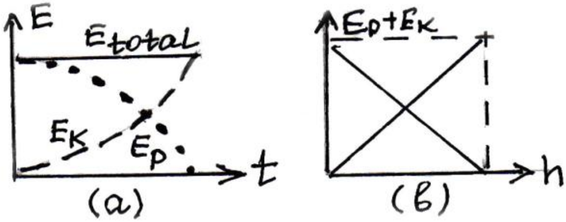


Fig.4.5

As can be seen from Fig.4.5a kinetic energy increases ,but potential energy decreases with time upon free fall.

Work, power and energy

4.1. Work

When work is done, the force that does the work is the net force. If the force is applied to an object at an angle with the direction of motion, the net force is the component of the force that acts in the direction of motion (Fig.4.1) Work is done when an object moves some distance due to an applied force. The general expression for work is written in the form

$$W=Fs\cos\alpha \qquad (4.1)$$

Here α – is the angle between directions of the displacement and the force. The product **scos** α is the projection of the displacement in the direction of the force. Denoting this projection by s_F, we get another expression for work

$$W=F s_F \qquad (4.2)$$

When the directions of the force and displacement coincide $(\alpha = 0)$, work is equal to the product of the magnitudes of the force F and displacement s

$$W=Fs \qquad (4.3)$$

The work becomes negatively when vectors of force and displacement are in opposite directions. $(\alpha = 180^{\circ})$

If the force and displacement are at right angle ($\alpha = 90°$) to each other, the work done by the force is zero **(** W= 0 **).**

Work one is related to the change in both kinetic and potential energies:

$$W = E_{k,2} - E_{k,1} = \Delta E_k \quad (4.4)$$

$$W = -(E_{p,2} - E_{p,1}) = -\Delta E_p \quad (4.5)$$

Equation ( ) expresses the work- energy theorem. The net work done is equal to the change in kinetic energy. This theorem is applied to all kinds of forces.

Equation ( ) is applied to the potential forces (force of gravity, force of elasticity and Coulomb's forces) only.

Work done by forces of gravity and elasticity is independent on the object's path , therefore work done along the closed path is zero. Such force known as *conservative* forces.

For upward motion :

$\Delta E_k = W = -mgh$ (E_k *decreases as* mgh)

$\Delta E_p = -W = mgh$ (E_p *increases as* mgh)

For downward motion:

$\Delta E_k = W = mgh$ (E_k *increases as* mgh)

$\Delta E_p = -W = -mgh$ (E_p *decreases as* mgh)

For horizontal motion $\Delta E_p = 0;$ if $v = const.$ then $\Delta E_k = 0$

Work done by the *force of gravity* is defined by $W = mgh$ for both uniform and accelerated (decelerated) motions.

However work done by tractive force is defined by

$W = m(g + a)h$ (upward motion)

$W = m(g - a)h$ (downward motion)

The work of gravitational force is determined as

$$W_{grav.} = GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \quad (4.6)$$

r_1 and r_2 are position vectors and characterize the initial and final positions of removing point.

If the distance between points increases the work is negative ($W < 0$, $r_2 > r_1$), and when the points approach the work done is positive.($W > 0$, $r_2 < r_1$)

If the trajectory over which the point is removed is closed ($r_1 = r_2$) the work is equal zero.

The work done by the force of friction is calculated as

$$W = \mu NS \qquad (4.7)$$

where N – the force of reaction and μ is the coefficient of friction. When a body moves upward on the inclined plane work done is opposite to the gravitational and frictional forces. In this case the work done is expressed by the formula:

$$W = mgl\,(\sin\alpha + \mu\cos\alpha) \qquad (4.8)$$

where l is the length of the inclined plane.

Example 4.1: ***A 50 kg mass is uniformly lifted over the inclined plane, with 60 cm height, 2 m length and 60% efficiency. What force was used?***

Given: $m = 50\ kg$

$h = 60cm = 0.6\ m$

$l = 2\ m$ *Find: F*

$\epsilon = 0.6$

$g = 10\frac{m}{s^2}$

Solution: Since for the inclined plane a useful work output is the work done by the force of gravity and work input is work done along the inclined plane then efficiency is given by

$$\epsilon = \frac{mgh}{Fl}\cdot 100\%$$

Solving this formula for *F* gives

$$F = \frac{mgh}{\epsilon l} = \frac{50\cdot 10\cdot 0.6}{0.6\cdot 2} = 250\ N$$

Answer: $250\ N$

When a constant torque M acts through an angle φ in rotational motion the work done is given as

$$W = M\varphi \qquad (4.9)$$

The work performed by an elastic spring is given by

$$W = \frac{kx_1^2}{2} - \frac{kx_2^2}{2} \qquad (4.10)$$

When the spring goes away from the equilibrium state the x and E_p increase, therefore $W_{elas} < 0$. Contrary when the spring approaches to the equilibrium state the x and E_p decrease and $W_{elas} > 0$

The ratio of useful work to the total work is called the **mechanical efficiency**

$$\epsilon = \frac{useful\ work\ output}{work\ input} = \frac{W_{us.out}}{W_{inp}} \cdot 100\% \qquad (4.11)$$

The SI unit of work is the work done by the force of one newton over the displacement of one meter. This unit is known as a joule (J). 1 J = 1 N x 1 m

4.2. Power

Power is the rate of doing work. The average power is the work done divided by the time it takes to do the work, or work per unit time:

$$P = \frac{W}{t} \qquad (4.12)$$

When power is constant the work is calculated as the area of rectangle between the power graph and the time axis (Fig.4.3).

When force is constant we can take into consideration the formula W=Fs and the ration **S / t** represents velocity υ. Then instead of (4. 12) we get

$$P = F\upsilon \qquad (4.13)$$

If the directions of velocity and the force coincide, the power is equal to the force multiplied by the velocity of the point of application of the force. A useful power output due to force of gravity is given by

$$P = mgv \qquad (4.14)$$

If the power input of a engine is N then its mechanical efficiency is calculated by

$$\epsilon = \frac{mgv}{N} \cdot 100\% \qquad (4.15)$$

Example 4.2: ***A 1500 kg mass is lifted to 20 m height by a machine with*** 40% ***mechanical efficiency. What is the power of its engine?***

Given: $m = 1500\ kg$

$h = 20\ m$

$\epsilon = 0.4$ *Find: N*

$t = 50\ s$

$g = 10\frac{m}{s^2}$

Solution: A formula (4.15) is used;

$$\epsilon = \frac{mgv}{N} \cdot 100\%$$

Solving for N gives

$$N = \frac{mgv}{\epsilon} = \frac{mg(\frac{h}{t})}{\epsilon} = \frac{mgh}{\epsilon t} = \frac{1500 \cdot 10 \cdot 20}{0.4 \cdot 50} = 15000W$$

Answer: $15kW$

In particular, for the power, developed by the engine of machine we get

$$P = 2\pi\nu M \qquad (4.16)$$

where ν is the frequency, M=Fr – the torque, and r– is the radius of pulley.

Power is measured in watts (W). A watt is one joule per second: 1 W = 1 J / 1 sec.

Example 4.3: ***A 48 kg mass is lifted by the moveable pulley. What is the mechanical efficiency of pulley if the a force of*** $\mathbf{300\ N}$ ***was required.***

Given: $m = 48\ kg$

$F_{tr} = 300\ N$ Find: ϵ

$g = 10\frac{m}{s^2}$

Solution: Using Eq.(3.36) and Eq. (3.35) gives

$$\epsilon = \frac{m_l g h_l}{F_{tr} \cdot h_{rope}}(100\%) = \frac{m_l g h_l}{F_{tr} \cdot 2h_l}(100\%) =$$

$$= \frac{48 \cdot 10}{2 \cdot 300} \cdot 100 = 0.8 \cdot 100 = 80\%$$

Answer: 80%

Mechanical efficiency of inclined plane is expresses by

$$\epsilon = \frac{m_I g h}{F_{trac} s} \cdot 100\% = \frac{1}{1+\mu ctg\alpha} \qquad (4.17)$$

where μ – is the coefficient of frictional force.

4.3. Potential and kinetic energies

Energy is a physical quantity characterizing the property of an object to do work. When an object changes its state the energy changes as well.

Energy is measured in the units of work, viz. in joules. In mechanics are considered two type of energy: **potential and kinetic** energies.

Potential energy is associated with location, or position, because a force is required and work is done in moving an object from one position to another. A body lifted above the Earth′s surface has a potential energy, equal to the product of the force of gravity and the height to which it has been lifted. For the height very smaller as compared to the Earth′s radius the potential energy is expressed as

$$E_P = mgh \qquad (4.18)$$

When the height h can be compared with the Earth ′s radius R_E the acceleration due to gravity g depends on the distance. Then dependence of mg upon the h will have the form as illustrated in Fig.4.4 When g is constant the E_P is equal in magnitude to the area of polygon shown in Fig.4.5.

Potential energy of elastic spring is determined by the formula

$$E_p = k\frac{x^2}{2} \qquad (4.19)$$

where x – is the change of length of the specimen and k is the coefficient of elasticity (or it is called spring constant)

Kinetic energy. Any moving body has a certain property to do work, i.e. a certain energy due to its motion, which is called a kinetic energy. Consider as an example an object of mass ***m*** resting on a frictionless surface. A constant force F acts on it through a displacement **s** . The force will accelerate the object in accordance with Newton's second law $F=ma$. If we multiply both sides of this equation by displacement s , the left hand side of this equation represents work done on an object.

$$F\,s = mas \qquad (4.20)$$

The speed of an object starting from rest is $\upsilon^2 = 2as$ This expression can be rearranged to read as $\boldsymbol{as = \upsilon^2/2}$. Substituting into the equation $Fs = mas$, gives

$$F\,s = \frac{\mathbf{m}\,\upsilon^2}{\mathbf{2}} \qquad (4.21)$$

The expression (4.17) relates the work done on the mass to its resulting speed. The right hand side of the equation states that amount of work done on mass ***m*** moving with velocity υ can do as it is brought to rest. The energy the object has because of its velocity equals the work that was done to give the mass its velocity. Thus, the quantity $\frac{mv^2}{2}$ is called the *kinetic energy* E_k of an object.

$$\mathbf{E}_{\mathbf{K}} = \frac{\mathbf{m}\,\upsilon^2}{\mathbf{2}} \qquad (4.22)$$

Fig. 4.6 shows the dependence of E_k versus υ^2. Here tg $\theta = \frac{m}{2}$. The dependence of E_k on υ is demonstrated in Fig. 4.7 as a parabola. Kinetic energy can be presented by one of following expressions

$$E_k = \frac{mv^2}{2} = \frac{pv}{2} = \frac{v^2}{2m} \qquad (4.23)$$

where *p*-is the linear momentum.

If the object starts moving with a certain initial velocity then the work done is defined by the difference in kinetic energies

$$W = E_2 - E_1 \qquad (4.24)$$

4.4. Law of conservation of mechanical energy

The total mechanical energy of the body is the sum of its kinetic and potential energies.

$$E_t = E_k + E_p = \frac{mv^2}{2} + r \qquad (4.25)$$

The law of conservation of energy states that the total energy of a system cannot change, unless work is done on the system. Within an isolated system energy can change from one form to another, but the total amount of energy always remains the same. Energy can never be lost by a system. Let us consider the free falling motion of a body. In Fig.4.8 are given time dependencies of kinetic an potential energies. The dependencies of these energies versus the height ar shown in Fig.4.9. It is seen that in both cases a decreasing in potential energy causes an equal increase in kinetic energy.

If an object moves from one position to another the law of conservation of mechanical energy is written as

$$\frac{mv_1^2}{2} + mgh_1 = \frac{mv_2^2}{2} + m \qquad (4.26)$$

At the Earth's surface $E_p = 0$, therefore $E_k = E_{total} = \frac{mv_0^2}{2}$

At the highest point $E_k = 0$; therefore $E_p = E_{total} = mgh_{max}$

Hence, one may conclude that

$$\frac{mv_0^2}{2} = mgh_{max} \quad \text{or} \quad v_0 = \sqrt{2gh_{max}} \qquad (4.27)$$

Example 4.4: ***A body is thrown vertically downward from a certain height with $v_1 = 8\frac{m}{s}$ initial velocity . Find the height if the final velocity of the body is $v_2 = 10\frac{m}{s}$.***

Given: $v_1 = 8\frac{m}{s}$

$v_2 = 10\frac{m}{s}$. *Find: h*

$g = 10\frac{m}{s^2}$

Solution: We use Eq. (4.26) and since body's second position is the Earth's surface then $h_2 = 0$. Second term in the right-hand side of (4.26) is ignored. Therefore

$$\frac{mv_1^2}{2} + mgh = \frac{mv_2^2}{2}$$

In this equation masses are cancelled out. Equation for *h* is given by

$$h = \frac{1}{2g}(v_2^2 - v_1^2) = \frac{1}{20}(100 - 64) = \frac{36}{20} = 1.8\ m$$

Answer: $1.8\ m$

For an object of mass ***m*** suspended from the spring with constant ***k*** total mechanical energy is the sum of kinetic energy of an object and potential energy of spring.

$$E_{total} = \frac{mv^2}{2} + \frac{kx^2}{2} \qquad (4.28)$$

The spring's potential energy may be given by one of the following expressions;

$$E_p = \frac{kx^2}{2} = \frac{F \cdot x}{2} = \frac{F^2}{2k} \qquad (4.29)$$

If $x = x_{max}$ $E_k = 0$ and $E_p = E_{max} = E_{total}$

And when $x = 0$, $E_p = 0$ and $E_k = E_{max} = E_{total}$

From these formulas it follows that

$$\frac{kx_m^2}{2} = \frac{mv_0^2}{2} \quad \text{or} \quad v_m = \sqrt{\frac{k}{m}}\, x_m \qquad (4.30)$$

Note that for both cases (force of gravity and force of elasticity) law of energy conservation states that total mechanical energy is the sum of instantaneous values of kinetic and potential energies:

$$E_{total} = E_{k,ins.} + E_{p,ins.} \qquad (4.31)$$

Example 4.5: ***An object is thrown vertically upward with initial velocity*** $\boldsymbol{v_0 = 20\ m/sec}$ ***. At what height its potential energy*** $\boldsymbol{E_{p,ins.}}$ ***is equal to*** 25% ***of kinetic energy*** $\boldsymbol{E_{k,ins.}}$ ***?***

Given: $\boldsymbol{v_0 = 20\ m/sec}$

$\boldsymbol{E_{p,ins.} = 0.25 E_{k,ins.}}$ *Find: h*

Solution: As can be seen the initial kinetic energy plays the role of total mechanical energy. Hence, using the formula (4.31) gives

$$E_{total} = E_{k,ins.} + 0.25\, E_{k,ins.}$$

Or

$$\frac{mv_0^2}{2} = 1.25 \frac{mv^2}{2} \qquad (a)$$

On the other hand

$$E_{p,ins.} = 0.25E_{k,ins.} = 0.25\ \frac{mv^2}{2} \qquad \text{(b)}$$

From equation (b) we get

$$\frac{mv^2}{2} = 4mgh \qquad \text{(c)}$$

Substituting Eq. (c) in Eq.(a) gives

$$h = \frac{v_0^2}{2 \cdot 4 \cdot 12.5} = \frac{20^2}{100} = \frac{400}{100} = 4\ m$$

Answer: $4\ m$

CHAPTER 5

Mechanical oscillations

§5.1. Principal definitions

Motion of bodies repeated exactly or approximately through equal time interval is called the mechanical oscillation. As an examples of mechanical oscillations the motion of mathematical pendulum or a motion of the bob suspended from a spring may be considered. Oscillation process is characterized by its frequency (f) and period (T).

Period is called the minimum time interval through which the motion of body is repeated. Period is measured in seconds.

Frequency is the number of oscillations per unit time and is inversly proportional to the period.

$$f = \frac{1}{T} \quad . \qquad (5.1)$$

The frequency is measured in SI by Hz: 1 Hz=1 sec $^{-1}$.

Cyclic (or angular) frequency is related with the frequency ν as

$$\omega = 2\pi f \qquad \text{or} \qquad \omega = \frac{2\pi}{T} \quad . \qquad (5.2)$$

§5.2. Harmonic oscillations

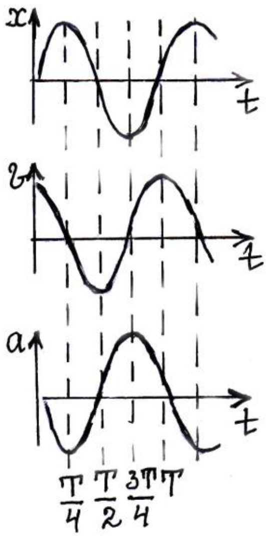
The analytical description of oscillations is given by the dependence of displacement (x) as a function of time x =f(t) (Fig.5.1)

The oscillations which are described by means of equation

$$\mathbf{x = x_m \sin(\omega t + \varphi_0)} \qquad (5.3)$$

are called the harmonic oscillation. In (1.3) the x_m is the maximum displacement from the position of equilibrium which is called the **amplitude**. A quantity ($\boldsymbol{\omega t + \varphi_0}$) is called the phase, $\boldsymbol{\varphi_0}$ – the phase at the initial moment of time (t=0) known as **phase constant** .

Velocity of a body in harmonic oscillations is determined as $\upsilon(t) = x'(t)$ and is given by



$$\upsilon_x = \upsilon_{xm} \sin\left(\omega t + \frac{\pi}{2}\right) \;, \qquad (5.4)$$

Fig.5.1

where υ_{xm} = $\mathbf{x_m \omega}$ is the maximum value (amlitude) of velocity.

Acceleration of a body in harmonic oscilations is equal to

$$a_x = a_m \sin(\omega t + \pi) \;, \qquad (5.5)$$

where $a_{xm} = x_m\omega^2$ is the amplitude of acceleration.

Temporary dependences for (a) displacement , (b) velocity and (c) acceleration are given in Fig. 5.1

§5.3. Free oscillations of a spring pendulum

The oscillations appeared due to internal forces are called free oscillations. For example, the motion of a body is performed only under action of the force of elasticity is considered to be the free oscillation. Let consider the motion of the bob suspended from the spring (Fig .5.2a) An equation of its motion can be written as

$$ma = -kx \quad , \qquad (5.6)$$

where k – is the coefficient of elasticity of spring, m – is the mass of the bob. Frequency and period of such motion are determined by the equations:

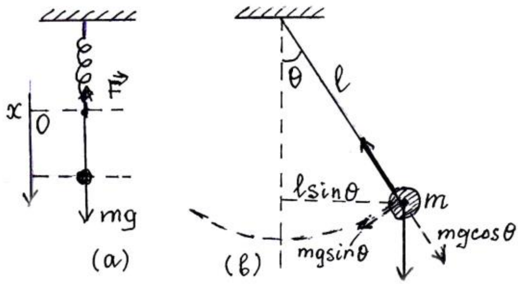


Fig.5.2

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad , \qquad T = 2\pi\sqrt{\frac{m}{k}} \; . \qquad (5.7)$$

If the mass of spring m_s is small as compared to the mass m, but cannot be neglected then the equivalent mass of such oscillating system is equal to $m + (1/3)\, m_s$ and for period we get

$$T = 2\,\pi\sqrt{\frac{m + (1/3)m_s}{k}} \; . \qquad (5.8)$$

When springs are seriesly connected the equivalent coefficient of elasticity is determined by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots\ldots + \frac{1}{k_n} \quad . \qquad (5.9)$$

If the springs are connected in parallel the total coefficient of elasticity can be calculated as

$$k = k_1 + k_2 + \ldots\ldots + k_n \qquad (5.10)$$

§5.4. Mathematical pendulum

The body of small sizes suspended from non-stretched thread which mass is smaller as compared to mass of the body is called the mathematical pendulum. The oscillation becomes harmonic when the deflections from the position of equilibrium are small and the resultant of gravitational and elasticity forces directed towards the equilibrium position appears. (Fig.5.2b). The period s dependence on the free falling acceleration has the form

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (5.11)$$

If the fulcrum from which the pendulum is suspended moves relatively the Earth then the period changes in dependent upon the acceleration.

a) a and g are in the same directions: $T = 2\pi\sqrt{\frac{l}{g-a}}$. (5.12)

If a = g then oscillations will be nondamped.

b) **a** and **g** are opposite: $T = 2\pi\sqrt{\frac{l}{g+a}}$. (5.13)

c) **a** directed horizontally: $$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$ (5.14)

d) If a certain angle there is between **a** and **g.** Then

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + a^2 + 2\,g\,a\sin\varphi}}} \;. \qquad (5.15)$$

§5.5. Physical and torsion pendulums

A real body doing an oscillations under action of its weight is called the physical pendulum. Such a body is shown in Fig..5.3a from which can be seen that force of gravity acts to the center of gravity, located at a distnace b from point of support. The torque acting on the physical pendulum about the point O is given by

$$\tau = -\,mgb\sin\vartheta \;. \qquad (5.16)$$

On the other, according to the Newton s second law for rotational motion the torkue is

$$\tau = I\,\alpha \;, \qquad (5.17)$$

where I is the moment of inertia and α is the angular acceleration.

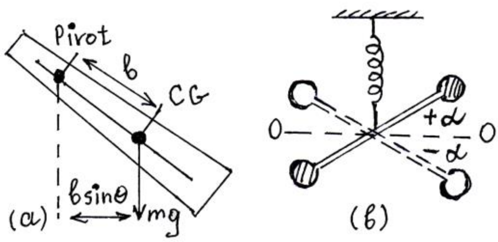


Fig.5.3

Solving (5.16) and (5.17) for the period of physical pendulum we obtain

$$T = 2\,\pi\sqrt{\frac{I}{mgb}} \;. \qquad (5.18)$$

The point located at the distance L = I / (mb) from point of support on the line, passing through the center of gravity is called the **rocking center**. The rocking centre of specimen of length l is placed at a distance L = $(2/3)\,l$ from the rotational axis.

A comparison of formulas (5.11) and (5.18) shows that a period of mathematical pendulum of length L=I / mb is the same as a period of physical pendulum..

A torsional pendulum (Fig.5.3b) is a body which performs rotary- oscillatory motion under the action of a spring. Under a certain condition such motion can also be considered harmonic. The period of a torsional pendulum is given by the formula:

$$T = 2\pi\sqrt{\frac{I}{K}} \qquad (5.19)$$

where I is the moment of inertia of the body about the axis of rotation, and K is the **torsional rigidity**, equal numerically to the torkue required to turn the body through unit angle.

§5.6. Energy conversions in oscillations

When a pendulum performes oscillations a mutual conversion of kinetic and potential energies takes place. Consider the mathematical pendulum. When the motion is directed toward the position of equilibrium its velocity rises, i.e. the kinetic energy increases. Increment of kinetic energy is due to decreasing of potential energy as a result of reducing of the distance from the Earth’s surface. When motion of a spring pendulum is directed upward or updown from equilibrium position its potential energy increases. Energies of both pendulums reach maximum value at the equilibrium position. Kinetic and potential energies are given by

$$E_k = \frac{m\upsilon_0^2}{2}\cos^2 \omega t \ , \qquad E_p = \frac{k\,x_m^2}{2}\sin^2 \omega t \ . \qquad (5.20)$$

Using the formulas (5.3) and (5.4) for the total mechanical oscillatory energy we get;

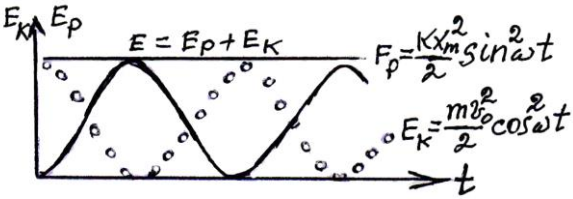


Fig.5.4

$$E = \frac{k}{2}x_m^2 = \frac{m}{2}\upsilon_0^2 \quad . \qquad (5.21)$$

Temporary dependences for (a) kinetic, (b) potential and (c) total total mechanical energies of oscillations are presented in Fig. 5.4

§5.7. Addition of coinciding and perpendicular oscillations

Suppose, displacements of two coinciding harmonic oscillations are given as

$$\mathbf{x_1 = x_{m\,1} \cos (\omega\, t + \varphi_{01})} \ , \qquad (5.22)$$

$$\mathbf{x_2 = x_{m\,2} \cos (\omega\, t + \varphi_{02})} \ . \qquad (5.23)$$

The displacement of resultant harmonic motion is

$$\mathbf{x = x_m \cos (\omega\, t + \varphi)} \qquad (5.24)$$

where x_m – refers to the amplitude of resultant oscillation, **φ -is** the phase. The quantitues x_m and **φ** are given by the following formulas:

$$x_m = \sqrt{x_{m1}^2 + x_{m2}^2 + 2x_{m1}x_{m2}\cos(\varphi_{02} - \varphi_{01})} \,, \qquad (5.25)$$

$$\varphi = \operatorname{arctg} \frac{x_{m1}\sin\varphi_{01} + x_{m2}\sin\varphi_{02}}{x_{m1}\cos\varphi_{01} + x_{m2}\cos\varphi_{02}} \,. \qquad (5.26)$$

Two perpendicular oscillations can be described by

$x = x_{mx}(\omega_x + \varphi_{0x})$; $y = x_{my}(\omega_y + \varphi_{0y})$.

Consider the cases:

$\omega_x = \omega_y = \omega$.

a) Phases are equal: $\varphi_x = \varphi_y = \varphi$.

Particle' s motion is described by the line on the xy plane, whose slope is equal to x_{my} / x_{mx}

b) $\varphi_y - \varphi_x = \pm\frac{\pi}{2}$; $x_{mx} = x_{my} = x_m$.

This motion is circular.

g) $\varphi_y - \varphi_x = \pm\frac{\pi}{2}$; $x_{mx} \neq x_{my}$.

This motion occurs along the ellipse .

d) $\varphi_y - \varphi_x \neq 0, \neq \frac{\pi}{2}, \neq \pi$.

In this case the patticle s motion is occur also over the ellipse, but amplitudes may be either equal or unequal.

§5.8. Damped oscillations

The really mechanical oscillations do not occur without energy loss. A certain part of mechanical energy is converted into the internal energy of atom s or molecule′s heat movement. Amplitude of oscillations decreases stage by stage and through any time interval is stopped.

Free mechanical oscillations are always damping. Equation of damped oscillations is expressed by

$$ma = -kx + F_{fric.} \quad , \qquad (5.27)$$

where $F_{fric.}$ is the frictional force depending on the velocity as $F_{fric.} = -r\upsilon$. Solution of (5.27) gives

$$x = x_m\cos(\omega t+\varphi) \qquad (5.28)$$

where $x_m = x_{m0}\, e^{-\delta t}$ is the amplitude,

$\delta = r/2m$ is the damping factor

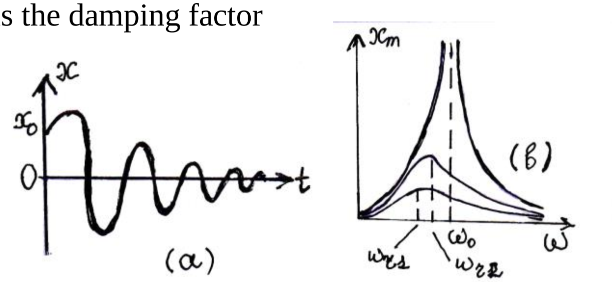


Fig.5.5

e – the base of the natural system of logarithms and $\omega = \sqrt{\omega_0^2 - \delta^2}$ the cyclic frequency.

The period of damped oscillations is

$$T = \frac{2\pi}{\sqrt{\omega_0^2 - \delta^2}} \quad . \qquad (5.29)$$

Dependence of x_m versus **t** is shown in Fig.5.5a

The damping process is characterized by the **logarithmic decrement** of damping. This quantity is determined by the ratio of two seriesly amplitudes of oscillations, splitted with the time interval equal to the oscillation s period T and is

$$\lambda = \ln \frac{x_m(t)}{x_m(t+T)} \quad . \tag{5.30}$$

The quantity λ is releated with the damping factor δ by the equation

$$\lambda = \delta T \quad . \tag{5.31}$$

§5.9. Forced oscillations

The oscillations of a body under the action of periodic driving force are called the **forced oscillations**.

An equation of forced oscillations has the form.

$$ma = -kx + F_{fric.} + F_{forced.} \quad . \tag{5.32}$$

The solution of (5.32) has the form

$$x = x_m \cos(\omega t + \varphi_0) \ , \tag{5.33}$$

where

$$x_m = \frac{f_0}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}} \tag{5.34}$$

and

$$\mathrm{tg}\,\varphi_0 = -\frac{2\beta\omega}{\omega^2 - \omega_0^2} \quad . \tag{5.35}$$

When the frequency of external driving force approaches the frequency of natural oscillations of the body, the amplitude of the forced oscillations increases sharply. This phenomenon is called **resonance.**

Cyclic frequency of forced oscillatios upon resonance equals to

$$\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2} \quad . \qquad (5.36)$$

In case of resonance for the amplitude of forced oscillations we get:

$$x_{mres.} = \frac{f_0}{2\beta\sqrt{\omega_0^2 - \beta^2}} \quad . \qquad (5.37)$$

The amplitude of forced oscillations x_m as a function of cyclic frequency is presented in Fig.5.5b

CHAPTER 6

Mechanical waves

§6.1. Velocity and wavelength of wave

A process of propagation of oscillation in space is called the wave. Mechanical waves propagate in elastic medium. In such medium there are forces of interaction, opposited any deformation among of particles. For example. pressure of gas to the valls of vessel provide up to resist the changing of volume. Mechanical waves are transversed and longitudinal. Waves which vibrations are occur perpendicular to the direction of propagation are called transversed waves. Transversed waves propagate in solids under action either of force of elasticity (shear deformation) or of forces of gravity and surface tension. Waves which wibrations are occur in the direction of propagation are called longitudinal waves. Longitudinal waves can arrise in gases, liquids and also in solids. Harmonic oscillations of... in the gas or liquid filled tube are transmitted by means of particles of matter due to force of elasticity, thus the longitudinal elastic wave propagates along the tube. It presents itself the system of compressions and rarefactions changing periodically their positions.

The propagation process both transversed and longitudinal waves is not accompanied by the transfer of substance in direction of wave propagation. The medium′s particles performe the oscillations relative the position of equilibrium in each point. However the propagation of waves is accompanied by energy transfer from one point of medium to another.

A quantity numerically equal to the distance which any point of wave surface passes per unit time. is called velocity of wave propagation

$$\upsilon = \frac{\Delta \mathbf{x}}{\Delta \mathbf{t}} \quad .$$

Vector of velocity is directed along the perpendicular to the wave surface.

Wavelength. From the definition of velocity is resulted that wave length is the distance passed by the wave surface during time interval equal the period T

$$\lambda = \upsilon T \quad . \qquad (6.1)$$

Since T=1/ f , then the (5.33) takes the form

$$\upsilon = \boldsymbol{\lambda} \mathbf{f} \quad . \qquad (6.2)$$

There is another definition of wave length: The distance between nearest one to another points oscillating with the identic phases is called **wavelength**. Phase changing Δφ during period T is equal to 2π . Hence,

$$\frac{\Delta \varphi}{\Delta \mathbf{t}} = \frac{\mathbf{2}\,\pi}{\mathbf{T}} \quad \text{or} \quad \Delta \varphi = \frac{\mathbf{2}\,\pi}{\lambda} \Delta \mathbf{x} \quad . \qquad (6.3)$$

Final expression is the relation between phase shift and difference in geometrical paths travelled by wave.

The velocity of longitudinal waves in a rod is expressed by the formula:

$$\upsilon_{\text{long.}} = \sqrt{\frac{E}{\rho}} \quad , \tag{6.4}$$

where E is the Young′s modulus and ρ is the density of medium.

§6.2. Equation of plane wave. Energy and intensity of wave

Wave equation has the form:

$$x = x_m \cos\left[(\omega\,\mathrm{t} - \mathrm{kx}) + \varphi_0\right] \tag{6.5}$$

A quantity $\mathrm{k} = \frac{\omega}{\upsilon} = \frac{2\pi}{\upsilon \mathrm{T}} = \frac{2\pi}{\lambda}$ is called the **wave number**. It is equal to the number of wave lengths placed at the distance equal to 2π unit of length. Wave energy is expressed by the formula:

$$\mathrm{E} = \frac{1}{2}\mathrm{m}\omega^2 \mathrm{x}_\mathrm{m}^2 \quad , \tag{6. 6}$$

where **m** the mass of the fixed volume.

In accordance with the (6.6) the volumic density of energy is given as

$$\varpi = \frac{1}{2}\rho\,\omega^2 \mathbf{x}_\mathbf{m}^2 \quad , \tag{6. 7}$$

where ρ - is the density of medium .

Intensity of wave is the energy transferred by it per unit time accross unit area of surface oriented perpendicular to the direction of propagation:

$$J = \frac{1}{2}\rho\upsilon\omega^2 x_m^2 \qquad (6.8)$$

here υ is the wave velocity

In accordance with (6.8) a flux of wave is given by

$$\Phi = JA \;, \qquad (6.9)$$

where A - refers to the area of surface perpendicular to the direction of propagation of wave.

§6.3. Standing or stationary waves

Consider a rubber cord which is clumped by one end. If wiggle its other end by holding it in hand we observe that the wave that was set up along the cord subsides after stopping the motion of the hand.By continuing the process a different wiggling frequency waves will be arrise in both directions so the incident wave will interferre with reflecting one. As a rule a perfect irregularity is appeared. But if the cord will be wiggled at a particular frequency *f*.. the cord even after the motion of the hand has been stopped, continues to oscillate as shown in Fig... a. In the present case we do not see any wave moving along the cord, but instead, the whole of it continues to oscillate in the loop, with its two ends being at rest and the central portion vibrating with maximum amplitude. A wave has been set up on the cord that keeps it oscillating in this fashion. These waves are known as stationary or standing waves because no wave moving on the cord is visible. Similarly as the wiggling frequency is further increased, stationary waves are set up at frequencies $2f_1$, $3\,f_1$, .., etc.. then the cord oscillates in two and three loops correspondingly.The points N_1 , N_2 , N_3 , always remain at rest and the points A_1, A_2, A_3, oscillate with the maximum amplitude. Generally we can say that if the wiggling frequency is **n**ν_1**,** stationary waves will be set up with the cord oscillating in **n** loops where n is an integer equal to 1,2,3,...etc. The points on the cord which do not oscillate at all are known as nodes and the points which oscillate with the maximum amplitude are known as antinodes.

Now consider a string of length L which is kept stretched by clamping its two ends so that the tension in the string is T. If the string is plucked at its middle point, two transverse waves will originate from the point.. One of them will move towards the left end of the string and the other towards its right end. When these wave reach the two clamped ends, they are reflected back thus giving rise to stationary waves. The string will vibrate with such a frequency so that nodes are formed at the new clamped ends and the middle point of the string becomes an antinode. Thus the string vibrates in one loop as shown in Fig.6.1a It should be noted that the string was plucked from the position of the antinode. As the distance between two consecutive nodes is half the wavelength and λ_1.. is wavelength in this mode of vibration so

$$\lambda_1 = 2\,l \ . \qquad (6.10)$$

Fig.6.1

If v is speed of either of the component waves, then from the equation

$$\upsilon = f_1\lambda_1 \quad \text{or} \quad f_1 = \upsilon / 2\,l \ . \qquad (6.11)$$

If m is the total mass of the string, it can be shown that the velocity v of the wave along the string is given by

$$\upsilon = \sqrt{\frac{T\,l}{m}}\,, \qquad (6.12)$$

where T is the tension in the string. Substituting the value of v in equation (6.11) we get

$$f_1 = \frac{1}{2\,l}\sqrt{\frac{T\,l}{m}} \quad . \qquad (6.13)$$

If the same string is plucked from one quarter of its length then again stationary waves will be set up but now the string will vibrate with another frequency ν_2 and the position of nodes and antinodes are as shown in Fig.6.1b. Wavelength in this mode of vibration is

$$\lambda_2 = l \quad . \qquad (6.14)$$

A comparison of this equation with the equation... shows that the wavelength in this case becomes half that in the first case. Now from the equation $\upsilon = \lambda f$ it follows

$$\upsilon = f_2\, l \quad \text{or} \quad f_2 = \upsilon / l \qquad (6.15)$$

Comparing with equation (2.11) gives

$$f_2 = 2f_1 \qquad (6.16)$$

Thus when the string vibrates in two loops, its frequency is double than when it vibrates in one loop. Thus we can generalise that if the string is made to vibrate in n loops, then the frequency at which the stationary waves will be set up on the string will be

$f_n = n\, f_1$, and the wavelength $\lambda = (2/n)\, l$.

Stationary waves can be set up on the string only with a discrete set of frequencies f_1, $2f_1$, $3f_1$ etc. The lowest of these is known as **fundamental** and the others which are the integral multiples of the fundamental are known as **overtones** or harmonics.

Similar situation is occur in a column of air. Two sort of air column can be distinguished. The former is which both ends are open. Open tube has the antinodes at both ends. The air particles vibrate likely longitudinal or compressional waves in a tube. Note, that in order to producing the standing waves one node must arrised be at least in a tube. Since a distance between consecutive nodes or antinodes is always equal to half the wavelength $\lambda/2$ then the half of wavelength can place in tube: $L=\lambda/2$ and the fundamental frequency $f_1 = \upsilon/\lambda = \upsilon/2L$, where υ is the sound velocity in air. The standing wave, having two nodes is the first overtones or the second harmonics of vibrations.

If the tube is closed at one side, the antinode produces at the open end, while the node at the closed end. Since a distance between the node and nearest antinode is equal to $\lambda/4$, the quarter the wavelength with fundamental ffrequency will be placed in the tube: $L=\lambda/4$. Thus, the fundamental frequency equals to $f_1=\upsilon/4L$.

§6.4. Sound waves

The mechanical vibrations of elastic media with frequencies ranging from 16 Hz to 20.000 kHz produce the sensation in the human ear. These wibrations are called the sound waves. Sound waves with frequencies low 16 Hz are called infrasonic, but waves with frequencies above 20.000 Hz are called ultrasonic. In gases sound waves travel in the form of compressions and rarefactions. In the equation (6.4) the elasticity E is thus the bulk modulus which is given by the ratio of stress to volumetric strain A stress here means the change in pressure.Thus, if take into account that **E= γp,** we get

$$\upsilon_{\text{sound}} = \sqrt{\gamma \frac{\text{p}}{\rho}} \qquad (6.17)$$

Velocity of propagation of elastic sound waves in ideal gases depends on the absolute temperature

$$\upsilon_{\text{sound}} = \sqrt{\gamma \frac{R}{\mu} T} \qquad (6.18)$$

Velocity of sound in gas is approxomately equal to the velocity of heat movement of molecules. At standart conditions (T=273 K) for gas γ =1,4 and **v**=20 $\sqrt{T}$ =330 m/ s.

Sound waves are characterized by **loudness** and **pitch.**

The loudness depends on the intensity and is determined by the amplitude of oscillations in the sound wave. Human ear has ability to receive the sound with intensities ranging from 10^{-12} to 10^{-11} Wt/ m^2

The pitch depends on the frequency. The greatest sensation of human ear is to the frequencies from $7x10^{2}$ to $6x10^{3}$ Hz.

A measure of sensation of human ear to sound wave at given intensity is the intensity level L, defined as

$$\text{L}_{\text{dB}} = 10 \lg \frac{\text{J}}{\text{J}_0} \quad . \qquad (6.19)$$

where $J_0 = 10^{-12}$ Watt/m^2 – is the standart threshold of audibility. Threshold of audibility is called a smallest intensity of sound wave which is audible for human ear.

Threshold of feeling is the smallest intensity of wave sound which produce the painful sensation in the ear. Since intensity of a sound is directly proportional to its squared pressure we get

$$L = 20 \lg \frac{P}{P_0}$$

where $P_0=2x10^{-5}$Pa - is the threshold of audibility. Below the threshold of audibility sound is no longer audible to the human ear. Note, maximum pressure at which sound is audible is P_{max}=60 Pa called threshold of feeling.

CHAPTER 7

Mechanics of liquids

§7.1. The hydrostatic pressure

When a force F acts perpendicular to a surface of area A, the pressure acting on the surface is the ratio between the force and the area:

$$P = \frac{F}{A} \quad . \qquad (7.1)$$

Pressure is measured using a monometer or other type of gauge. A barometer is used to measure atmospheric pressure. Standard atmospheric pressure is 1.013×10^5 N/m^2.

SI unit of pressure is the **pascal** (Pa): 1 Pa = 1 N / 1 m^2

A liquid, as distinct from solids, accepts the shape of vessel in which is contained. Therefore the surface of the liquid always is perpendicular to the force acting on it. Pressure of a column of liquid is due to the weight of the column and is calculated by the formula:

$$p = \rho g h \ , \qquad (7.2)$$

where ρ – is the density of liquid and h - the height of the column.

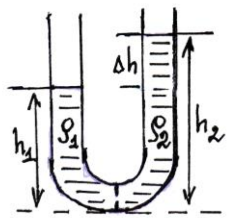
Pressure determined by (7.2) is called the **hydrostatic pressure.**

§7.2. Pascal's principle. Law of communicating vessels

Pascal's principle states that an external pressure applied to a confined fluid is transmitted throughout the fluid.. For instance, pressures acting on all pistons in hydraulic machine are equal. However, as a result of variety of pistons " areas, forces acting those are not equal. Since $p_1 = p_2$ and then

$$\frac{F_1}{F_2} = \frac{A_1}{A_2} \quad . \qquad (7.3)$$

Law of communicating vessels states that in the gravitational field heights of liquid′s columns are inversely proportional to their densities (Fig.7.1) . From the Pascal ′s principle ($p_1 = p_2$) and formula (7. 2) it follows that



$$\frac{\mathbf{h}_1}{\mathbf{h}_2} = \frac{\rho_2}{\rho_1} \quad . \qquad (7.4)$$

Fig.7.1

If the liquid is homogeneous ($\rho_1 = \rho_2$) the heights are equal

$$h_1 = h_2 \quad . \qquad (7.5)$$

§7.3. Archimedes' principle. Float conditions

A body submerged wholly or partially in a fluid is exerted by a buoyant force, equal in magnitude to the weight of fluid displaced by it. This force is known as a Archimedes ' force. and is exerted to the pressure center of immersed part of the body or to the mass center of the body, if it is perfectly immersed . . Archimedes‘ force is given by

$$F_A = \rho_{liq.} V_{body} g \quad . \qquad (7.6)$$

Float conditions. In dependence of the buoyant force a body may be in one of three situations.

a) mg $< F_A$. An object will float in a fluid if the density of the object is less than the density of the fluid.

b) mg = $F_{A.}$ An object will be in equilibrium at any submerged depth in a fluid if the densities of the object and the fluid are equal.

g) mg $> F_A$ An object will sink in an fluid if the density of the object is greater than the density of the fluid.

§7.4. Motion of ideal fluid. Equation of continuity

Motion of a fluid occurs under acting of force of gravity and difference in pressures. Velocity of each particle in flux of a fluid has a certain magnitude and direction for any moment of time. A space occupied by fluids‘ particles is called flux. In order to determine the direction of velocities the current lines are used. Tangents of these at arbitrary points give the direction of flowing. A fluid is called an ideal if the internal frictional forces can be neglected. On the contrary, when one layer of a fluid moves over another layer forces of friction arise. Such fluids are known as a viscous fluid. A flowing of fluid is called **laminar** if the velocity and pressure at each point are independent on time. A flowing is described graphically be current lines.

A surface formed by current lines passing through contour in a fluid is called **current pipe.** When a flow is steady, a mass of fluid passing through any cross-sectional area per unit time remains constant
(Fig.7.2a) Fluid can not be stored in various sections of the pipe. It permits to write the equation as

$$m=\text{const.} \qquad \rho\,\Delta l\,A=\text{const.} \qquad \text{or} \qquad \upsilon A=\text{const.}$$

In particular

$$\upsilon_1 A_1 = \upsilon_2 A_2 \qquad (7.7)$$

Expression (7. 7) is known as the **equation of continuity.**

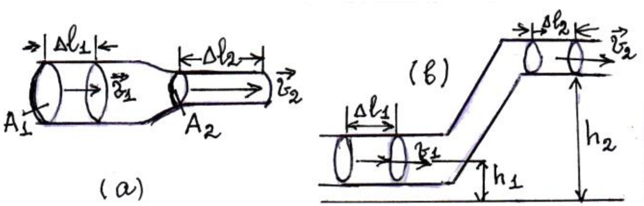


Fig.7.2

§7.5. Bernoulli′s equation

Consider a steady flow of incompressible fluid. Let us calculate the work needed to displace a certain volume of a fluid from one section to another (Fig.7.2b). A displacement of fluid through cross sectional area A_1 by a distance causes to remove it through the cross section A_2. by distance . A fluid in left side of A_1 produces a pressure P on the considered volume and performers a certain work $W_1 = P_1 A_1 l_1$. In cross- section A_2 the work performed is equal to $W_2 = -P_2 A_2 l_2$ (here the work done is opposed to the direction of flow). The work done in the field of gravity is written as $W = -mg(h_2 - h_1)$. The total work done onto the fluid is equal to the change in kinetic energy. Thus

$$\frac{1}{2}m\upsilon_2^2 - \frac{1}{2}m\upsilon_1^2 = P_1A_1\Delta l_1 - P_2A_2\Delta l_2 - mgh_2 + mgh_1 \qquad (7.8)$$

Dividing (7. 8) by the mass $m = \rho A\Delta l$ of fluid we get

$$P_1 + \frac{1}{2}\rho\, \upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\, \upsilon_2^2 + \rho g h_2 \qquad (7.9)$$

This equation is known as *Bernoulli' s equation* and can be written as

$$\mathbf{P} + \frac{1}{2}\rho\, \upsilon^2 + \rho g h = \text{const.} \qquad (7.10)$$

Thus the sum of static, dynamic and hydrostatic pressures remains the unchaingeable. Static pressure is due to fluids′ forces of elasticity. Dynamic pressure is due to kinetic energy of moving fluid.

If the pipe is placed horizontally the equation (7.10) can be rewritten as .

$$\mathbf{P} + \frac{1}{2}\rho\, \upsilon^2 = \text{const.} \qquad (7.11)$$

With increase in flux velocity the dynamic component of pressure increases while the static component decreases. The dynamic pressure of a fluid at rest is equal to zero. Then the total pressure is composed of static and hydrostatic pressures i.e., the pressures produced by piston and weight of fluid's column respectively.

A work done during time *t* by a pump with power N when it remove the fluid of volume V up to the height h is given as

$$\eta N t = \rho V g h \qquad (7.12)$$

where η- is the coefficient of useful work (efficiency) of a pump .

§7.6. Flux pressure measurement

Static pressure is measured by means of monometer whose cross-sectional plane is placed parallel to the direction of flow.

Total pressure is measured by means of monometer whose cross-sectional plane is placed perpendicular to the direction of flow (Pito's pipe)The difference of total and pressures is measured by the Prandtle′s pipe The difference of two static pressures is measured by the Ventury′s pipe By knowing difference of the static pressures the velocity can be calculated as

From Eq. (7 11)

$$\Delta p = \frac{\rho}{2}\left(\upsilon_2^2 - \upsilon_1^2\right) . \qquad (7.13)$$

Substituting υ_2 from Eq. (7. 7) into (7.13) gives

$$\Delta p = \frac{\rho}{2}\upsilon_1^2\left((A_1/A_2)^2 - 1\right) . \qquad (7.14)$$

Hence it follows

$$\upsilon_1 = \sqrt{\frac{2\Delta p}{\rho\left((A_1/A_2)^2 - 1\right)}} . \qquad (7.15)$$

§ 7.7.Torricelli′s formula

Using the Eq. (7.10) velocity of the fluid emerging from the small orifice can be easy calculated. Since the area of open surface A_1 is more greater as compared to the cross- section A_2 then in accordance with the equation (7.7) $\upsilon_1 = 0$. Thus equation (7.10) is rewritten as

$$\frac{1}{2}\rho\,\upsilon_1^2 + \rho g h_1 = \rho g h_2.$$

Hence we get:

$$\upsilon = \sqrt{2g(h_2 - h_1)}\quad . \qquad (7.16)$$

This expression is known as Torricelli′ s formula.

§7.8. Internal resistance of fluid

The resistance of a fluid includes two types; resistance due to pressure and resistance related to the friction. The first component is determined by the difference of pressures at the end (edges) of the body. The second one appears due to great gradient of velocity:

$$F = \eta \frac{\Delta \upsilon}{\Delta \mathbf{x}} A \qquad (7.17)$$

where η– is the *dynamic viscosity* of fluid, $\Delta \upsilon$– is the change of velocity, Δx – the distance between layers , and A – is the area of fluid's surfaces in contact. Eq. (7.17) is known as Newton ′s formula.

The SI unit of measurement of η is Pa • sec. Another unit of η- is called the poise: 1 poise = 0.1 Pa • sec.

The quantity determined by the ratio

$$\nu = \frac{\eta}{\rho} \qquad (7.18)$$

is called a *kinematic viscosity*. Here ρ is the density of a fluid.

The resistance due to difference in pressures depends on the density of fluid, its velocity and maximum cross – sectional area, perpendicular to the flux.

$$R=C\frac{\rho \upsilon^{2} A}{2} , \qquad (7.19)$$

where C- is the coefficient of proportionality.

§7.9. Stock′s and Poiseuille′s formulas. Reynolds′ number

A sphere of radius r moving in a viscous fluid is acted by the forces of gravity $G = mg = (4/3)\pi r^{3} \rho_{body} g$, byoyant force $F_A=\rho_{liq.} V_{body} g=(4/3)\pi r^{3}\rho_{liq.} g$ and force of friction, which is directly proportional to the first order of velocity and dimensions of a body.

$$F_{fric.}= 6\pi \eta r \upsilon , \qquad (7.20)$$

where, r- is the radius of sphere and v is its velocity.

Equation (7.20) is known as Stocks′ formula.

When sphere moves with constant velocity its motional equation is

$$F_{Ar.} + F_{f.} = G.$$

Velocity of sphere is expressed by

$$\upsilon = \frac{2}{9}\frac{g R^2}{\eta}\left(\rho - \rho_{fl.}\right) . \qquad (7.21)$$

The volume of a fluid transferred per unit time through the cross- sectional area of cylindrical pipe of radius **r** is given by the Poiseuille′s formula

$$V = \frac{\pi r^4}{8 \eta l}\left(p_1 - p_2\right) , \qquad (7.22)$$

where *l* – is the length of the pipe, $p_1 - p_2$ the pressure difference at the its ends.

The ratio of forces (7.19) and (7.20) determines the quantity called Reynold′s number.

$$R_e = \frac{\rho \upsilon D}{\eta} , \qquad (7.23)$$

where D - is the pipe′s diameter.

CHAPTER 8

Molecular Physics and Heat

§ 8.1. Movement of molecules

A particle of substance with minimum sizes storing its chemical properties is called *molecule***.**

Molecules of all bodies are in uninterrupted movement and therefore they possess kinetic energy.

In solids molecules oscillate relatively a certain position in crystal lattice.

In liquids a molecules oscillate relatively time varying instant position .

In gases between molecules there is a weak interaction and therefore they move with higher speeds. The given molecule moves rectilinearly between collisions with other molecules or molecules of obstacle .

Movement of molecules in liquids and gases is observed by means of microscope. Small particles of the weighed substance under action of impacts of liquid′ s molecules move chaotically on a zigzag trajectory. Such movement refers to *brownian movement.*

Size of molecules is considerable smaller. For example , a size of hydrogen molecule is about (3 –4) x10^{-10} m.

§ 8.2. Measurement of molecules′ speed

One of the first experiments in which speed of movement or separate molecules of gas was directly measured was executed by Otto Shtern. In experiment the device consisting of two cylinders with the common axis of rotation was used. On an axis of the cylinder the platinum wire covered with silver was located. When an electrical current passes through a wire as a result of its heating there occurs an evaporation of silver atoms from a surface of wire. Then on the internal wall of the second cylinder forming on it appreciable strip.

When cylinders were resulted in rotation with identical frequency a strip appeared in the other place. By the angle ϑ between these two positions of a strip, distance $l = R_2 - R_1$ and frequency f of rotation of cylinders it was possible to define speed of atoms of silver as

$$\upsilon = \frac{l}{\Delta t} = \frac{2\pi f l}{\vartheta} \qquad (8.1)$$

Strip of silver turned out at rotation of cylinders there was dim. It testifies to that that atoms of silver evaporated from the wires surface have different speeds.

§ 8.3. Mean free path

The average distance which the molecule travels between two successive collisions is called **mean free path.** Mean free path may be found as a ratio of a distance L passed (length of cylinder) to the number of collisions N over Δt time. (Fig.8.1) On the other hand, a number of collisions is equal to the number of molecules, whose centers placed within a cylinder or it is the product of molecules’ concentration *n* by the volume V. Thus,

$$l = \frac{L}{N} = \frac{\upsilon \Delta t}{n V} \quad . \qquad (8.2)$$

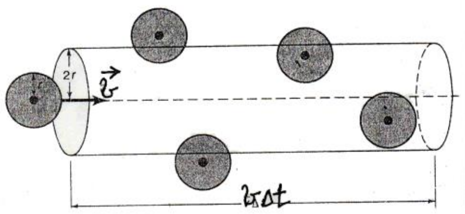


Fig.8.1

Volume of cylinder is equal to $\pi(2r)^2\ \upsilon_{rel.} \Delta t$ and $\upsilon_{rel.} = \sqrt{2}\ \upsilon$. Here $\upsilon_{rel.}$ – the relative velocity and υ the mean arithmetical velocity. Then (8.2) will have form

$$l = \frac{1}{4\sqrt{2}\pi r^2 n} \quad . \qquad (8.3)$$

Mean free path may be expressed also by the formula

$$l = \upsilon / z \quad ,$$

where z- is number of collisions per unit time,

At a constant temperature the mean free path is inversely proportional to the pressure

$$p_1 l_1 = p_2 l_2 \quad , \qquad (8.4)$$

Free path time is given as

$$\tau = 1/\upsilon \ .$$

§ 8.4. Mean arithmetical velocity

Mean arithmetical velocity of molecules is determined as

$$\upsilon_{m.a.} = \frac{\upsilon_1 + \upsilon_2 + ... + \upsilon_n}{n} = \sqrt{\frac{8RT}{\pi\mu}} \ , \qquad (8.5)$$

where μ- the mass of one mole, υ - the modules of velocities, N-number of molecules, R=8.31 J/mole. K-universal gas constant and T the absolute temperature.

The **root mean square velocity** is determined by the formula

$$\mathbf{u}=\sqrt{\frac{3RT}{\mu}} \quad . \qquad (8.6)$$

§ 8.5. Basic items of molecular – kinetic theory of gas. Pressure. Principal equation

The basic items of m.k.t. of gas are:

1.Gas molecules have small sizes compared with distance between them.

2.Molecules move irregular and have different speed or direction.

3.Collisions among molecules and walls of container are elastic.

4.Gas molecules exert almost no forces on one another, when they are not collide.

The **pressure** of a gas on the walls of its container is the result of bombardments by billions of molecules. When a gas is compressed into a smaller volume, its molecules strike the walls at the container more often than before, leading to an increase in pressure. Pressure is determined by the expression:

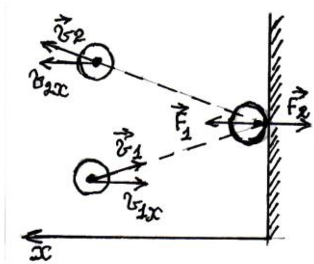


Fig.8.2

$$p=\left(\frac{N}{t}\right)x\left(\frac{1}{A}\right)x\ Ft \quad , \qquad (8.7)$$

where N/t the number of bombardments of walls by molecules, S the area of wall, Ft is the impulse of force. On the other hand the number of strikes N is equal to the number of molecules , which may be written as $N= n_0V=n_0Aut$ Here u is the root mean square velocity. One can suppose , that (1/3)N of molecules move along the considered axis . However, half , i.e. (1/2)(1/3)•N of these molecules move , for example, only in positive direction of axis. For simplicity, only horizontal molecular motion is shown in Fig.8.2 It is seen that the impulse of force is equal to the change in momentum in the direction of force action.

$$F_{1x}\, t = m\upsilon_x - (-m\upsilon_x) = 2m\upsilon_x \ .$$

At the moment of bombardment the molecule exerts on the wall the force F_2 equal to the force F_1 in magnitude and opposite in direction. With accounting mentioned assumptions above the pressure of a gas is determined by the so called principle *equation of kinetic theory:*

$$P =(1/3)n_0mu^2 \ , \qquad (8.\ 8)$$

where n_0-is number of molecules per unit of volume, m-is the mass of molecule.

§ 8.6. Partial pressure. Dalton ′s law. Atmospheric pressure

The pressure that is exerted by an single gas occupying the total volume of mixture is called the *partial pressure* of a gas.

Dalton′s law: The pressure of the mixture of gases is equal to the sum of their partial pressures:

$$P = p_1+ p_2 + p_3 +... + p_n \ . \qquad (8.\ 9)$$

The pressure of the atmosphere decreases with increasing distance (h) from the surface of the Earth. If it is assumed that the temperature of the atmosphere is independent of the height, then

$$p = p_0\, e^{-(\mu g h / RT)} \quad , \qquad (8.10)$$

where μ- is the average molecular weight of the mixture of gases comprising the atmosphere. This expression is called Boltzman ′s law.

To measure the air pressure both of **mercury barometer** and **aneroid barometer** are used. Upon the first way of measurements the height of a column of a mercury supported by the atmosphere is used. At the second way a measurement of pressure is related with the changes in the size of an evacuated chamber. The measured value of standard atmospheric pressure is equal to
760 mm. Hg (mercury column.) or 101325 kPa.

§ 8.7. Amount of substance. Relative molecular mass. Mole. Avogadro′s law. Molar mass

Amount of substance is called a physical quantity determined by the number of structural elements - molecules, atoms or ions. Note, the same amount of substances has different mass.

Relative molecular mass: The SI unit of the amount of substance is taken a mole. The relative molar mass is the ratio of the mass of molecule to the 1/12 portion of mass of carbon isotope ^{12}C. Mole is the amount of substance containing the number of particles equal to the number of atoms in 0.012 kg of the carbon isotope ^{12}C. For *r*elative mass of atom or molecules we get:

$$M_r = \frac{m_0}{(1/12) m_{0C}} \quad , \qquad (8.11)$$

where m_0 the mass of atom of substance, m_{0C} is the mass of ^{12}C atom.

Avogadro′s law: This law states, that an equal volumes of different gases contain the equal number of molecules at the same pressure and temperature. Moles of different substances also contain the equal number of molecules. It is called the Avogadro′s number: $\boldsymbol{N_A=6.02x10^{23}}$ ***mol***

$$N_A=\frac{N}{\nu} \quad , \qquad (8.12)$$

where N is the number of atoms or molecules of substance, ν the amount of substance .

Molar mass: The ratio of mass of substance to the amount of substance is called the molar mass.

$$M=\frac{m}{\nu} \quad . \qquad (8.13)$$

Fron (8.12) and (8.13) it follows:

$$N=\frac{m}{M}N_A \quad . \qquad (8.14)$$

The concentration of particles of substance of density ρ can be found by means of formula:

$$n=\frac{\rho}{M}N_A \; . \qquad (8.15)$$

From Avogadro’s law it follows, that under equal conditions the moles of different gases have equal volumes equal to $\mathbf{V=22.4x10^{-3}m^3/mol}$. In particular for helium (He):

Relative molecular mass M_r=44

Molar mass M=0.044 kg/mol.

Density ρ=0.179kg/m^{3} –

Consequently, the volume of a mole of helium is

$$V_0=\frac{M}{\rho}=\frac{\boldsymbol{0.044} kg/mol}{\boldsymbol{0.179} kg/m^3}=22\text{x}10^{-3}\ m^3/mol.$$

If the volume V of gas is known, the amount of substance is equal to

$$\nu=\frac{V}{V_0}.$$

§ 8.8. Diffusion. Fick's equation

The process of mixing due to chaotic motion of molecules is known as a diffusion. In any case the diffused substance removes from the region with higher concentration into the region with lower value of concentration. This process results in equality both of temperature and concentration at all points of substance. The flux of molecules experimentally was observed to be direct proportional the variation of concentration per unit of length or concentration gradient.

$$J=-D\,\frac{n_2-n_1}{\Delta x}\quad, \qquad (8.16)$$

where J – the flux, measured by the number of molecules passing through unit cross sectional area per unit second (mole / m^2 sec.) , $n_{1,2}$ – the concentration (mole / m^3). The expression (8. 16) is known as Fick s equation. The sign of minus in (8 .16) means that the flux is opposite to the concentration gradient . The diffusion coefficient D is expressed by mean free path l of gas molecules by the simple relation:

$$D=\frac{1}{3}\upsilon\, l\quad,$$

where υ – is the mean arithmetical velocity.

If the concentration will be expressed by the partial pressure of concrete component of gas , from the equation (8. 16) we get:

$$J = -\frac{D}{kT}\frac{\Delta P}{\Delta x} \quad . \qquad (8.17)$$

§8.9. Laws of ideal gas. Isoprocesses. Clapeyron – Mendeleyev′ s equation

A processes in which one of quantities as temperature, pressure and volume remains constant are called iso-processes. Let us consider individually to each of these:

1. Isothermal process (Temperature remains constant, T=constant)

Boyle′s law: **At constant temperature the volume of a sample of any gas is inversely proportional to the pressure applied to it.**

$$\frac{P_1}{V_2} = \frac{P_2}{V_1} \quad . \qquad (8.18)$$

Fig.8.3 demonstrates the processes at constant temperature (isobars) for coordinates : a) P -V ($T_2 > T_1$) b) P-T ; c) V-T ; d) $\rho - T$

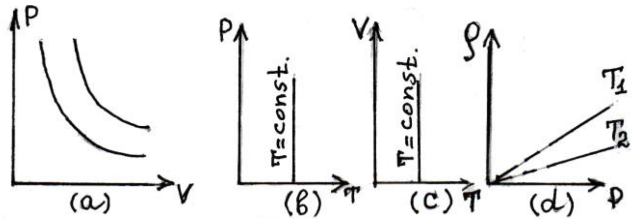


Fig.8.3

2. Isobaric process (Pressure remains constant , P=const.)

Gey-Lussak′ s law: **At constant pressure the volume gas is directly proportional to its temperature.**

$$\frac{V_1}{T_1}=\frac{V_2}{T_2} \quad \text{or} \quad V=V_0(1+\alpha t)\,, \qquad (8.19)$$

where V_0 volume at T_0=273,16K and $\alpha=1/T_0$ thermical coefficient of volume expansion.

Fig.8.4a shows the proportionality between volume and absolute temperature for gases at constant pressure ($P_2 > P_1$). If the temperature of a gas could be reduced to absolute zero, its volume would fail to zero. Actual gases pass into liquid at temperatures above absolute zero. Fig.8.4b,c,d illustrate the processes at constant pressure n with different coordinates, b) PT, c) PV, d) ρ, n - V, T

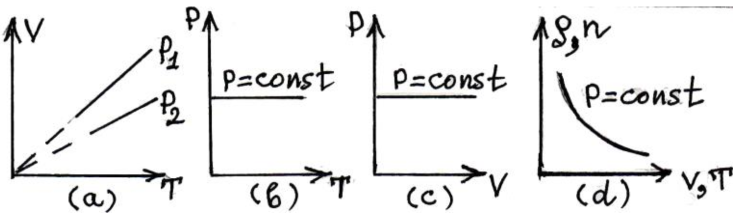


Fig.8.4

3. Isochoric process (Volume remains constant , V=const.)

Charle′s law: At a constant volume the pressure of a sample of any gas is directly proportional to its absolute temperature.

$$P=P_0(1+\alpha_P\, t) \quad \text{or} \quad P/T=P_0/T_0\,, \qquad (8.20)$$

where P_0 – the pressure of gas at T_0=273.16K and $\alpha_P=P/P_0T$- thermical coefficient of pressure.

Fig.8.5 illustrates the processes at constant pressure for different coordinates : a) P-T , b) P-V, c) V-T d) ρ, n - P,T

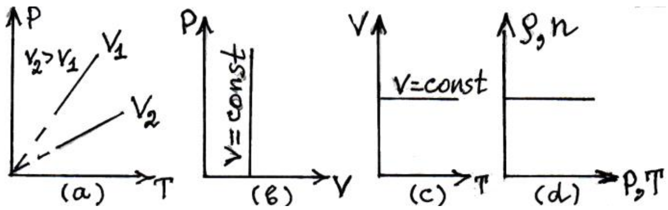


Fig.8.5

Combined law of state of ideal gas for a given mass is written by the expression:

$$PV=\frac{m}{M}R\,T \qquad (8.21)$$

This relation is called the Clapeyron –Mendeleyev′s equation**.** R – is the universal gas constant. It is determined with the work performed by a gas of one mole under a constant pressure its temperature is raised for one unit.

From equation (8.21) the density **ρ** of gas may be calculated as

$$\rho=\frac{P\,M}{R\,T}$$

In order to calculate the parameters of mixture of gases the mean values of density and gas constant are used. Both of quantities depend on the mass of components and therefore the calculation is carried out as follows:

$$\rho_{\mathbf{mean}}=\frac{\rho_1 m_1+\rho_2 m_2+...}{m_1+m_2+...}\ ;\ R_{\mathbf{mean}}=\frac{R_1\,m_1+R_2\,m_2+...}{m_1+m_2+...}.$$

.

§8.10. Temperature scales. Absolute zero. Thermometers

Basically, temperature scales are classified into three major kinds:

1. Celsius. 2. Kelvin. 3. Fahrenheit

Both the Celsius and Kelvin scales split the region from freezing to boiling of water into 100 parts. Each part called a degree.

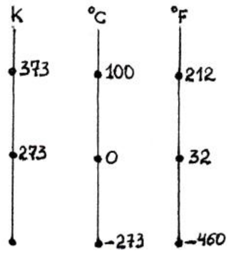


Fig.8.6

In the **Celsius scale** the freezing point of water is given the value 0°C and the boiling point of water the value 100°C. (Fig.8.6)

Absolute zero is that temperature at which the substance remains in gaseous state, exerts zero pressure (under constant volume) or has zero volume under constant pressure). It is the lowest temperature in nature. In fact at temperature much above the absolute zero all gases undergo a change of gaseous state and become liquid. The *absolute temperature scale* has its zero point at –273°C. Temperatures in this scale are designated K (Kelvin). Any Celsius temperature t_c can be converted to its equivalent absolute temperature T_K by adding 273.

$$T_K = t_c + 273$$

The *Fahrenheit scale* divides this region from freezing to boiling in 180 parts. In this scale the freezing and boiling points of water are given the values 32°F and 212°F respectively. A degree is larger than a Fahrenheit degree by 9/5. A simple equation conversion between these scales is

$$T_F = \frac{9}{5} t_C + 32^\circ \quad \text{or} \quad t_C = \frac{5}{9}(T_F - 32^\circ)\,. \qquad (8.22)$$

To measure the temperature the following types of thermometers are used: 1 . Liquid thermometers, 2. Metal thermometers, 3. Resistivity thermometers.

The ordinary liquid (mercury) thermometer uses the thermal expansion of mercury as a measure of temperature. Mercury is used as a thermometric substance because over a wide range of temperatures its expansion is quite a linear function of the temperature and that it does not stick to the walls of the capillary tube.

§8.11. Total energy. Transfer methods of internal energy

Total energy consists of:

a) kinetic energy of macroscopic motion.

b) potential energy due to external field(for ex. electrical or gravitation field)

c)internal energy.

The internal energy consists of:

a) kinetic energy of irregular motion of particle.

b)potential energy due to the interaction between molecules .

c) energy of electrons at the atomic or electron levels.

d)internal energy of nucleus.

The internal energy is accepted to be equal to zero at absolute zero (T= 0).

The internal energy of one mole of single atomic gas depends only on its temperature:

$$U=\frac{3}{2}RT \quad . \qquad (8.23)$$

Then a change of internal energy is expressed as

$$\Delta U=\frac{3}{2}R\Delta T \quad \text{where} \quad \Delta T=T_2-T_1 \ .$$

1. The internal energy may be transferred by means of both forms: work and heat. When the internal energy is transferred as a work the energy of regular motion of particles is converted to the energy of irregular motion .The work of the system is considered to be “plus” when it is done on the external forces. When the ideal gas is expended the work done is “plus”. When the gas is compressed the work done is “minus”.

2. Heat –is the such form of energy transfer when the direct energy conversion among interacting particles or bodies occurs.

As a comparison can be said that, when the energy is transferred in the form of heat then it raises only the internal energy, but when the form of transfer is the work done, then the other types of energy may be increased as well.

At the really conditions the both of forms of energy transfer are performed; when the metal specimen is heated its internal energy increases and the thermal expansion also takes place.

Mechanical equivalent of heat

$$I=4{,}19\ J/cal\ .$$

The inverse quantity of I is equal to $(1/I)=0{,}239$ cal/J and is called the *heat equivalent of work***.**

§ 8.12. Work done due to variation in volume

The work performed at **isobaric expansion** of gas is expressed by the formula:

$$W=p\,(V_2-V_1)\ . \qquad (8.24)$$

The area limited by the graph and axis V equals to the work performed. (Fig. 8.7a)

Work performed under due to isothermal change in volume:

Expansion: Direction of force (or pressure) coincides with the direction of displacement of a piston in a vessel. Therefore work done by gas is positive ($W>0$ while work done by external forces is negative($W'<0$) (Fig.8.7b)

Compression. Direction of exterior force coincides with the displacement of a piston, therefore work performed by external forces is positive ($W' >0$), while work done by a gas is negative ($W< 0$).(Fig.8.7c)

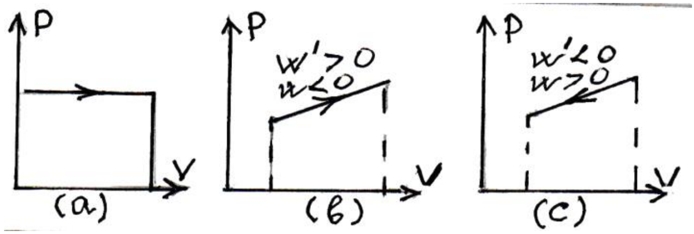


Fig.8.7

§8.13. First law of thermodynamics

The first law of thermodynamics expresses the law of energy conservation. A change in internal energy of gas (ΔU) is equal to the sum of amount of heat (Q) gained by the gas and work (W) performed on it.

$$\Delta U = Q + W \quad . \qquad (8.25)$$

Another form of this law is:

$$Q = \Delta U+W' \quad \text{here} \quad W = -W'.$$

The amount of heat gained by the gas is expended to the increase of internal energy of gas and to the work (W')done by the gas.

The first law of thermodynamics can be written for the various iso-processes as follows:

1. Isothermal: T=const. $\Delta U=0$ $Q=W'$
2. Isobaric: P=const. $W=P\Delta V$
   $Q=\Delta U+P\Delta V$
3. Isochoric: V=const. $W'=0$ $Q=\Delta U$
4. Adiabatic: $Q=0$ $\Delta U=W'$. (Fig.8.8)

The first law of thermodynamics permits to calculate amount of heat needed to increase the temperature of gas:

1. Single-atomic gas: $Q=\frac{5}{2}\nu R\Delta T$.
2. Double-atomic gas: $Q=\frac{7}{2}\nu R\Delta T$.
3. Multi-atomic gas: $Q=4\nu R\Delta T$.

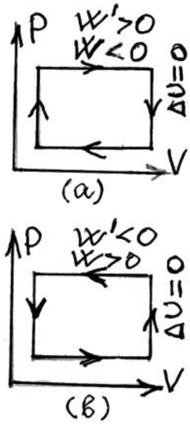


Fig.8.8

§8.14. Heat capacity. Specific heat capacity. Molar heat capacity

The quantity of heat required to rise the temperature of a body for one degree is called the **heat capacity**

$$C=\Delta Q/\Delta T \quad . \qquad (8.26)$$

The quantity of heat required to rise the temperature of a body of unit mass for one degree is called the **specific heat capacity**

$$c=\Delta Q/ m\Delta T \quad . \qquad (8.27)$$

The specific heat of a body depends on the mass, temperature, chemical composition and on the conditions under which it is heated mass, temperature, chemical composition and on the conditions under which it is heated.

The values of specific heat capacity at constant pressure c_p and at constant volume c_v, are related (Mayer′s formula)

$$c_P = c_V + R \;, \qquad (8.28)$$

where R – is the universal gas constant.

To determine the specific heat capacity the equation of heat balance usually is used. Suppose that a body with mass m_1 specific heat capacity c_1, has a temperature T_1 , and it takes part in heat transfer with the body of mass m_2, s.h.c. c_2, and temperature T_2 . Then the amount of heat $Q_1 = m_1c_1(T_1 - T_0)$ lost by the hot body is equal to the amount of heat $Q_2 = m_2 c_2(T_0 - T_2)$. gained by the cold body Here T_0 is the final temperature. On the basis of the energy conservation we have $Q_1=Q_2$ or

$$m_1c_1(T_1 - T_0) = m_2 c_2 (T_0 - T_2) .$$

This expression is known as the law of heat exchange , in other words heat lost = heat gained.

Hence (4.6) it follows that,

$$c_1 = \frac{m_2\left(T_0 - T_2\right)}{m_1\left(T_1 - T_0\right)} c_2 \;. \qquad (8.29)$$

The average temperature is

$$T_0 = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} \;. \qquad (8.30)$$

If the mixed substances are the same and homogeneous ($c_1=c_2$), then ( ) simplifies

$$T_0=\frac{m_1T_1+m_2T_2}{m_1+m_2} \quad \text{or} \quad T_0=\frac{V_1T_1+V_2T_2}{V_1+V_2}\ ,. \qquad (8.31)$$

where V_1 and V_2 are the volumes of substances respectively.

The heat capacity of a mole of a substance is known as its *molar heat capacity* and denoted by C_μ. It is connected with the specific heat capacity **c** through the relation $C_\mu = c\mu$ where μ- is the molar mass.

CHAPTER 9

Mutual conversion of liquids and gases

§9.1. Evaporation. Latent heat of vaporization

The processes in which the molecules of a liquid escape into the air are called *evaporation***.** The number of molecules escaped from 1cm^2 surface area of liquid per second characterizes the *speed of evaporation* . Evaporation speed depends on the attractive forces between liquid′s molecules. When these forces are smaller the liquid evaporates more rapidly, for example, the alcohol evaporates more quickly than the water because the attraction of its particles is smaller than for one another and greater number of molecules can escape. Hence the average kinetic energy of the remaining molecules is lower and the liquid temperature drops. Evaporation speed depends also upon external conditions; such as it increases with increasing of the open surface area of liquid and with decreasing of pressure over the liquid. Number of escaping molecules increases with the raising of temperature.

Evaporation occurs only at a liquid′s surface and at all temperatures.

An evaporation of the liquid from the open vessel can proceed until all of the liquid is converted into vapor.

On the contrary, in the closed vessel evaporation proceeds until a state of equilibrium between the mass of the liquid and that of the vapor is reached. At this stage evaporation and condensation compensate each other. Such a state is referred to *dynamic equilibrium***.**

The process of evaporation causes the cooling.

The amount of heat needed to convert 1 kg of a substance at the its boiling point from the liquid to the gaseous state is called *latent heat of vaporization***.** (Fig.9.1, d – e **)**

$$r = Q/m \quad . \qquad (9.1)$$

The latent heat of vaporization is equal to the latent heat of condensation (converting from gaseous state to the liquid state)
The SI unit of latent heat of vaporization is J / kg.

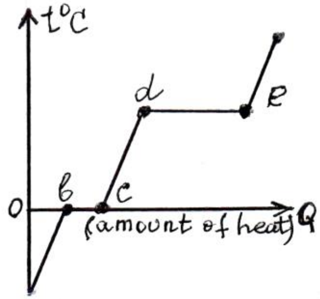


Fig.9.1

§9.2. Saturated vapor. Vapor isotherms

A vapor in dynamic equilibrium with its liquid is said to be *saturated.* For example, a vapor within the bubbles of gas is saturated. A pressure of saturated vapor is not dependent on its volume and is determined only by the absolute temperature.

$$P = nkT \quad . \qquad (9.2)$$

A pressure of saturated vapor rapidly is increased with increasing of temperature. In Fig.9.2a the P – T diagrams for saturated vapor (1) and ideal gas (2) are given. Fig.9.2b shows the densities of liquid (curve 1) and its saturated vapor (curve 2) versus their temperature.

The point at which densities of liquid and its saturated vapor become equal is called the *critical point***.** The value of temperature, corresponding to critical point is called *critical temperature.*

When $T = T_{crit.}$ a difference between liquid and gas states of substance disappears.

When $T > T_{crit.}$ even under greater pressures a conversion of gas to liquid state is impossible.

At $T=T_{crit.}$ the latent heat of vaporization and coefficient of surface tension is equal to zero.

A dependence of pressure on volume at a constant temperature is called the *isotherm of vapor*. In Fig.9.3a the part 0 –1 corresponds to ideal gas. The horizontal part of isotherm of vapor is due to the process of conversion of vapor into saturated vapor. The part 2–3 corresponds to liquid state. Under decreasing of volume a pressure of a saturated vapor remains unchangeable due to conversion its any part into liquid. The total volume of saturated vapor is perfect condensed at the point 2. Further decreasing of volume leads to the sharp increasing of pressure. It is explained with the small compressibility of liquid.

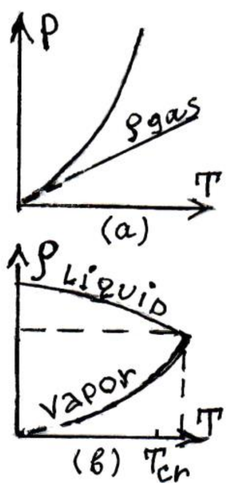


Fig.9.2

The process of passing of ideal gas into saturated vapor may be performed not only by isothermal compressing, but also by decreasing of temperature. The isotherm 3–2–1–0 may be considered as a curve of liquid continuously passing into saturated vapor. The process of conversion of saturated vapor into unsaturated can be occur not only by isothermal expansion, but also by increase in temperature. unsaturated vapor , obtained by this way is called the *superheated* vapor.

In the absence of condensation centers are absent by means of slow isothermal condensation one can obtain the *supersaturated* vapor, a pressure of which exceeds the pressure of saturated vapor at a given temperature.

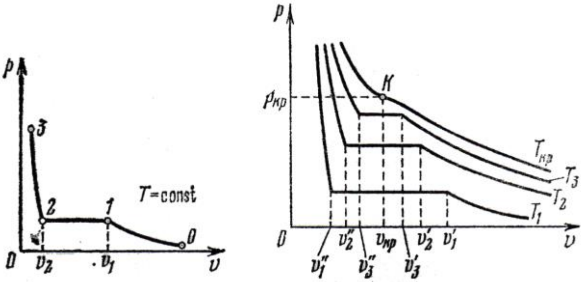


Fig.9.3

Isotherms plotted for various high temperatures T_1, T_2, T_3, ($T_3 > T_2 > T_1$) show, that state of saturated vapor is realized at smaller values of specific volume $v'_3 < v'_2 < v'_1$ (Fig.9.3b). It is explained as follows: A pressure of saturated vapor raises rapidly with increasing of temperature and to compare it with the pressure of unsaturated vapor a decreasing of vapor′ s volume is need. Its reason is the heat expansion of liquids.

§9.3. Boiling. Melting. Latent heat of fusion. Specific heat of combustion

The processes of evaporation in which the molecules is escaped at definite temperature is called *boiling***.** Boiling occurs in the entire volume of liquid. According to (9.2) a pressure of saturated vapor within of gas bubbles increases as the temperature raises. Consequently the volume of gas bubbles increases as well. It leads to increasing of Archimed′s force acting to the bubbles and makes easy to raise upward. The external pressure is the sum of atmospheric p_0 and hydrostatic ρgh. More over the gas bubbles are compressed by the force of surface tension. If the depth is negligible then the total pressure may considered to be equal to the pressure over the liquid surface. A pressure of saturated vapor becomes greater than p_0 at any temperature, which is called *boiling temperature or boiling point*. Boiling begins when the following condition takes place:

$$p_{s.v.} = nkT \geq p_0 + \rho gh \quad , \qquad (9.3)$$

where h-is the height of bubble of gas from the surface of liquid. Boiling temperature depends upon the external factors; it is unchangeable at a constant pressure, and increases with the raising of pressure over the liquid (atmospheric pressure). It also is decreased with reducing of external pressure. For example, at the height about 3 km where atmospheric pressure is about 500 torr. a boiling point of water is 89^0 C. If the liquid is a mixture, then boiling point layers between the boiling points of components.

The processes of converting of matter from solid to liquid at definite temperature is called *melting.* Accordingly, this temperature is called *melting point*.

The amount of heat needed to convert 1 kg of substance at its melting point from the solid to the liquid state.(Fig.9.1, b – c) is called the *latent heat of fusion*.

$$\lambda=Q/m \qquad (9.4)$$

The latent heat of fusion is equal to the latent heat of crystallisation (converting from liquid state to the solid state)

The SI unit of latent heat of fusion is J/kg.

Specific heat of combustion is defined with the amount of heat which is dedicated when 1 kg of fuel is burning:

$$k=Q/m_{fuel.} \qquad (9.5)$$

§9.4. Examples of heat exchange equations

The heat exchange equations (Look the equation 8.28) for some cases are given here-below:

1. **The fuel burns and any (η) fraction of gained heat is expended to heating of vessel and the liquid within it:**

$$\eta k m_{fuel} = c_{ves.} m_{ves.} (\theta - t_{ves.}) + c_{liq.} m_{liq.} (\theta - t_{liq.}) \qquad (9.6)$$

2. The hot and cold liquids are mixed:

$$c_{hot} m_{hot} (t_{hot} - \vartheta) = c_{cold} m_{cold} (\vartheta - t_{cold}) \qquad (9.7)$$

3. The hot body is placed in the cold liquid:

$$c_{hot} m_{hot} (t_{hot} - \vartheta) = c_{cold} m_{cold} (\vartheta - t_{cold}) + $$
$$+ c_{v\,es.} m_{v\,es.} (\theta - t_{v\,es.}) \qquad (9.8)$$

If the cold body is placed in the hot liquid an equation will obtain the form:

$$c_{body} m_{body} (\vartheta - t_{body}) = c_{liq} m_{liq.} (t_{liq} - \vartheta) + $$
$$+ c_v m_v (t_v - \vartheta) \quad . \qquad (9.10)$$

4. **The hot body is placed within the liquid and some part of liquid is vaporized:**

$$c_{hot} m_{hot} (t_{hot} - \vartheta) = c_{liq.} m_{liq.} (\vartheta - t_{liq.}) + $$
$$+ c_v m_v (\theta - t_v) + r m_{vap.} \quad . \qquad (9.11)$$

5. The vapor passes through the liquid:

$$r m_{vap.} + c_{liq.} m_{vap.} (t_{boi.} - \vartheta) = c_{liq.} m_{liq.} (\vartheta - t_{liq.}) + $$
$$+ c_v m_v (\vartheta - t_{v.}) \quad . \qquad (9.12)$$

6. A piece of ice is in the water:

$$c_{water}\ m_{water}\,(t_{water} - \vartheta) + c_{v.}m_{v.}\,(t_v - \vartheta) =$$
$$= c_{ice}m_{ice}(t_{melt.} - t_{ice}) + \lambda m_{ice} + c_{water.}m_{ice}(\vartheta - t_{melt.}) \quad (9.13)$$

7. A fraction of mechanical energy is expended to the heating of a body:

$$\eta mgh = cm\,(t_{melt} - t_1) \quad \text{or} \quad \eta\frac{mv^2}{2} = cm(t_{melt} - t_1) \quad (9.14)$$

If the energy is expended to both heating and melting processes, then the following equations are true:

$$\eta mgh = cm\,(t_{melt} - t_1) + \lambda m_{melt.}\ , \quad (9.15)$$
$$\eta\frac{mv^2}{2} = cm(t_{melt.} - t_1) + \lambda m_{melt.}\ . \quad (9.16)$$

In these expressions ϑ – the common temperature, r –the, latent heat of vaporization, λ– is the latent heat of fusion.

§9.5. Humidity. Dew point

Humidity. In the atmospheric air always exists certain quantity of water vapor. The partial pressure of water vapor can not be exceed the certain value depending on temperature and pressure of saturated water vapor. In the given volume at a certain temperature may exists only definite greatest quantity of water vapor.

The greatest amount of water vapor which can be existed in 1 m^3 of air at each temperature is called maximum humidity or amount of saturated vapor.

$$f_{max.} = m/V\ , \quad (9.17)$$

where m the possible maximum mass of water vapor in the air , V is the volume of air.

Usually, amount of vapor in the air is less than the possible maximum value.

Amount of vapor, existing in 1 m^3 of air in fact is called **absolute humidity**.

The ratio of absolute to the possible maximum humidity is called the **relative humidity** of air.

$$\varphi = f / f_{max.} \quad . \qquad (9.18)$$

Since $f_{max.}$ depends upon temperature the relative humidity changes with temperature even though when the absolute humidity remains constant. If the relative humidity is known for temperature T_1 we can find its value for temperature T_2.

$$\varphi_2 = \frac{f_{max.}(T_1)}{f_{max.}(T_2)} \varphi_1 \quad . \qquad (9.19)$$

Dew point. A condensation of vapor existing in the air begins under cooling until definite temperature. This temperature is called the *dew point*.

The relatively humidity reaches 100% under cooling until dew point.

At the temperature is below than the dew point, any part of water vapor is subjected to condensation. If the relative humidity φ_1 is known for temperature T_1, then we can find a mass of dew for any temperature below the dew point.

$$\Delta m = V(f_{max.}(T_{dew.}) - f_{max.}(T_2)) \quad \text{or}$$

$$\Delta m = V(\varphi_1 f_{max.}(T_1) - f_{max.}(T_2)) \quad . \qquad (9.20)$$

Measurement of humidity. For measurement of air relative humidity the hair hygrometers are used. In them the skim hairs which length varies with the change of humidity are applied. Other device for measurement of humidity of air is psychrometer, consisting of two identical thermometers. In one of them a glass ball with mercury is wrapped up by a wet cloth. Water evaporating cools this thermometer and it shows lower temperature than dry. Difference of temperatures serves as a measure of relative humidity. If $\varphi = 100\%$, then $\Delta T=0$.

§9.6. Convection , conduction and thermal radiation

A heat is transferred by means of convection, conduction and thermal radiation processes.

Convection. The density of a warmer fluids and gas is less than colder ones. Therefore they always try to rise upward.
Then moving stream of fluid or gas transfers with its heat. It is called a *convection.*

Conduction. The molecules of that parts of the body, which has a higher temperature have a greater energy and transfers it to surrounded molecules with lower energy. It leads to equality of temperature within of body. In contrary to the convection the *conduction* is not related with transfer of particles. The amount of heat transferred through a layer of substance of thickness l and cross-sectional area A which has a temperature difference ΔT on its planes in a time t is given by

$$Q = \alpha \frac{\Delta T}{l} At \quad . \qquad (9.21)$$

where α-is the *thermal conductivity*. It is measured in Wt/mK. The ratio Q/ t is called a *heat flux*.

Heat transport. When substances are in contact with solids they will give or gain a heat quantity

$$Q = \lambda A t \Delta T \quad , \qquad (9.22)$$

where λ – coefficient of heat transport.

Equivalent coefficient of heat transferring k is related with the coefficients α and λ by expression:

$$\frac{1}{k} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\alpha} \qquad (9.23)$$

In these processes the sum of separate temperature differences is equal to the total temperature difference.

Thermal radiation. The energy, emitted by heated body per unit of time is proportional to the fourth power of the absolute temperature T. A quantity of emitted energy is proportional as well as to the area of radiating solid′s surface, so that energy emitted per unit time is given by

$$\frac{\Delta Q}{\Delta t} = e\sigma A T^4 , \qquad (9.24)$$

where σ- is the coefficient of proportionality, called the Stefan-Boltzman′s constant. The number e value of which has diapason $0 < e < 1$ is called the irradiating ability. For black bodies value of e is near to unit.

Any body not only emits energy but also absorbs the energy radiated by other bodies. Therefore, resultant heat flux irradiated by body is written as.

$$\frac{\Delta Q}{\Delta t} = e\sigma A \left(T_1^4 - T_2^4 \right) , \qquad (9.25)$$

where T_1 the temperature of the body, T_2 the temperature of surrounding medium. As can be seen from (9.25) the heat transfer in any direction disappears only in case, when temperatures of both of bodies and medium are the same.

§9.7. Cyclic processes. Carnot cycle

Set of processes as a result of which the system comes back in an initial condition refers to as *circular processes*. In a basis of work of all cyclic thermal machines lay circular processes. On the P V diagram circular process is represented the closed curve. (Fig 9.4a,b,c) Points 1 and 2 incorporate two various curves. Work made by system at transitions from one state to another is computed by the area under corresponding curve. Carnot has found that in order to achieve the most full transformation of thermal energy into mechanical a cycle consisting of four consecutive thermodynamic process should be done by gas: (Fig. 9.4d)

1. Isothermal expansion (1 – 2)

T_1 = const. $V_2 > V_1$, $p_2 < p_1$.

The gained heat: $Q_{12}= mRT_1 \ln\frac{V_2}{\mathbf{V_1}}$ (9. 26)

Work done by the system $W_{12} = Q_{12}$

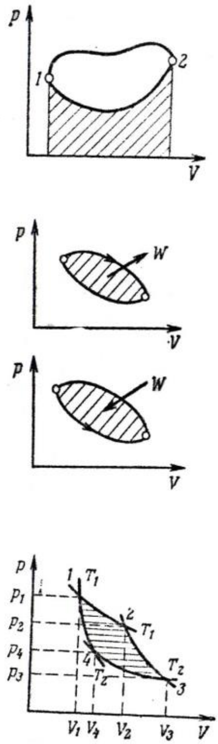


Fig.9.4

2.Adiabatic expansion (2 – 3)

$T_2 < T_1$ $V_3 > V_2$ $p_3 < p_2$.

The obtained heat: Q_{23} =0

Work done $W_{23}=\frac{m\mathbf{R}}{\gamma - \mathbf{1}}(T_1 - T_2)$. (9. 27)

3. Isothermal compression (3 – 4)

T_2 =const. $V_4 < V_3$, $p_4 > p_3$.

Lost heat: $$Q_{34}=mRT_2\ln\frac{V_4}{V_3} \quad . \qquad (9.28)$$

Work done over the system : $W_{34} = Q_{34}$.

4.Adiabatic compression. (4 – 1)

$T_1 > T_2$ $V_1 < V_4$, $p_1 > p_4$
.

Work done on the system:

$$W_{41}=\frac{m\,\mathbf{R}}{\gamma - 1}\left(T_2 - T_1 \right) \qquad (9.29)$$

In these formulas γ is determined as $= C_P / C_V$, where C_P and C_V are molar capacities under constant pressure and volume respectively.

§ 9.8. Heat engines. The thermal efficiency

A device that turns heat into mechanical energy or work is called a *heat engine***.** A heat engine converts part of the heat flowing from a hot reservoir Q_1 to a cold one Q_2 into work W.(Fig.9.5).

Thus

$$W= Q_1 - Q_2 .$$

The thermal efficiency η of a cyclic heat engine is defined to be the ratio of the net work W done by the engine in each cycle to the heat Q_1 absorbed in each cycle, or

$$\eta = \frac{W}{Q_1} = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\% \quad , \qquad (9.30)$$

where Q_2 is the heat rejected in each cycle. From (9. 30) it is obvious that a heat engine will have 100 percent efficiency if $Q_2 = 0$, that is if no heat were exhausted by the engine so that all the heat absorbed were converted to work.

Maximum efficiency of an idealized (without friction and other sources of energy loss) heat engine depends only on the absolute temperatures $T_{hot.}$ and $T_{cold.}$ of the hot operates:

$$E_{eff.max.} = (1 - T_{cold.}/T_{hot.}) \times 100\% \quad . \qquad (9.31)$$

The value of $E_{eff.max.}$ is usually more than 50 percent, however the actual efficiency is less than 40 percent in mostly cases.

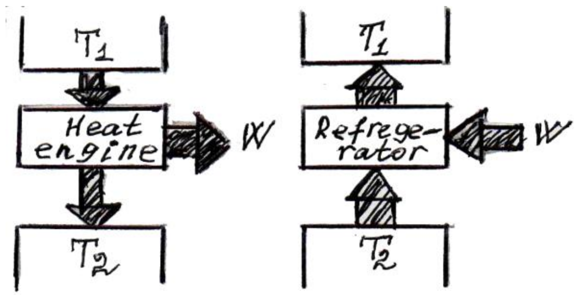


Fig.9.5

The efficiency of internal combustion engines is expressed by the formula:

$$\eta = \frac{W}{km} = \frac{Pt}{km} \qquad (9.32)$$

§ 9.9. Refrigerator. Coefficient of performance

A heat engine operating in reverse transfers heat from a cold reservoir to a hot reservoir with the help of external work. This external work is also converted into heat and is given over to the hot reservoir. A heat engine operating in reverse is called a *refrigerator* or a heat pump. In a refrigerator the interest lies in the amount of heat that is removed from a cold reservoir, while in a heat pump the interest is the amount of heat rejected to the hot reservoir. Both these devices cause heat to flow from a low temperature reservoir to a HTR at the cost of the external mechanical work that must be supplied to the system. The quantity $Q_2 + W$ is the amount of heat Q_1 which is rejected to the hot reservoir. From the first law of thermodynamics

$$Q_1 = Q_2 + W \ .$$

For refrigerator the coefficient of performance E is defined as

$$E = \frac{desired\ out\ \ put}{required\ in\ \ put} = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2} \ , \qquad (9.33)$$

where T_2 is the temperature of the refrigerator and T_1 is the exhaust temperature or the room temperature.

§ 9.10. Thermal expansion

1) Thermal expansion of solids is characterized by the following quantities:
α – coefficient of linear thermal expansion, β – coefficient of surface expansion, and γ – coefficient of volume expansion .

a) **Linear expansion** is calculated by the formula

$$L_2 = L_1 (1 + \alpha \Delta t) \qquad \text{hence} \qquad \alpha = \frac{L_2 - L_1}{L_1 (t_2 - t_1)} \ , \qquad (9.34)$$

where L_1 – the initial length at temperature t_1 , L_2 – the finally lentgh at temperature t_2 , and $\Delta t = t_2 - t_1$.

2) **Surface expansion:** This type of expansion can be considered as a linear expansion in two dimensions:

$$S_2 = S_1(1 + \beta\Delta t) \quad , \qquad (9.35)$$

where $\beta = 2\alpha$

3) **Volume expansion**: Volume expansion is considered as a linear expansion in three dimensions:

$$V_2 = V_1 (1 + \gamma\Delta t) \quad , \qquad (9.36)$$

where $\gamma=3\alpha$.

Expansion of liquids.

A liquid is extended in all dimensions. As a result of great mobility of liquid′ s molecules it accepts the shape of vessel. Expansion of liquids is expressed by the formula:

$$V_2 = V_1 (1 + \gamma\Delta t) \quad . \qquad (9.37)$$

Hence it follows that coefficient of volume extension γ is equal to ratio of relatively extension when temperature rises as 1°C.

The density of liquids decreases with the rising of temperature, since a volume and a density are inverse proportional: $\rho_1 / \rho_2 = V_2 / V_1$

Taking into account formula (9. 36) gives:

$$\rho_2 = \frac{\rho_1}{1 + \gamma \Delta t} \quad . \qquad (9.38)$$

CHAPTER 10

Properties of liquids

§10.1. Surface energy and surface tension

Surface tension is due to force of attraction among the molecules. Force of attraction within of liquids are mutually compensated (molecule A , Fig.10.1). On the molecules placed near the surface (one of those is molecule B) acts the resultant force directed from surface into the bulk of liquids. Therefore in order to remove the molecule from depth to surface the work opposite this resultant force must be performed. The internal energy of the body is proportional to the area of the surface and therefore is known as the surface energy. Owing to the tendency of molecules to go into the bulk of a liquid from its surface the liquid acquires such a shape that its free surface has minimum possible area. In all cases the interaction of a solid with a drop is weak as compared to that forces acting among the parts of a liquid and the tendency of the liquid to reduce its surface is clearly manifested; the spherical shape of the drops corresponds to their minimum surface area. When drops are small the effect of the forces of gravity distorting their shape is insignificant.

The surface energy per unit surface area is called the surface tension and denoted by σ . In order to increase the surface of a liquid by A units of area without changing the state (for example the temperature) of the liquid the work equal to σA must be done:

$$\sigma = W / A \quad . \qquad (10.1)$$

Another definition states that a surface tension is a force exerted by the surface layer per unit length of the contour bounding this layer:

$$\sigma = F / l \quad . \qquad (10.2)$$

In the SI system surface tension is measured in newton per meter (N/ m).

Surface tension may be determined by measuring the force, which should be applied to increase the surface area of liquids; for this aim the wire frame is used.

§10.2. Excess pressure

Under acting of forces of surface tension the surface layer is curved and exerts excess pressure Δp relative to the external pressure. Resultant force is directed towards the curvature centre. Using the (10.1) the excess pressure within spherical drops of liquid and of gas bubbles within liquid may be calculated as follows: to increase the surface area of sphere (Fig.10.1b) the work equal to increment of surface energy must be performed:

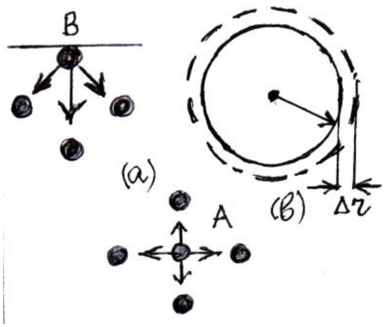


Fig.10.1

$$\Delta W = \sigma \Delta A = \sigma[4\pi (r + \Delta r)^2 - 4\pi r^2]$$

since $r^2 << 2r\Delta r$ we have

$$\Delta W = 8\sigma \pi r \Delta r \quad . \qquad (10.3)$$

By the other hand, the work done is

$$\Delta W = F\Delta r = p\Delta V = pA\Delta r = 4pr^2 \pi \Delta r \quad . \qquad (10.4)$$

Equating the expressions we obtain

$$\Delta p = \frac{2\sigma}{r} \quad . \qquad (10.5)$$

Surface tension is decreased with increase in temperature and vanishes at the critical temperature.

§10.3. Wetting and nonwetting

These phenomena may be observed at the boundary of contacting different medium.
Let us consider the behavior of drops of liquid over the surface of solid:

Wetting is characterized by means of quantity

$$\cos\vartheta = (\sigma_{32} - \sigma_{13}) / \sigma_{21} \quad .$$

here the angle θ is between solid's surface and tangent to the of liquid. It is called *contact angle*.

Let us consider the following cases:

1. $\sigma_{32} > \sigma_{13}$ $\theta < \pi/2$ (Fig.10.2) Then force of interaction among molecules of liquid and solid are greater as compared to the force between molecules of solid and gas (wetting). In this case the surface of solid is called hydrophil.

2. $\sigma_{32} < \sigma_{13}$ $\theta > \pi / 2$ (Fig. 10.2) This case is called nonwetting and the surface of solid is known as hydrophob.

3. $\sigma_{32} - \sigma_{13} = \sigma_{21}$ $\theta = 0$ Then intermolecular forces are completely compensated and the drops .spread out on the surface of solid, forming monomolecular layer. This case is called **ideal wetting.**

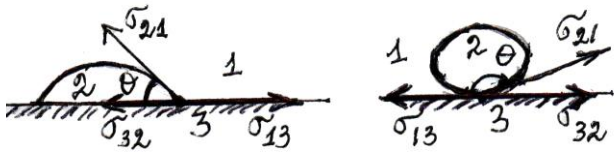


Fig.10.2

§10.4. Capillarity

In narrow tube the edges of the liquid form the entire surface of the liquid so that the surface has the shape resembling a hemisphere (meniscus). In wetting liquids the meniscus is concave upwards, while in nonwetting liquids it is convex downwards. The surface of the liquid becomes curved due to difference in pressure: the pressure under the concave meniscus is lower than under the flat surface. Therefore, if the meniscus is concave, the liquid rising up to the hydrostatic pressure compensates the pressure difference. From Fig. 10.3 it is seen that $r=R / \cos\theta$, where R – radii of capillary tube. Therefore we have $\Delta p=2\sigma\cos\theta / R$.

Then $\rho gh = 2\sigma\cos\theta / R$

Hence

$$h=\frac{2\sigma\cos\theta}{\rho g R} \quad . \qquad (10.6)$$

If the capillary tube moves with acceleration **a** then (10.6) takes the forms

$$h = \frac{2\sigma\cos\vartheta}{\rho\ R\,(g+a)}\ \text{(upward)},$$

$$h = \frac{2\sigma\cos\vartheta}{\rho\,R\,(g-a)}\ \text{(downward)}.$$

The mass of liquid rising in the capillary tube may be found from the condition *mg* = *σ l* by means of formula

$$m = \frac{2\pi\, r\,\sigma}{g} \quad . \qquad (10.7)$$

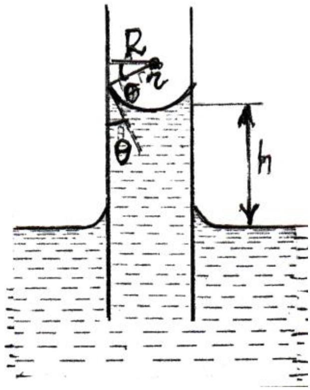


Fig.10.3

The surface tension of any liquid in the capillary one can find comparing it with the known surface tension σ_1 and by measuring the rising heights and number of drops:

$$\sigma_2 = \sigma_1\,\frac{\rho\ h_2}{\rho_1\,h_1} \quad \text{or} \quad \sigma_2 = \sigma_1\,\frac{\rho_2\,N_1}{\rho_1\,N_2}\ , \qquad (10.8)$$

where $\rho_{1,2}$ – the densities, $N_{1,2}$ – the numbers of drops and $h_{1,2}$ – are the rising heights.

CHAPTER 11

Static electricity

§11.1. Electric charge. Law of conservation of electric charge

A physical quantity owing to which the electrical interaction occurs is known as *electric charge*. The existence of two types of electric charges is suggested. One of these is positive and other is negative. Like charges repel each other and unlike charges attract each other. Electric charges are measured by means of device called *electrometer*. It consists of metal shank and needle which may rotate about horizontal axis.

Numerous experiments have showed, that electric charges equal in magnitude and opposite in sign arise as a result of electrization upon conductor over the objects. Appearance and disappearance of electric charges over the objects are explained in more cases by the transitions of elementary charged particles – electrons from one to another.

The SI unit of charge is the coulomb (C) One coulomb is defined as the charge flowing through the cross section of a wire with a current one ampere per sec:

1 C=1A x 1sec.

The coulomb is the amount of charge found on 6.25×10^{18} electrons.

1 micro – coulomb (1 μC) = 10^{-6} C.
1 nano –coulomb (nC) = 10^{-9} C.
1 pico – coulomb (pC) = 10^{-12} C.

In the closed system the algebraic sum of electric charges of all objects remains constant upon any interactions:

$$q_1 + q_2 + q_3 + \ldots + q_n = \text{const}.$$

This equation expresses law of *conservation of electric charge*.

§ 11.2.Interaction of electric charges. Coulomb′s law

The first quantitative measurement of the force between two electric charges was first made by a Augustin de Coulomb. Coulomb′ s law states : A force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them. In the SI system Coulomb′ s law is mathematically written as:

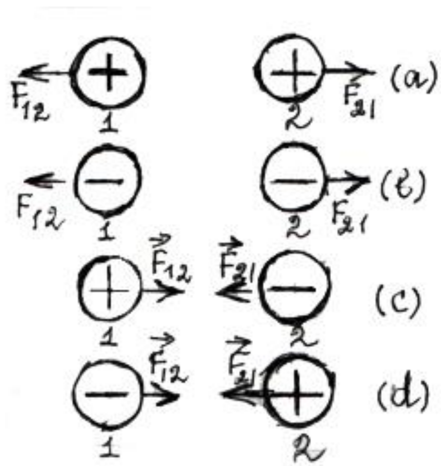


Fig.11.1

$$F = \frac{q_1\, q_2}{4\,\pi\,\varepsilon_0\, r^2}\ , \qquad (11.1)$$

where **r** is measured in meters F in newtons and q_1 , q_2 in coulombs. Here $\boldsymbol{\varepsilon_0} = \mathbf{8{,}85x10^{-12}}$ $\left(A^2 sec^2\right)/m^2$ is the electric constant: This force is directed along the line of connecting charged objects. Like charges exert forces of repulsion (Fig. 11.1a,b; $F > 0$, such as $q_1q_2 > 0$) and unlike charges exert forces of attraction (Fig.11.1c,d ; $F < 0$, such as $q_1q_2 < 0$)

§ 11.3. Electric field. Electric field strength. Superposition principle

According to Michael Faraday the interaction between charges q_1 and q_2 separated by some distance is explained by introducing the electric field concept: the charge q produces an electric field in the space surrounding it and the field interacts with a charge q which is brought in the field and produces a force F on it.

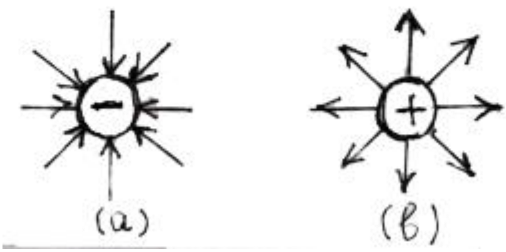


Fig.11.2

A physical quantity determined as the ratio of force acting on the point electric charge to the value of this charge is called the *electric strength* of field. If force experienced by a point positive charge q_2 in an electric field of q_1 is F , then

$$E = \frac{F}{q_2} \quad . \qquad (11.2)$$

Using the Coulomb's law we obtain an expression of a magnitude of electric field strength at some point of space:

$$\mathbf{E} = \frac{\mathbf{q}}{\mathbf{4}\,\pi\,\varepsilon_{\mathbf{0}}\mathbf{r}^{\mathbf{2}}} \quad . \qquad (11.3)$$

Intensity is a vector quantity. If **$q < 0$** then field lines are directed towards the charge (Fig.11.2a,) if **$q > 0$** then field lines are directed apart the charge (Fig. 11.2b).

If the different charged particles produce electric fields with intensities E_1, E_2, E_3 and etc. at the given point of space, then the resultant vector of intensity is equal to the vector sum of separate intensity vectors:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n$$

For example, magnitude of intensity of the field produced by two charges is given by

$$E = \left(E_1^2 + E_2^2 + 2E_1 E_2 \cos\varphi \right)^{1/2} , \qquad (11.4)$$

where **φ** is the angle between vectors $\vec{E}_1$ and $\vec{E}_2$

In particular , (Fig.11.3a) if the vectors coincide then

$$E = E_1 + E_2 .$$

If vectors have opposite directions (Fig.11.3b), then

$$E = E_1 - E_2 .$$

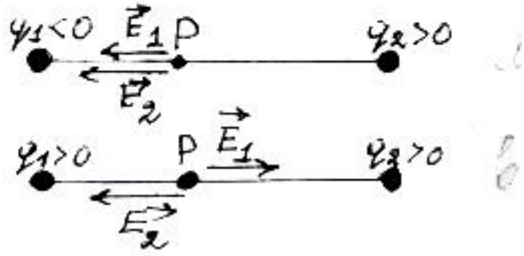


Fig.11.3

Intensity of the electric field of point charge in dielectric is equal to

$$E = \frac{1}{4\pi\ \varepsilon_0}\frac{q}{\varepsilon\, r^2}\ , \qquad (11.5)$$

where ε- is the *relative dielectric permittivity* of medium.

Solid sphere of charge. An electric charge q is distributed uniformly throughout a nonconducting sphere of radius r_0. Magnitude of the electric field as a function of the distance r from the center of a uniformly charged solid sphere is given Fig.11.4a. Inside the sphere the field increases linearly with r until $r = r_0$.

$$E = \frac{1}{4\ \pi\,\varepsilon_0}\frac{q}{\varepsilon\, r^2}\ , \quad r \geq r_0 \quad (r \leq r_0 \ , E=0) . \qquad (11.6)$$

Outside electric field of the charged solid sphere coincides with the field of point charge (equal to charge of sphere) placed at the center of sphere.

Spherical shell of charge. Electric field inside a uniform spherical shell of charge is zero. Again the field outside a spherical shell of charge is the same as that for a point charge of the same magnitude located at the center of the sphere.
(Fig.11.4 b)

A uniformly charged infinite plate produces homogeneous field whose strength is defined by

$$E = \frac{\sigma}{2\ \varepsilon_0\ \varepsilon}\ , \qquad (11.7)$$

where σ– the charge ′s surface density given by

$$\sigma = \frac{q}{A}\ .$$

Two opposite charged parallel infinite plates with equal

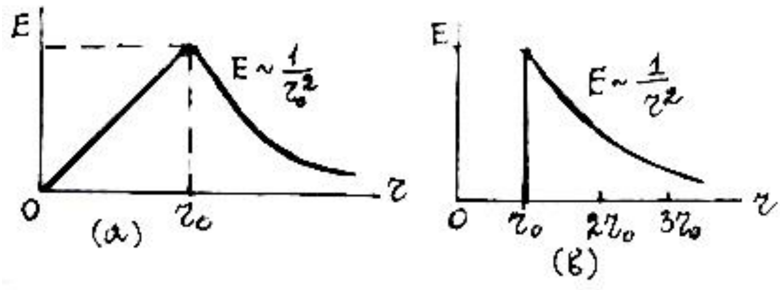


Fig.11.4

charge densities σ creates a homogeneous electric field. The magnitude of intensity of this field is

$$E=\frac{\sigma}{\varepsilon_0 \varepsilon} \quad . \tag{11.8}$$

Now suppose, that a drop having a charge q is at rest or in uniformly motion between horizontally placed plates. Since the drop is acted by the electric field its charge can be found from equality:

$$mg = qE \tag{11.9}$$

A number of excess electrons in a drop is calculated as

$$N_e=\frac{mg}{eE} \tag{11.10}$$

Now suppose, that charged drop is in an accelerated motion. Equations of drop's motion can be given as

a) downward then $ma = mg - eE$

b) upward then $ma = eE - mg$

§11. 4. Conductors in an electric field

Conductors are the substances, in which a regular movement of electric charges may occur or the electric current can carried out.

If the metal conductor is placed in an electric field then a regular movement will be added to the irregular movement of electrons and those will remove in opposite direction to the field intensity lines.

Electrons in a metal conductor will be redistributed under the action of an external electrostatic field so, that field intensity E will be equal to zero within of it: **E=0**. Uncompensated charges are distributed over the outer surface of a charged object.

A phenomenon of charge redistribution in a conductor is due to an electrostatic field and is called *electrostatic induction*.

If the conductor has an internal cavity then the field intensity within it is equal to zero independent whether the conductor is charged or not. The internal cavity in the conductor is screened from external electrostatic fields.

Potentials φ (sec. 3 1.6) of all points within conductor are identical. Surfaces of charged conductor serves to be *equipotential surfaces* where potentials of all points are equal.

§11.5. Electric dipole.

A system of two unlike charges, equal in magnitude and placed at a distance one from another is called – **electric dipole**. (Fig.11.5a) Dipole′ s electric field intensity E is equal to the geometrical sum of individual intensities produced by each of charges (Fig.11.5b).

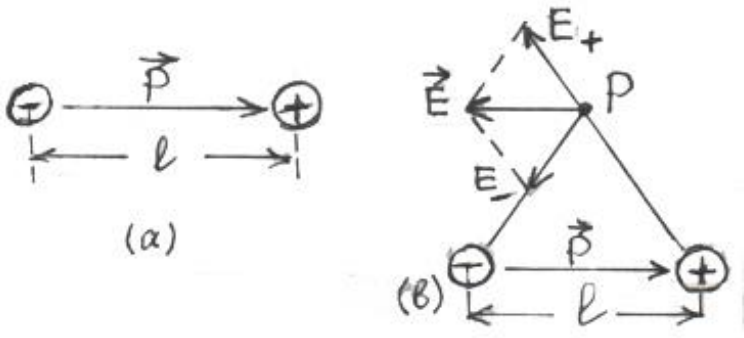


Fig.11.5

$$E = E_{+} + E_{-} \quad , \qquad (11.11)$$

where $E_{+} = E_{-} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{(r^2 + l^2/4)}$.

The dipole is characterized by a vector quantity called the electric moment (p_e)

$$\vec{p}_e = q\vec{l} \quad , \qquad (11.12)$$

where l – is the distance between the charges. The vector of p is directed from – q to + q.

§11.6. Work done by electrostatic field. Potential and difference in potentials

1. A work done upon removing of positive charge between points 1 and 2 in homogeneous field with intensity E is equal to

$$W = -qE(x_2 - x_1) \quad , \qquad (11.13)$$

where x_1 and x_2 are the x axis

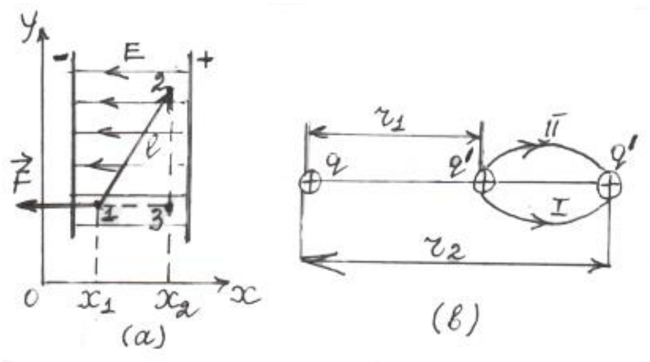


Fig.11.6

coordinates of points 1 and 2 respectively (Fig.11.6a). Sign of minus indicates that a directions of force and displacement are opposite. Work performed upon the removing of charge is independent on the kind of trajectory and depends only on initial and finally positions of charge. Thus, the works performed upon removing over the trajectories 1 – 2 and 1– 2 – 3 are equal.

2. Work done upon the removing of charge between points B and C is equal to

$$W=\frac{q\,q'}{4\pi\varepsilon_0\varepsilon}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \qquad (11.14)$$

where r_1 and r_2 are the distances of points B and C from charge q. The work performed by electrostatic forces, when a charge removes along the closed trajectory is equal to zero(Fig.11.6b).

A physical quantity equal to the ratio of potential energy of charge and its magnitude is called the *potential* of electric field. Potential is measured also by the work done to remove the unit positive charge from Earth' s surface to infinity.

$$V = \frac{W}{q}$$

.

Hence it follows that electric potential due to single charge *q* at a distance *r* away from it is given by

$$V = \frac{q}{4\pi\varepsilon_0 \text{r}} \qquad (11.15)$$

Electric potential is a *scalar* quantity. If the electric field at any point of space is produced by several charges then the resultant potential is equal to the algebraic sum of individual potentials:

$$V = V_1 + V_2 + \ldots + V_n \; . \qquad (11.16)$$

For example the dipole′s (Fig.11.7) potential is equal to the sum of potentials produced by each of its charges:

$$V = \frac{\mathbf{1}}{\mathbf{4}\,\pi\,\varepsilon_0}\frac{\mathbf{p\cos\theta}}{\mathbf{r^2}} \quad , \quad (\text{r} >> 1)$$

Potential difference is numerically equal to the work which is performed by the forces of electrostatic field when positive unit charge moves between those points.

$$V_1 - V_2 = \frac{W}{q} \qquad (11.17)$$

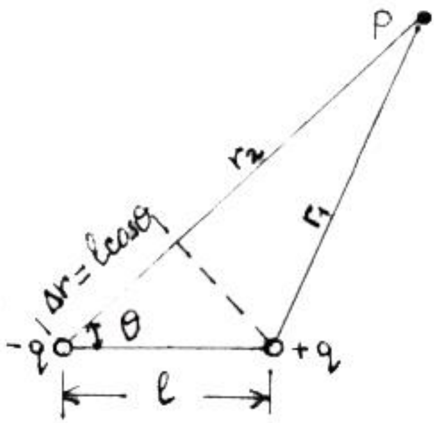


Fig.11.7

Unit of potential is the volt: $1\,\text{V} = \dfrac{1\,\text{J}}{1\,\text{C}}$.

Potential difference between two points 1 and 2 placed far from uniformly charged infinite plane is expressed by the formula:

$$V_1 - V_2 = \frac{\sigma}{2\,\varepsilon_0 \varepsilon}(x_2 - x_1) \; . \qquad (11.18)$$

Potential difference between two infinite parallel plates is

$$V_1 - V_2 = \frac{\sigma\,\mathbf{d}}{\varepsilon_{\mathbf{0}}\;\varepsilon} \; , \qquad (11.19)$$

where d – is the distance between plates

The intensity at an arbitrary point of electrostatic field equals in magnitude to the changing of potential per unit length of line of force.

$$E = \frac{\Delta V}{\Delta l} \; . \qquad (11.20)$$

The SI unit of field strength is volt / meter (V/m).

§11.7. Electric capacitance. Combination of capacitors

A device used for storing electric charge is called a capacitor**.**

A physical quantity determined as ratio of charge q of single conductor to its potential φ is called the electric capacitance of conductor and characterizes the capability of a capacitor to store charge:

$$C = \frac{q}{V} \; . \qquad (11.21)$$

Electric capacitance of geometrically similar conductors direct proportional to their linear sizes, relative permitivity of medium where a conductor is placed.

A mutual capacitance of two conductors is called a physical quantity, which is equal in magnitude to the charge per unit difference in potentials.

$$C=\frac{q}{V_1 - V_2} \qquad (11.22)$$

Taking into account formula for electric potential of sphere $V=\frac{1}{4\pi\varepsilon_0}\frac{q}{\varepsilon R}$ gives

$$C = 4\pi\varepsilon_0 R \quad (\text{for free space}) \qquad (11.23)$$
$$C = 4\pi\varepsilon_0 \varepsilon R \quad (\text{for medium})$$

When two spheres are in contact the electric charge flows from one to another up to equality of potentials. Then the sum of charges remains unchangeable (conservation of charge). The finally charges of spheres one can find using the following equations:

$$q_1 + q_2 = q_1' + q_2' \qquad V_1' = V_2'$$

where $\mathbf{q_1}$ and $\mathbf{q_2}$ are the initial charges of spheres.

The reciprocal of equivalent capacitance for capacitors in series (Fig.11.8a) is the sum of reciprocals of individual capacitances :

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+...+\frac{1}{C_n} \quad . \qquad (11.24)$$

In particular , if the battery consists of two capacitors, then

$$C = \frac{C_1 C_2}{C_1 + C_2} \quad . \qquad (11.25)$$

The charges of capacitors connected in series are equal to

$$q_1 = q_2 = ... = q_n = q \quad . \qquad (11.26)$$

The voltage of the ends of battery is equal to the sum of voltages of individual capacitors:

$$V = V_1 + V_2 + ... + V_n \quad . \qquad (11.27)$$

The total capacitance of capacitors connected in parallel (Fig.11.9b) is equal to the sum of individual capacitors:

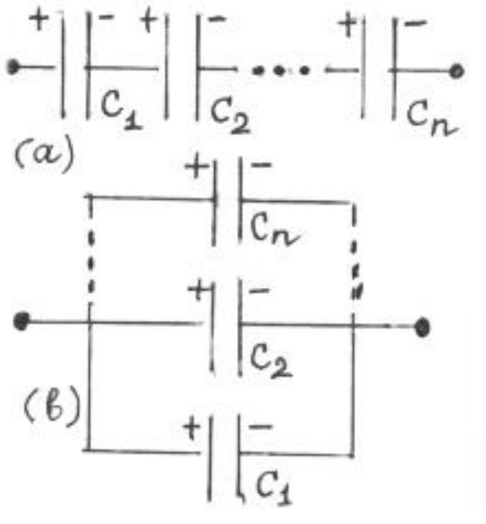


Fig.11.8

$$C = C_1 + C_2 + ... + C_n \qquad (11.28)$$

When the capacitors are connected in parallel the voltages between plates are equal

$$V = V_1 = V_2 = ... = V_n \qquad (11.29)$$

In this case the total charge of the system is equal to the sum of charges of individual capacitors:

$$q = q_1 + q_2 + ... + q_n \qquad (11.30)$$

§11. 8. Parallel plate capacitor

. g) The capacitance of parallel plate capacitor is given by the formula (n=2)

$$C = \frac{\varepsilon\varepsilon_0 A}{d} \qquad (11.31)$$

where A– the surface area of a single plate, d –the distance of separation between plates, ε– the relative dielectric permitivity.

d) Multiple capacitor

$$C = \frac{\varepsilon\,\varepsilon_0\, A}{d}(n-1) \qquad (11.32)$$

where n – is the number of plates of capacitor.

There are some also another types of capacitors like as variable capacitor, tubular capacitor, electrolytic and miniature capacitors; Variable capacitors (gang) are commonly used in tuning the radio circuits and television receivers. This is again aan example of parallel plate capacitor with variable capacitance. A tubular capacitor consists of plates of thin metal foil separated by a sheet of paper or plastic film and rolled up into a small package.

§11.9. Energy of electric field

The energy of the electric field between plates of charged capacitor can be calculated by one of the following formulas:

$$W = \frac{CV^2}{2} \quad ; \quad W = \frac{qV}{2} \quad \text{or} \quad W = \frac{q^2}{2C} \qquad (11.36)$$

The energy density of an electric field is determined by the ratio of energy to the volume of capacitor:

$$w = \frac{W}{A\,d} \, , \qquad (11.37)$$

where A – is the area of plates , d- refers to the distance of separation between plates

Energy density is expressed by the charge density as

$$\mathbf{w} = \frac{\sigma^2}{2\,\varepsilon\,\varepsilon_0} \; . \qquad (11.38)$$

Here $o = q / A$-refers to the charge density, ε -is the dielectric permittivity of medium

CHAPTER 12

Direct electric current in metals

§12.1. Electric current. Current density

The orderly motion of charge carriers (in metals they are electrons) is called *electric current*. The direction of the current is defined as the direction opposite to that in which the negative charges move.

The quantity of charge, which flows across any cross – section of the conductor per unit time is called current intensity.

$$I = \frac{q}{t} . \qquad (12.1)$$

Current is measured in **ampere** one of the basic units of SI system.

The ratio of current to the cross – sectional area of the conductor is the current density.

$$J = \frac{I}{A} = \frac{I}{\pi r^2}$$

The unit of measurement of current density in SI system is ampere/m^2 .

Current density is related to the concentration **n** of charge carriers and their velocities **v** as follows:

$$J = e\,n\,\upsilon, \qquad (12.2)$$

where **e** – is the electric charge of electron which is equal to the elementary charge: $\mathbf{e=1.6 \times 10^{-19}}$ C.

The current is a scalar quantity, while current density a vector quantity.

§12.2. Work done due to motion of a charge in conductors. Electromotive force. Voltage

When a charge moves along the conductor a work is performed by the both coulomb and outside forces.

$$W=W_{coul.} + W_{out.} .$$

Electromotive force is a quantity numerically equal to the work performed by outside forces upon removing of positive unit charge.

$$\varepsilon = \frac{W_{out.}}{q} \qquad (12.3.)$$

Voltage. The voltage is a quantity defined as a ratio of work performed upon the movement of positive charge, from one point to another to the magnitude of charge. Thus, by using formulas (11.17) and (12.3) we get

$$U= V_1 - V_2 + \varepsilon . \qquad (12.4)$$

If the electric field is nonpotential then a voltage is equal to the potentials difference between two points.

$$\Delta V = V_1 - V_2.$$

The SI unit both of voltage and difference in potentials is volt (V) .

Relationship between voltage and field intensity is given by the expression:

$$\Delta V = Ed\,, \qquad (12.5)$$

where d – is the distance between electrodes.

§12.3. Ohm′s law for a section of circuit. Resistance and resistivity

Ohm′s law states that current power in circuit is direct proportional to the potential difference and inverse proportional to the resistance:

$$I = \frac{V}{R} \qquad (12.6)$$

where R is the resistance depending upon the type of material, its geometry and temperature. Resistance is measured in ohm: 1 ohm=1 V / 1A. Resistance of cylindrical wire is calculated by the formula:

$$R = \rho \frac{l}{A} \qquad (12.7)$$

where l – the length of a wire, A – the cross – sectional area, ρ – is the resistivity. Unit of measurement of resistivity is ohm x meter (ohm m).

Resistivity is a function of temperature as follows:

$$\rho = \rho_0 \left(1 + \alpha t\right) , \qquad (12.8)$$

where α – the temperature coefficient of resistivity, ρ_0 – resistivity at 0 °C temperature (Fig.12.1) The unit of α is K^{-1}.

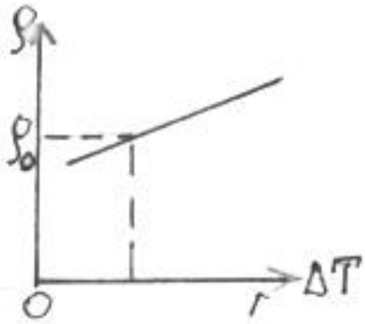


Fig.12.1

Electric field intensity within of a wire is determined as

$$E = \frac{\Delta V}{l} = \frac{IR}{l} = \frac{I\rho}{A} .$$

§12.4. Combination of resistors

a) Connection in series .

When two or more wires are connected in series (Fig.12.2a)

The current power flowing through individual wires is the same:

$$I_1 = I_2 = ... = I_n I \quad (12.9)$$

In this case the voltage at the ends of wire is equal to the sum of voltages of individual wires:

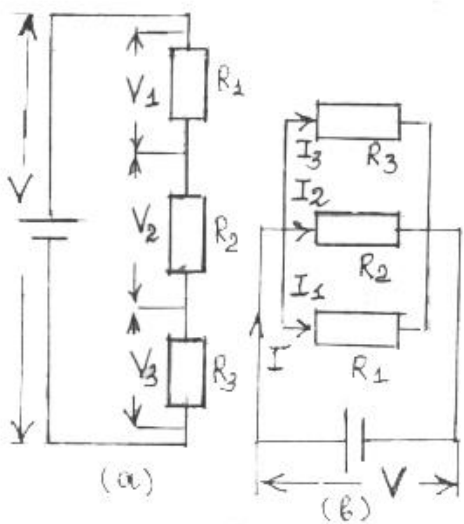


Fig.12.2

$$V = V_1 + V_2 + ... + V_n \qquad (12.10)$$

The equivalent resistance is determined as the sum of individual resistances:

$$R_{ser.} = R_1 + R_2 + ... + R_n \qquad (12.11)$$

If the individual resistances are identical then

$$R_{ser.} = nR\ , \qquad (12.12)$$

where n –the number of resistors in series.

In this type of connection the voltages of conductors are directly proportional to their resistances:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}\ . \qquad (2.13)$$

b) Connection in parallel .

When wires are connected in parallel (Fig.12.2b) the voltage at the ends of individual wires are the same:

$$V_1 = V_2 = ... = V_n = V\ . \qquad (12.14)$$

In this case the total current is equal to the sum of individual current powers:

$$I = I_1 + I_2 + ... + I_n\ . \qquad (12.15)$$

The reciprocal of equivalent resistance of circuit connected in parallel is the sum of reciprocals of individual resistances

$$\frac{1}{R_{par.}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \quad . \qquad (12.16)$$

When resistances are equal, then $R_{par.} = \frac{R}{n}$, where **n** is the number of resistors connected in parallel.

Currents in sections connected in parallel inversely proportional to their resistances

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \quad . \qquad (12.17)$$

§12.5. Ohm′ s law for a closed circuit

A motion of charge q over the closed circuit is due to the work **W** performed by the exterior forces. A ratio of this work to the magnitude of charge **q** is called the electromotive force (e.m.f.) Fig.12.3

$$\varepsilon = \frac{\mathbf{W}}{\mathbf{q}} \quad . \qquad (12.18)$$

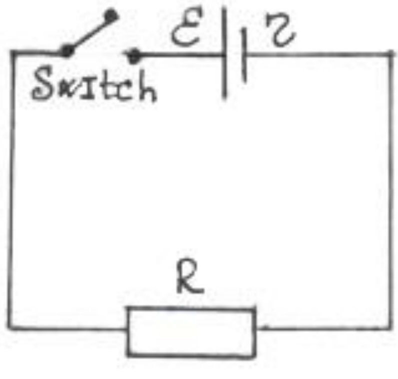


Fig.12.3

E.m.f. is measured in SI system in volts (V).

Power of current flowing in closed circuit can be found by the Ohm′ s law:

$$\mathbf{I} = \frac{\varepsilon}{R + r} \quad , \qquad (12.19)$$

where ε – the e.m.f. of current source, r – the internal resistance, R – the external resistance.

The expression (12.19) can be written as

$$\varepsilon = IR + Ir \quad . \qquad (12..20)$$

The right side of final expression is the sum of voltage drop in an external circuit as well as in source. Thus the e.m.f. is equal to voltage drop across the entire circuit:

$$\varepsilon = \mathbf{U_R + U_r} \; .$$

When the external resistance R=0 the current power takes greatest value which is called the short circuit.

$$I = \frac{\varepsilon}{r} \quad .$$

When the resistors R_1 and R_2 are connected with the current source then the e.m.f. and internal resistance are found by means of system of equations:

$$\varepsilon = I_1 (R_1 + r) \quad , \qquad (12.21)$$

$$\varepsilon = I_2 (R_2 + r) \quad ,$$

where I_1 , I_2 are the current powers in resistors R_1 and R_2 respectively.

If the powers liberated in currents I_1 and I_2 are P_1 and P_2 respectively then Σ and r are found using the system of equations

$$P_1 = \left(\frac{\varepsilon}{R_1 + r} \right)^2 R_1 \quad ,$$

$$P_2 = \left(\frac{\varepsilon}{R_2 + r} \right)^2 R_2 \quad . \tag{12.22}$$

The efficiency is determined as a ratio of power in external circuit to the power in total (closed) circuit. Thus we get

$$\eta = \frac{R}{R + r} \tag{12.23}$$

In case of combination of emf sources in series the equivalent emf is equal to the sum of individual emf (Fig. 12.4a)

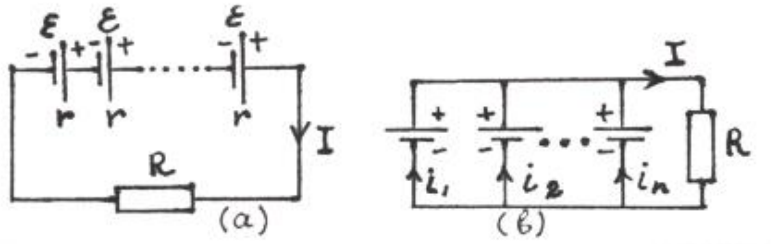


Fig.12.4

When the emf sources are connected in parallel total emf is found by subtracting and when the emf sources are connected in series total emf is found by adding the individual emf.

§12.6. Multi-loop circuits. Kirchoff′s rules

The electric circuit is a complex of conductors and sources of currents. In general case the circuit is multiloop and has the branch points. To calculate the current powers of each section in branched circuits the both rules of points and contours are used:

1. Kirchoff′s first rule.

Consider any section of arbitrary branched circuit. Let the branched point be A. From the condition that current is constant it follows, that a quantity of charge entering into point A per sec is equal to the charge flowing from A per sec. In other words, the sum of currents flowing into a point is equal to the sum of currents flowing from point. For the point A from Fig.12.5.a we obtain:

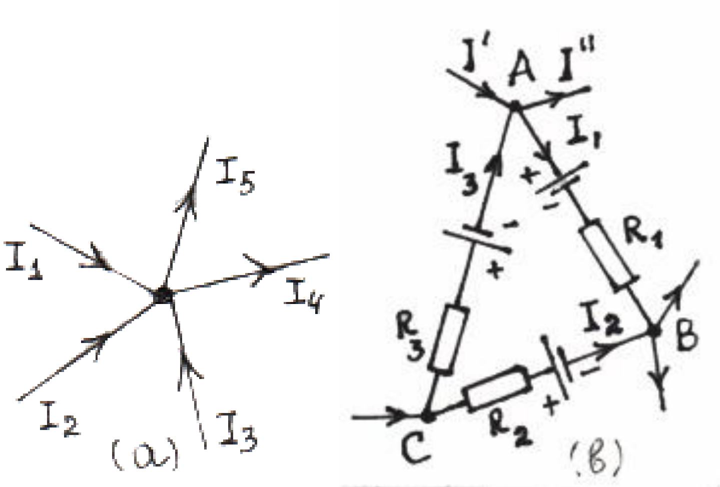


Fig.12.5

$$I' + I_3 - I'' - I_1 = 0 . \qquad (12.24)$$

Algebraic sum of all currents at the point A is equal to zero.

2. Kirchoff′s second rule.

Let us apply the Ohm′s law to individual current loops of ABC circuit (Fig.12.5b) First choose the reference direction of closed circuit. We will consider that the e.m. forces creating the currents in the direction of reference are positive, e.m.f which create opposite currents are negative.

All currents, coinciding with the reference direction of closed circuit are positive. Sign of voltage drop is determined by the sign of current, flowing through the resistors.

Suppose, V_A , V_B , V_C are the potentials of points A, B, C respectively. Then we can write:

$$V_B - V_A + I_1 R_1 = -\varepsilon_1,$$
$$V_C - V_B - I_2 R_2 = \varepsilon_2, \qquad (12.25)$$
$$V_A - V_C + I_3 R_3 = -\varepsilon_3,$$

where I_1, I_2, I_3 – the currents, R_1, R_2, R_3 – the resistances, ε_1, ε_2, ε_3 –electromotive forces of corresponding circuits. Summation of equations above gives

$$I_1 R_1 - I_2 R_2 + I_3 R_3 = -\varepsilon_1 + \varepsilon_2 - \varepsilon_3 \,. \qquad (12.26)$$

As can be seen an algebraic sum of voltage drops in the closed circuit is equal to the algebraic sum of e.m.f. acting in this circuit. This is the *Kirchoff′s second rule* .

§12.7. Work done due to current. Joule-Lenz′s law

When a current flows through conductor it performs a certain amount of work. The work performed during of time interval **t** is given by the formulas for section of circuit.

$$W = \frac{U^2}{R} t = I^2 R t, \qquad P = \frac{U}{R} t = I^2 R. \qquad (12.27)$$

and for closed circuit

$$W = \frac{\varepsilon^2}{R + r} t, \qquad P = \frac{\varepsilon^2}{R + r}. \qquad (12.28)$$

Joule –Lenz′ s law: A quantity of heat (energy) developed in the resistor is calculated by the formulas:

$$Q = IUt \qquad \text{or} \qquad Q = \frac{U^2}{R}t \qquad (12.29)$$

If the electric quantities are measured in SI system and heat in calories then formulas takes the form:

$$Q = 0{,}24\, I^2 Rt$$

If the efficiency of an electric engine is η the mechanical power N is closed to the electric power $P = VI$ by the relationship

$$N = \eta P \quad . \qquad (12.30)$$

CHAPTER 13

Current in various media

§13.1. Current in electrolytes. Ohm′s law

Substances whose solutions are carrying out the electric current are known as electrolytes. Electric current in electrolytes is due to motion of ions .Pure water and crystals of copper chloride do not carry out the current, however solution of copper chloride is considered to be a good conductor.

When current flows through the solution the chlorine is liberated on the positive electrode while copper is liberated on the negative electrode.

Density of current due to ions of both signs is given by

$$\mathbf{j = n_+ e\, \upsilon_+ + n_- e\, \upsilon_-} \quad , \qquad (13.1)$$

where $\mathbf{n_+}$ ($\mathbf{n_-}$) is the concentration of positive (negative) ions, υ_+ (υ_-) the drift velocities of the positive(negative) ions, and e – is the charge of electron, The mobility of ions is defined as the average drift velocity attained by ion in a field of 1V/cm strength.

$$u = \frac{\upsilon}{E} \quad . \qquad (13.2)$$

The current density can be expressed in terms of ion mobilities $\mathbf{u}_+$ and $\mathbf{u}_-$:

$$J = (n_+ u_+ + n_- u_-) eE \quad . \qquad (13.3)$$

This expression is called the ***Ohm′s law*** for electrolytes.

§13.2. Electrolysis. Faraday′s laws

The process of decomposition of an electrolyte by an electric current is called electrolysis.

Faraday′s first law: The mass of any substance liberated at the electrode as a result of electrolysis is proportional to the total quantity of charge **q** passed through electrolyte:

$$m=kq \quad \text{or} \quad m=kIt \ , \qquad (13. 4)$$

where k – is called the *electrochemical equivalent* and is numerically equal to the mass of a given substance liberated when unit positive charge passes through electrolyte.

Faraday′s second law: The electrochemical equivalent of a given substance is proportional to its chemical equivalent:

$$k=\frac{1}{F}\frac{A}{Z} \ , \qquad (13. 5)$$

where F–is called Faraday’ s constant, which is the quantity of electric charge equal in round numbers to 96.500 coulombs, Z-indicates the valence. If this quantity of charges passes through a solution of an electrolyte a mass of substance equal to the chemical equivalent of the substance will be liberated. The quantity $\frac{\mathbf{A}}{\mathbf{Z}}$ is defined as the ratio of the atomic weight of an element to its valence and is called *chemical equivalent*.

Finally, general law of electrolysis is written as follows:

$$m=\frac{1}{F}\frac{A}{Z}q \quad , \tag{13.6}$$

From (13.6) a charge of any ion is determined as:

$$q=\pm\frac{Z\,F}{N_A} \, .$$

In order to liberate the net mass **m** a work done by current is equal to

$$W=q\,\Delta V=\frac{m}{k}\Delta V \, , \tag{13.7}$$

where ΔV - is the voltage between electrodes.

§13.3. Electric current in gases. Types of electric discharge

Thermionic ionization.

A gas becomes a conductor as a result of heating, therefore any fraction of its atoms or molecules is converted into ions.

In order to remove electron from the atom a certain work opposed to the coulomb attraction between nucleus and electrons must be done. The process of liberation of electron from atom is called the *ionization.* The minimum energy required to remove electron from an atom is called the *binding energy*.

Electrons may escape from atoms, when two atoms strike one another and their kinetic energy exceeds the binding energy of electron. The kinetic energy of heat movement of electrons is directly proportional to the temperature. The appearance of free electrons and positive ions as a result of collisions between atoms and molecules at high temperatures is called the *thermionic ionization.*

Induced discharge.

A substance in gas state is usually insulated, because of its atoms or molecules are consisted of the equal number of negative and positive electric charges and therefore totally are neutral. By means of experiments is shown that gas may become a conductor.

The phenomenon of the passage of an electric current through a gas, observed only under of any external action other than an electric field (for example. heating, X rays, etc.) is called an *induced discharge*.

Induced gas discharge is characterized with the dependence of current I versus voltage U between electrodes. In Fig.13.1 the curve OCA is called the volt-ampere characteristics. The value of current $I_{sat.}$ independent the voltage is called *saturated current*. $I_{sat.} = e N_0$ where N_0 the maximum number of pair of single valence ions, produced per unit volume and time. The AB part is related to the bombardment ionization.

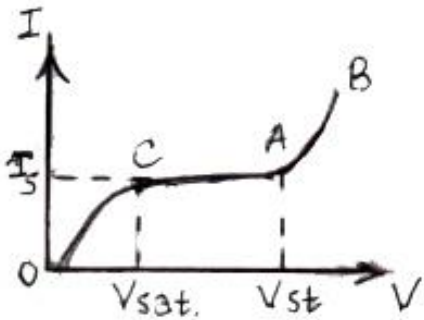


Fig.13.1

Plasma. A gas in which a sufficient number of atoms and molecules are ionized is called a *plasma*. It is characterized with the ionization power which is determined by the ratio of concentration of charged particles to the number of total concentration. The ionization power of plasma depends on temperature. Electrons and ions of plasma can be removed under action of an electric field. Thus, gas serves to be an insulator at low temperatures and is converted into plasma at high temperatures and becomes a conductor.

Depending on ionization degree a plasma may be:

a) weak ionized
b) partially ionized
c) perfect ionized.

Intrinsic discharge. When a strength of electric field E in gas increases up to definite value a current independent on external ionizators appears. This phenomenon is called *intrinsic discharge***.**

The basic mechanism of ionization of gas during intrinsic discharge is the ionization of atoms and molecules due to bombardment by electrons. This kind of ionization becomes possible when an electron acquires over free path the energy, exceeding the binding energy W with atom.

Kinetic energy acquired due to electric field is equal to the work done by it

$$E_k = Fl = eEl,$$

where l – is the mean free path.

From the condition for ionization it follows

$$eEl > W .$$

Binding energy is usually expressed in electron–volts (eV). An electron–volt is the energy acquired by an electron when it passes through 1 volt potential difference.

$$W = e\Delta V$$

$1\ eV = 1.6 \bullet 10^{-19}\ C \bullet 1V = 1.6 \bullet 10^{-19}$ Joule.

Mechanism of intrinsic discharge

An increase of the intrinsic discharge in gas occurs by the following way. Free electron acquires an acceleration due to action of electric field. If the field strength is higher, electrons increase their kinetic energy over free passing so, that it can ionizate the molecule upon collisions.

The former electron which causes ionization of the molecule and the second escaped electron acquire an acceleration directed from cathode towards anode. Each of those electrons may liberate one electron upon collisions, thus total number of free electrons becomes equal to four. Further they can reached up to 8, 16, 32 and etc.

The number of free electrons moving from cathode towards anode rises avalanche –like while they reach anode. Positive ions appeared in gas move from the anode toward the cathode due to electric field action. New electrons may be liberated from the cathode as a result of bombardments of latter by the positive ions and due to light radiated in discharge process. These electrons are accelerated by electric field and produce in their turn a new electron– ion avalanche. Therefore the process may be continued uninterruptedly.

Concentration of ions in plasma increases with rising in intrinsic discharge and a resistance of discharge band decreases. Current in intrinsic discharge circuit is usually determined only by internal resistance of source.

Spark discharge

If the current source is not able to support the intrinsic discharge for long time then the occurred intrinsic discharge is called *spark discharge*. A lightning is a good example of spark discharge. Current in lightning channel reaches up to $(1 - 2)\ 10^{4}$ A and pulse duration is measured in microseconds. With rising of current in a lightning channel a heating of plasma reaches up to temperatures more than 10^{5} K. A change in pressure of plasma channel of lightning with the rising of current causes the sound phenomenon, called *thunder.*

Grow discharge

When a gas pressure in a discharge gap is decreased the discharge channel becomes more widely and discharge tube is homogeneously is filled. This type of intrinsic discharge is called a *grow discharge*.

Arc discharge

If the current in an intrinsic gas discharge is sufficient, then bombardments of positive ions and electrons may cause a heating of both cathode and anode. At high temperatures, emission of electrons occurs, which provides the bearing of intrinsic discharge. Long time intrinsic electric discharge supported due to thermionic emission from the cathode's surface is referred to as *arc discharge*.

Corona discharge.

A special type of intrinsic discharge called *corona discharge* appears in powerful inhomogeneous electric fields created, for example, between corona and plane, or between conductor and plane. An ionization due to electron bombardments upon the corona discharge occurs only near one of the electrodes, where electric field strength is highest.

§13.4. Current in vacuum. Diode. Triode

Vacuum is called the state of gas in which the collisions between gas molecules can be neglected. In this state the electron' s mean free path exceeds the linear sizes of vessel ($l >> d$). A concentration of molecules in vacuum state is so small, that ionization processes can not provide for the required number of electrons and positive ions for electric current conduction. Conduction between electrodes in vacuum may be supported by means of charged particles produced due to *thermionic emission* phenomeneon.

Kinetic energy gained by electron during its passage from surface of cathode to anode is equal to the work done by an electric field.

$$\frac{m\upsilon^2}{2} = eV \quad \text{or} \quad \frac{m\upsilon^2}{2} = eEl\ , \qquad (13.8)$$

where m and e are the mass and charge of electron respectively, υ –speed of electron, V and l are voltage and distance between electrodes respectively.

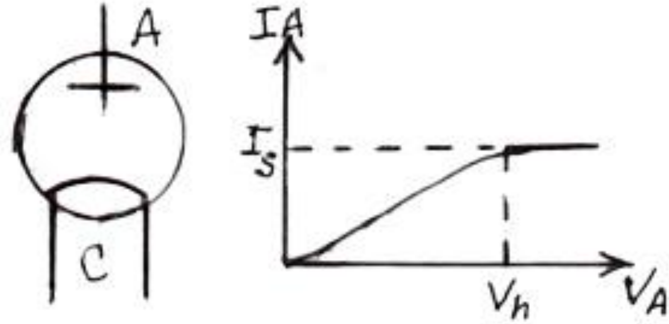


Fig.13.2

A force acting on electron in vacuum lamp is

$$F = eE = \frac{eV}{l} .$$

Hence, an acceleration attained by electron

$$a = \frac{F}{m} = \frac{eV}{ml} . \qquad (13.9)$$

The period of time during which electron reaches the anode is found from relation $l = at^2/2$. Thus

$$t = \sqrt{\frac{2ml^2}{eV}} \qquad (13.10)$$

Thermionic emission is used in various electron equipments, like as diode, triode and vacuum tube. **Vacuum diode** consists of two electrodes (Fig.13.2a):

cathode and anode. Its basic property is unipolarity conduction. When a current source is connected up with positive pole to the anode and with negative to the cathode the electrons emitted by heated cathode move to the anode, the direct thermionic current I_A flows in circuit. A dependence I_A on the voltage between electrodes V is called the volt ampere characteristics (Fig.13.2b). Current at the constant temperature depends on the dimensions and mutual displacement of electrons, work junction of electron s work junction from cathode and its temperature. At small values of V_A current I_A slowly increases with rising of V_A . It is explained as follows: when V_A is small not of all electrons liberated from cathode can reach the anode. Any portion of electrons produces an electron cloud between cathode and anode. This negative charge opposes the electron's motion. With increase in V_A the electron cloud and current I increases. When $V_A=V_{sat.}$ current I reaches the maximum value. This value is called ***saturated current***. If N is the number of electrons emitted from the cathode s surface per unit time at the given temperature, then $I_{sat.}=Ne$. If the positive terminal will be connected up to the cathode and the negative terminal to the anode, then the electric field will cancel the motion of electrons; current will disappear. This property is used to convert the alternating current into direct one.

Triode. In triode the controlling grid between cathode and anode is placed (Fig.13.3a). Dependence of I versus U at the given value of

V_A is called grid characteristic. of lamp (Fig.13.3b). If the V > 0 the electrons move to the anode with great speeds, more than speed at V =0. If V < 0 field of ... ... the electric field between anode and cathode and both of electron's speed and anode current decrease.

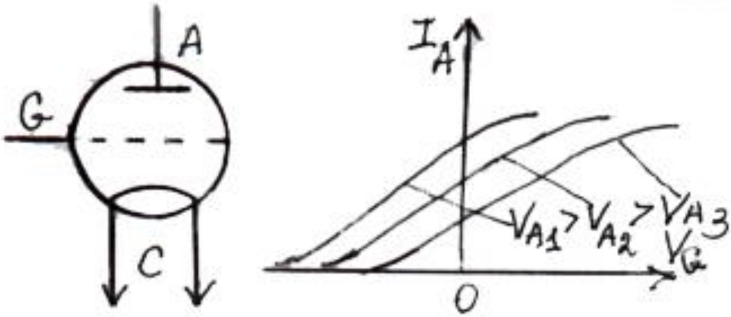


Fig.13.3

The negative V_G voltage at which the anode current is totally can cancel out . It is called the ***stopping voltage***. It rises with increasing of anode voltage. Thus change in the grid voltage can regulate the current of triode..

A vacuum tube contains a pair of metal electrodes . When a high voltage is applied across the electrodes, a stream of electrons moves across the tube from the negative electrode (cathode) to the positive electrode (anode).

§13.5. Energy band theory. Conductors. Insulators

Energy bands give an explanation of the electrical classification of solids. In case the highest energy band is unfilled the electrons can be excited from lower to higher energy level within the band. (Fig.13.4)

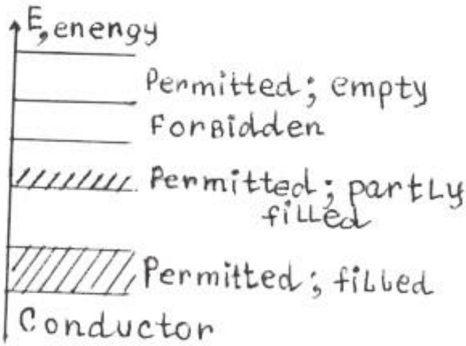


Fig.13.4

This is possible on application of even weak electric fields. A material with such energy band is a **conductor.** There are some materials in which valence and conduction bands overlap hence, electron from the valence shell can easily pass into conduction band. They are also conductors.

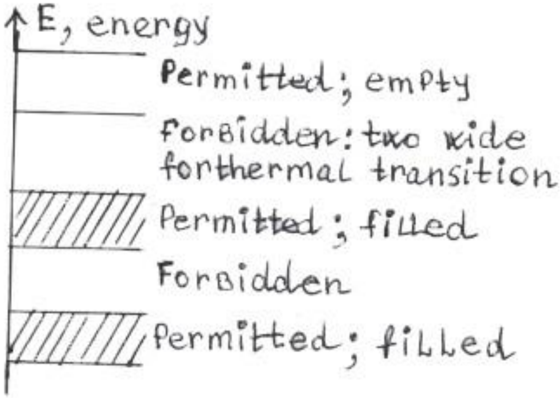


Fig.13.5

In an **insulator**, the highest occupied energy level is completely filled. Also the forbidden energy band above the occupied band is wide. The electron cannot jump from lower to upper permissible energy levels.(Fig.13.5)

§13.6. Semiconductors

Another category of solids is semiconductor. Here the gap between the filled energy band and next higher permitted energy band is small. The thermal energy, even at room temperature, is sufficient to allow some electrons from lower permitted band to pass to the upper permitted band.(Fig.13.6) The examples of semi conductors are Germanium and Silicon which belong to fourth group of Periodic Table. They have four electrons in their outermost shell called *valence electrons*. The valence electrons of each of these atoms are shared in pairs with four adjacent atoms. (Fig.13.7)

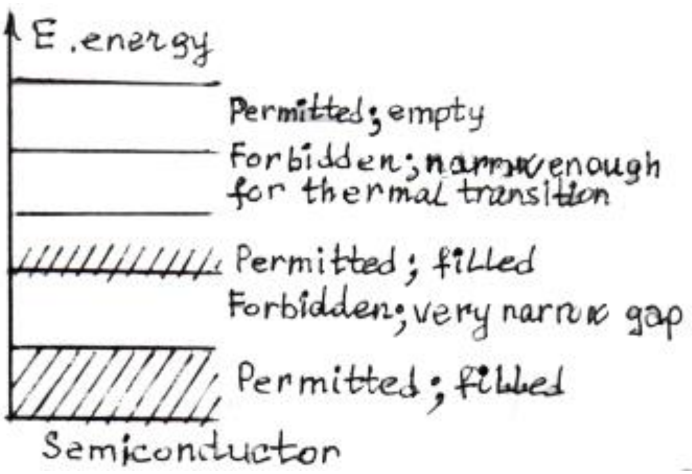


Fig.13.6

A pure semiconductor behaves like an insulator at temperature near absolute zero because the valence electrons of each atom are tightly held in covalent bonds with neighbor atoms. As the temperature rises to ambient range, the atom and the electrons absorb thermal energy. This energy appears as random vibration or agitation of these particles about their lattice locations. So, some electrons become free and mobile by acquiring sufficient energy to break the covalent bond. The energy required to break a covalent bond may also be provided by high voltage across the material or by exposing the material to photons of proper wavelength. When the bond is broken and an electron is free thereby, electron vacancy is left in the covalent bond. The vacant electron site is called a hole. The hole represents the absence of negative charge and is attractive to electrons. The hole appears to have a positive charge equal to the negative charge of electron in magnitude.

The hole also appears to be mobile.. A broken covalent bond has left a hole at the site A.. Another electron may move from a bond at the site B to the hole at A, under the force of a small voltage applied across the material.

When the electron fills the hole at the site A, then there is now a hole at the site B. Then another electron may move from a bond at the site C to the hole at the site B and the hole thus moves to the site C. Similarly, an electron may move from a bond at the site D to hole at the site C and so the hole has moved from A to D.

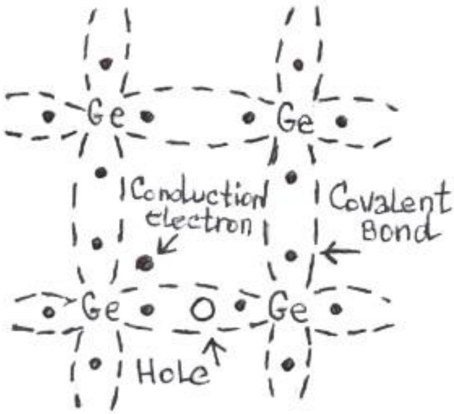


Fig.13.7

In order to obtain the desired conduction properties, the pure Silicon or Germanium is doped with minute amount of selected elements as controlled impurities. The impurities are added at the rates of one atom of doping element to 10^5 to 10^8 atoms of semiconductor. When the 5-th group element like Antimony is added to the Germanium crystal we have the n - *type* material. In an n - *type* semiconductor the conduction will be predominantly by electrons and the electrons are called the ***majority carriers***. There will be a few holes present in n - *type* material due to electron hole pair generation at usual ambient temperatures. The holes present in an n type material are known as the ***minority carriers.*** The use of trivalent impurity elements has created a material with free holes, so conduction occurs by hole transfer and impurity semiconductor is said to be ***p-type*** material. The trivalent impurity atoms accept electrons to fill their bond vacancies and are called acceptor atoms and the impurities as acceptor impurities.

The conduction in a ***p-type*** material will be primarily due to holes and they are known as ***majority carriers***. There will be present electrons in a ***p-type*** material due to thermal pair generation. These electrons in a ***p-type*** material are known as ***minority carriers***.

§ 13.7. Semi-conductor diode

A junction between p and n materials forms a semi-conductor diode as shown in Fig. 13.8a. It contains donor impurities on one side and acceptor impurities on the other side of a single crystal of Germanium or silicon during the process of its growth.

Initially there are holes (majority carriers) to the left of the junction and free electron to the right. A diffusion of electrons and holes takes place across the junction which stops after setting up a potential difference at the junction, the n-type being at a positive potential with respect to p-type as shown in Fig.13.8b

The semi-conductor diode has the property of one way conduction ,i,e,. it allows the current to flow only in one direction. The flow of current is practically zero in the opposite direction; it acts as a diode. Thus, if we want to send the current across a p-n junction, we have to reduce the height of the potential barrier at the junction.

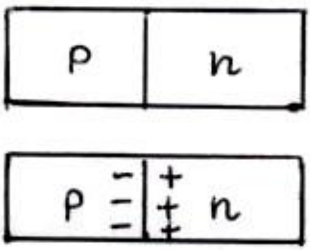


Fig.13.8

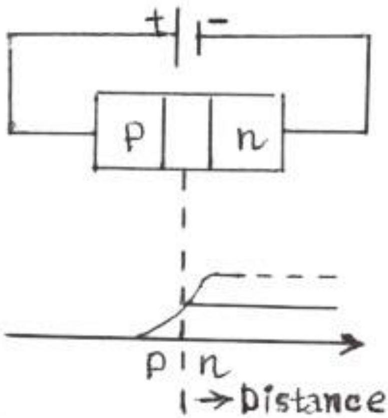


Fig.13.9

This can be done by connecting the p-type material of the junction with positive terminal of the battery and n-type with negative terminal as shown in Fig.13.9 The height of potential barrier has been reduced and the resistance offered by it decreases. It gives easy way to flow of electric current. In this case the diode is said to be forward biased. The depletion region is narrowed.

On the other hand, if the n-type material of the junction is connected to positive terminal of the battery and p-type with negative terminal, the height of potential barrier at the junction increases and it offers a very high resistance. (Fig.13.10) This makes the flow of current quite difficult across the p-n junction. In this state the diode is said to be reverse biased and practically no current flows in the circuit. The depletion region is widened in this case.

The characteristic of a diode is simply the relation between current and potential difference represented by a graph, showing the forward and reverse biasing of the diode respectively. (Fig.13.11.)

Thus the diode can act as a switch. It is due to this important property of diode, that it can be used for rectification, i,e., to convert alternating current into pulsating direct current. In most of the electronic devices the diode serves as a rectifier..

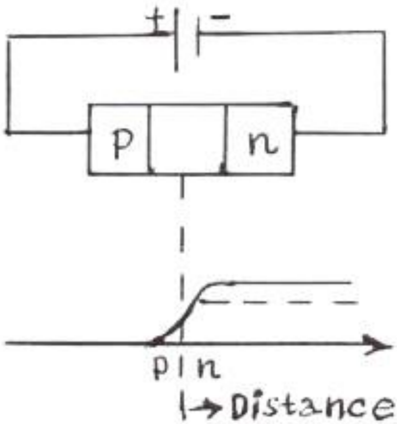


Fig.13.10

§13.8. Semi-conductor transistor

Transistors are of two types i.e., p-n-p or n-p-n transistors. In a p-n-p transistor, n-type material is sandwiched between two p-type materials Fig.13.12a

In the second type, p-type material lies in between two n-type materials (Fig.13.12b) The central portion of the transistor is known as base (B) and the regions on the left hand side and right are called emitter (E) and collector (C) respectively. The emitter is more heavily doped than the collector with the same impurities. These three portions of the transistor form two junctions, i.e., emitter –base junction (also knows as emitter junction) and collector-base junction (also known as collection junction). For normal operations the emitter-base junction is forward-biased and collector-base junction is reverse-biased as shown in Fig. 13.13.

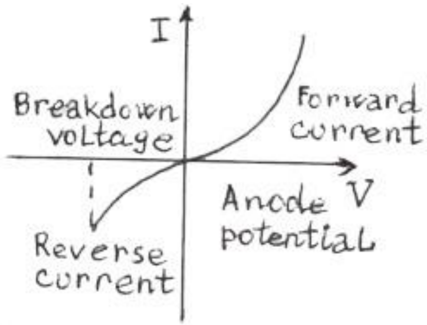


Fig.13.11

Emitter and collector are at lower potentials with respect to base. On this condition no current can flow from emitter to base or collector to base.

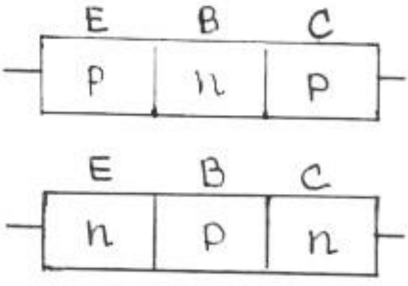


Fig.13.12

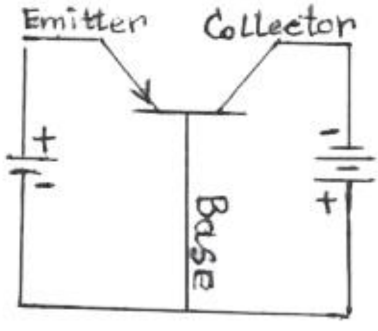


Fig.13.13

A small ac power source connected to the input produces changes in the base current. This produces much larger changes in the collector current. The collector current flowing through the load R produces a voltage drop. Under this condition the above circuit operates as an amplifier.

Transistor can be used as oscillators, switches, memory units and perform many other useful functions.

§13.9. Superconductivity

Falling in the temperature of a material reduces the thermal vibrations of the lattice ion and hence a decrease in the resistance can be observed in metal. It is, however, difficult to make the resistivity and hence the resistance, exactly equal to zero with temperature. A class of elements called superconductors show a remarkable variation in electrical resistivity with temperature as shown in Fig. 13.14

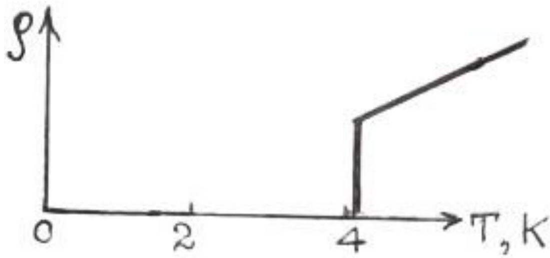


Fig.13.14

One can observe in this figure that at a certain critical temperature T_C (Curie temperature) the value of resistivity falls with shift to zero. For example, the value of resistance of some elements falls to zero below 7. 175 K At such temperatures, the ions are frozen at the lattice sites and therefore do not scatter or hinder the motion of electrons in a superconductor.

CHAPTER 14

Magnetism

§14.1. The magnetic field. Magnetic moment. Magnetic field induction

Magnetic field is one of the two parts of electromagnetic field. Its basic feature is to be produced by the current carrying conductors, moving electric charged particles , as well as by magnetized objects and time-varying electric fields. Appearance of magnetic field around the current carrying wire was observed in Oersted′s experiment. One of the wires used by him was laying across

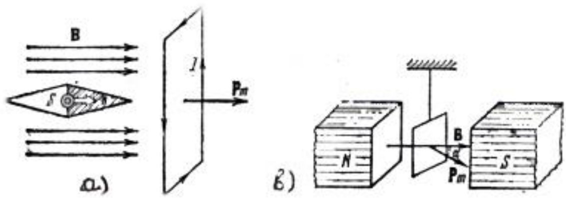


Fig.14.1

the top of a small compass. When he sent a current through the wire the compass needle moved. A compass needle is a small magnet. The magnetic field around a magnet will
interact only with another magnetic field. Thus the presence of an electric current in the wire was somehow causing a magnetic field around the wire. The electric current and magnetic field cannot be separated.

Magnetic moment P_m of current carrying plane loop (Fig.14.1a) is a vector defined by

$$\vec{P}_m = IA\vec{n}_0 \quad , \qquad (14.1)$$

where A– area limited by the loop, I - current intensity , $\mathbf{n_0}$– unit vector, normal to the surface**.**

Force characteristic of magnetic field is the **induction vector B** . If the current currying rectangular loop is placed in external magnetic field the torque exerted on it is equal to

$$M = P_m B \sin\alpha \quad , \qquad (14.2)$$

where **B** -the magnetic induction vector and α - the angle between the vectors $\mathbf{P_m}$ and **B** (Fig.14.1b) Magnitude of **B** is determined by the greatest value of torque – $M_{max.}$:

$$B = \frac{M_{max}}{P_m} = \frac{M_{max}}{IA} . \qquad (14.3)$$

If $\alpha \leq (\pi/2)$, the magnetic induction lines are on the plane of loop and its magnetic moment P_m is perpendicular to the induction lines B. This situation is called the **unstability equilibrium.** The condition in which the plane of loop is perpendicular to the induction lines is the **steady equilibrium**.

The unit of magnetic induction in the CGSM is the Gaus, in the system SI is **tesla** *(T)*.

A tesla (T) is defined as the induction of magnetic field that acts with newton-meter(Nm) revolving torque on the current currying loop of area $1m^2$ whose current is equal to 1 ampere.

$$1\ T = 1\ \frac{N}{Am}; \qquad 1\ Gauss = 10^{-4}\ T.$$

A magnetic induction B is related intensity H of magnetic field by

$$B = \mu H \ ,$$

where μ is the relative magnetic permeability of the medium.

Unit of magnetic intensity in the SI system is $\frac{A}{m}$

§14.2. Magnetic flux

Flux of magnetic induction Φ across the surface of small area A (Fig.14.2) is the scalar quantity, determined as

$$\Phi = BA\cos\alpha \qquad (14.4)$$

where α -is the angle between vectors **B** and $\mathbf{n_0}$. Hence the magnetic field equals the flux per unit area across an area at right angle to the magnetic field. Magnetic flux is positive when $\alpha \leq (\pi/2)$ and it is negative when $\alpha \geq (\pi/2)$. Magnetic flux may change because of:

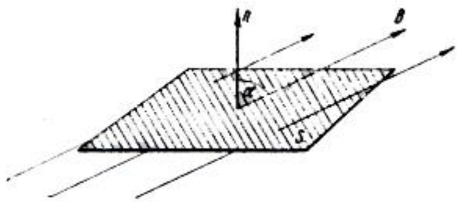


Fig.14.2

1) Reorientation of plane loop′s surface relative the direction of induction B:

$$\Delta\Phi = BS(\cos\alpha_2 - \cos\alpha_1) \;.$$

2) A change of the sizes of area A:

$$\Delta\Phi = B\Delta A\cos\alpha = B(A_2 - A_1)\cos\alpha \;.$$

3) A change in magnetic induction B :

$$\Delta\Phi = \Delta BA\cos\alpha = (B_2 - B_1)A\cos\alpha \;.$$

Change in magnetic induction B may be caused by variation in field intensity H and magnetic permeability μ.

The unit of magnetic flux in the CGSM system is the **maxwell** *(mks)*, in the SI system the **weber***(Wb)*: $\mathbf{1Wb = Nm\,A^{-1}}$.

§14.3. Biot and Savart′s law. Magnetic field due to motion of charged particle

The intensity of the magnetic field due to current in a conducting wire of length Δl at the distance r is given by

$$\Delta H = \frac{i\Delta}{r^2}\frac{l\sin\alpha}{} \quad , \qquad (14.5)$$

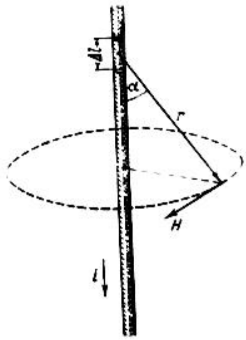


Fig.14.3

where α– the angle between direction of current and the direction pointing given point. (Fig.14.3)

The quantity **iΔl** is called **current element.** The field induction due to motion of charged particle (Fig.14.4) is equal to

$$B = \frac{e\upsilon\sin\theta}{4\pi r^2} \quad , \qquad (14.6)$$

Where υ is the velocity of the particle, r –is the distance from particle to the point of interest, θ -is the angle between the velocity vector and the line

Drawn from the particle to the field point.

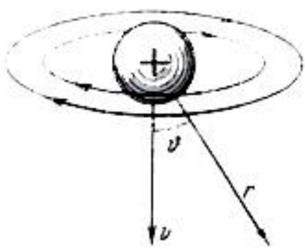


Fig.14.4

§14.4. Magnetic fields due to long straight current, current in solenoid and in toroid

A current I bearing straight longer wire (Fig.14.5) produces around itself at the distance R the magnetic field with induction

$$B=\mu_0\mu\frac{I}{2\pi}\frac{1}{R} \qquad (14.7)$$

where I-the current, μ_0 -the magnetic constant, equal to $4\pi\times10^{-7}$ and μ –relative magnetic permeability of a medium.

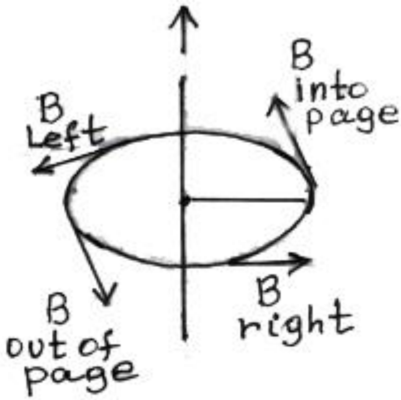


Fig.14.5

Direction of wire′s magnetic field is determined by means of left hand rule: **if the thumb of the left hand is placed along the wire pointing in the direction of the current, the curled fingers of the left hand will point in the direction of the magnetic lines of force.**

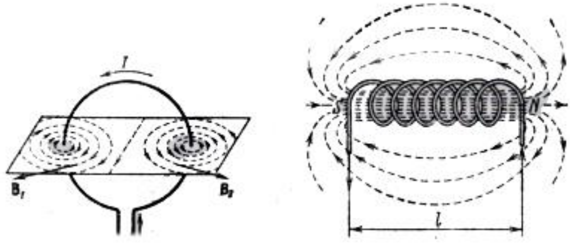


Fig.14.6

Circular wire of radius R produces magnetic field induction B at its center

$$B=\mu_0\mu\frac{I}{2R} \quad , \qquad (14.8)$$

Solenoid. A saolenoid is constructed by winding wire in a helix around a cylindrical surface (Fig.14.6). The magnetic field of the solenoid of length l is defined as

$$B=\mu_0\mu nI \quad , \qquad (14.9)$$

where n=N / l-the number of turns per unit length of solenoid and each turn carries a current I.

The direction of B is along the axis of the solenoid.

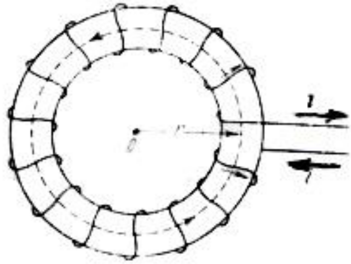


Fig.14.7

Toroid. A toroid is a solenoid that has been bent into a circle. (Fig.14.7). The magnetic field is given as

$$B=\mu_0\mu nI \quad , \qquad (14.10)$$

where n = N / 2π r is the number of turns per unit length, and 2π r is the circumference of the toroid.

§14.5. Force on the current carrying conductor. Ampere′s law.

On the small piece of the conducting wire of length l placed in the magnetic field acts the force **F** (Fig.14.), which is expressed by the formula

$$F=IlB\sin\alpha \quad , \qquad (14.11)$$

where α -is the angle between vector **B** and current direction. The force **F** is perpendicular to the both of wire and vector **B and is called Ampere′s force.**

Direction of **F** is determined by means of left hand rule: **if the open palm of the left hand is placed so that the lines of force of the magnetic field enter the palm, while the outstretched fingers point in the direction of the current, then the thumb will indicate the direction of the force acting on the conductor.**

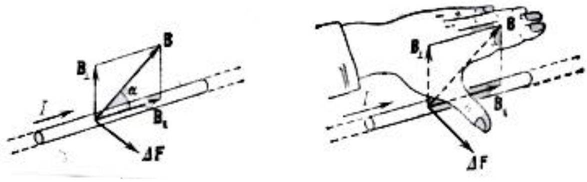


Fig.14.8

§14.6. Interaction between parallel currents

Between two parallel currents bearing conductors placed at a distance R one from other arrise force of attraction when the currents represents in the same direction and repulsive force when the currents are in opposite directions (Fig.14.9). This force is given by

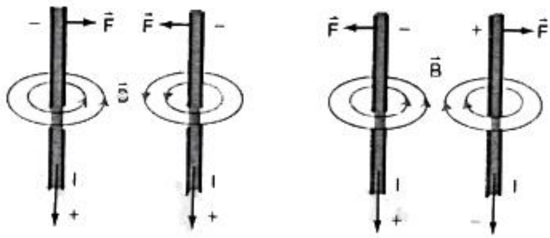


Fig.14.9

$$F=\mu_0\mu\frac{I_1I_2}{2\pi}\frac{}{R} \quad , \qquad (14.12)$$

where I_1 and I_2 are the currents.

§14.7. Motion of charged particle in a magnetic field. Lorentz′s force

If a single charged particle moves in a magnetic field Lorentz′ s force will exerted on it:

$$F_L = q\upsilon B\sin\alpha \quad , \qquad (14.13)$$

where q- magnitude of charge, υ-the particle's velocity, B-induction of magnetic field, α-the angle between vectors **B** and υ (Fig.14.10) In uniform magnetic field the vector **B** is perpendicular to the vector υ. The force $\mathbf{F_L}$ distorts the path of motion. The velocity changes only in direction but not in magnitude. A particle makes circular motion on the plane, perpendicular to the vector B. Since the centripetal force F_c is given by $F_c = m\frac{\upsilon^2}{R}$ where R is the radius of the circular path, the radius and period of this motion are given by

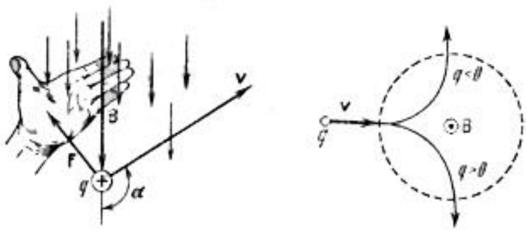


Fig.14.10

$$R = \frac{m}{q}\frac{\upsilon}{B} \quad , \qquad T = \frac{2\pi}{B}\frac{m}{q} \quad . \qquad (14.14)$$

A period T is regardless on the radius of circle and the velocity of the particle.

If the charged particle enters into the magnetic field under the angle α, the motion will occur over the curl (Fig.14.11) of radius R and step b respectively:

$$R=\frac{m\upsilon\sin\alpha}{qB}\ , \qquad b=\frac{2\pi m\upsilon\cos\alpha}{qB}\ . \qquad (14.15)$$

If the particle is simultaneously exerted by both electric and magnetic fields , the resultant force will be expressed as

$F_R=f_e + f_L= qE + q\upsilon B$. (14. 16)

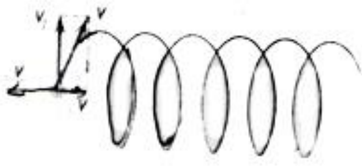


Fig.14.11

Suppose, that electron passing the potential difference $V_1 – V_2$, has the velocity υ and moves in the magnetic field, force lines of which are perpendicular to the vector υ . In this case Lorent'z force will act as centripetal force F_c.

If the radius of circle is R then $F_c = m\upsilon^2 / R$ and therefore

$$e\upsilon B=\frac{m\upsilon^2}{R}, \qquad \text{hence } \frac{e}{m}=\frac{\upsilon}{RB}\ . \qquad (14.17)$$

Since a work done by electric forces is expressed as $e(V_1 – V_2)$, then the kinetic energy of electron will be equal to:

$$\frac{m\upsilon^2}{2}=e(V_1-V_2)\ . \qquad (14.18)$$

Hence it follows

$$\upsilon=\sqrt{\frac{2e(V_1-V_2)}{m}}\ . \qquad (14.19)$$

§14.8. Magnetic properties of matter. Types of magnets

First consider the magnetic moment of electrons and atoms.

Each electron moving within atom around nucleus over closed orbit produce electronic current (Fig.14.12) Direction of this current is opposite to electron′s movement. Current due to electron's rotation is given by the formula

$$I=\frac{e}{T} ,$$

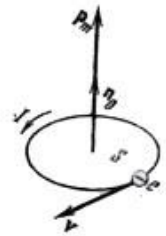
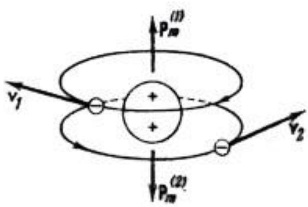


Fig.14.12

where e-absolute value of electron′s charge , T-rotational period of electron.

the magnetic is called the substance which can create their internal magnetic field $\mathbf{B_{int.}= \mu H_{int.}}$, when are placed in the external magnetic field. The total magnetic field existing within of substance is characterized by the magnetic induction vector which is equal to the vector sum of magnetic inductions of external $\mathbf{B_{ext.}=\mu H_{ext.}}$ and internal fields:



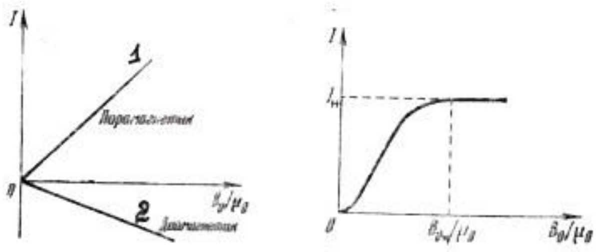
$$\mathbf{B=B_{int.} + B_{ext.}} . \qquad (14.20)$$

Fig.14.13

The degree of magnetization $I=(\mu -1)B_0$ of a magnetic material is characterized by the magnetic permeability μ which is given as

$$\mu = \frac{H_{ext.} \pm H_{int.}}{H_{ext.}} .$$

Substances, in which the induced internal field in the existence of external field is opposed to the internal field are called **diamagnetic.** For diamagnetic $\mu < 1$ Atoms and molecules of such substances have not their specific magnetic moments ($H_{int.}$= 0; $p_m = 0$) in the



Fiig.14.14

absence of external magnetic field. For example, the atom of He is diamagnetic. Nucleus of this atom has a charge equal to q=2e. Suppose, both of electrons revolve around nucleus over the same orbits with the same velocities in opposite directions. Then their revolving magnetic moments are equal in magnitude and opposite in sign. Fig.14.13 illustrates that the resultant magnetic moment for atom He is equal to zero.

Substance in which induced internal field is added to the external field are called **paramagnetic**,

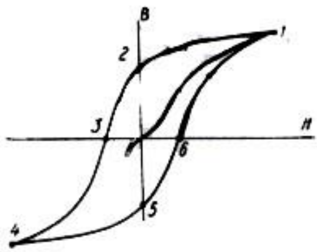


Fig.14.15

For paramagnetic $\mu>1$. In paramagnetic substances the permeability μ is varying with the temperature according the dependence $\mu=1+ C / T$, where T - the absolute temperature, and C – is Curie constant. In Fig.14.14 for both of paramagnetic (line 1) and diamagnetic (line 2) magnetization I is directly proportional to B_0. Materials, for which μ is more greater than unit ($B_{int.} >> B_{ext.}$ hence, $\mu >>1$) are called

ferromagnetic. For the ferromagnetic materials a dependence of I on the B_0 is nonlinear.(Fig.14.14)

The specific properties of ferromagnetic are observed at temperatures, below a certain T_C called the Curie temperature. When $T \geq T_C$ the ferromagnetic properties disappear and the ferromagnetic substance becomes paramagnetic.

The magnetization of a ferromagnetic material depends, in addition to the field intensity, on the magnetic history of the sample: the value of the induction lags behind that of the field intensity. This phenomenon is called **hysteresis,** and the curve depicting the dependence of B versus H in the process of remagnetization (Fig.14.15) is called a **hysteresis loop**.

The value of the residual magnetic induction of the ferromagnetic material after the magnetizing field has been reduced to zero and is called the **retentivity** (B_r).

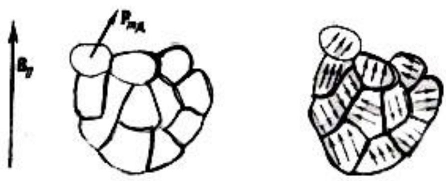


Fig.14.16

The **coercive force** *(H_r)* is the value of the magnetic field intensity needed to reduce the residual induction to zero (the direction of this field must be opposite to that of the retentivity).

The **saturation value** *(I_S)* is the greatest value of the magnetization I. When a ferromagnetic material has been magnetized up to the saturation value, further increase of the field intensity will practically have no effect on the magnetization.

At the temperatures $T < T_C$ ferromagnetic body is composed of **domains –** a small regions with linear dimensions 10^{-2} –10^{-3} cm, within of which the atoms are coupled and lined up in the same direction that leads to the larger magnetization, equal to saturation value **I_S.**

In the absence of external magnetic field vectors of magnetic moments of separated domains (Fig.14.16) within ferromagnetic are oriented perfect irregular and magnetic fields cancel one another. As a result the resultant magnetic moment of body is equal to zero. However, if the body is placed in a magnetic field a revolving moment of individual domains appear and they tend to align with the external field. If the magnetic field has high intensity the ferromagnetic body becomes perfectly magnetized

CHAPTER 15

Electromagnetic induction

§15.1. Faraday′s law of electromagnetic induction. Lenz′s rule

When the magnetic flux through a circuit is varying an electric current arrises in the circuit. This phenomenon is called **electromagnetic induction**, and the current generated thus is referred to as **induced current**.

Lenz′s rule: The direction of the induced current is always so that the magnetic field due to current opposes the change in flux by which the current is induced. (Fig. 15.1)

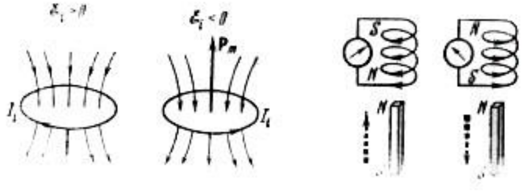


Fig.15.1

Induced electromotive force (e.m.f.) ε_i in the circuit is equal in magnitude and opposite in sign to the time rate of magnetic flux across the surface, limited by this circuit.

$$\varepsilon_i = -\frac{\Delta \Phi}{\Delta t}, \quad (15.1)$$

where $\Delta\ \Phi$ – is the change in magnetic flux.
The induced current is given as

$$I = \frac{\varepsilon_i}{R} = -\frac{\Delta \Phi}{\Delta t} \frac{}{R}. \quad (15.2)$$

The quantity of charge flowing in the circuit with resistance R is expressed as

$$\Delta q = -\frac{\Delta \Phi}{R}. \quad (15.3)$$

§15.2. Motional electromotive force

When a conductor of length l is sliding with velocity υ through in uniform magnetic field (Fig.15.2) the induced electromotive force (e.m.f.) in conductor is equal to

$$\varepsilon = Bl\upsilon \sin\alpha \quad (15.4)$$

where α – the angle between vectors $\vec{\upsilon}$ and $\vec{B}$. This is known as motional emf.

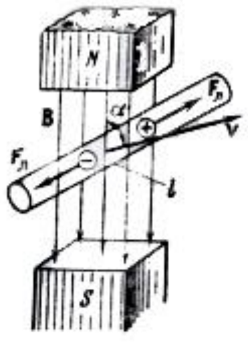


Fig.15.2

Induced e.m.f. is the energy given to electrons when a wire moves in a magnetic field.

Assume the plane loop, revolving in a magnetic field with the angular velocity ω, whose rotational axis is placed on the plane of loop and is perpendicular to vector **B** of external field. In this case the e.m.f. is given by

$$\varepsilon = BA\omega \sin\omega t \ ,$$

where A-is the area of the loop.

§15.3. Self-induction. Combination of inductors

Appearance of induced e.m.f. as a result of change of the current in the circuit. is called self – induction. The change of current causes a change in its specific magnetic field. The specific magnetic field of current produces the magnetic flux Φ_s across the surface area, limited by this loop. As is shown in Fig.15.3 a flux will be generated by the current and will be directed towards the left along the coil. By Faraday′ s law an e.m.f. will be induced in the coil and it will tend to produce flux to the right hand side along the coil. Hence an induced emf must be opposite to the emf of the battery. If the switch is suddenly opened, the induced emf will be added, rather than opposing the battery. In this case the magnetic flux is proportional to the current I:

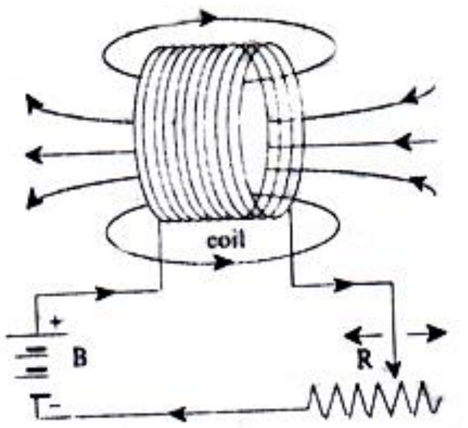


Fig.15.3

$$\Phi_s = LI \ , \qquad (15.5)$$

where a constant of proportionality L – is called the self inductance of the coil. Inductance L depends on the dimensions and geometry of the coil as well as relative permeability of the medium.

For the average self–induced e.m.f. we can write:

$$E_s = -L\frac{\Delta I}{\Delta t} \qquad (15.6)$$

where $\frac{\Delta I}{\Delta t}$ – is the rate of change of current through the coil.

The SI unit of inductance is the **henry (**Hn**).**

A henry is defined as the inductance of a conductor in which a change of current of 1 ampere per second induces an e.m.f of 1 volt.

The inductance of a solenoid with a core is equal to:

$$L = \frac{\mu_0 \mu N^2 A}{l} = \mu_0 \mu n^2 V \qquad (15.7)$$

where *N* – is the number of turns, *A*–is the cross – sectional area of solenoid, *l* – the length of solenoid.

The equivalent inductance of circuits connected in series is equal to the sum of individual inductances.

$$L = L_1 + L_2 + L_3 + \ldots + L_n \qquad (15.8)$$

The reciprocal of equivalent inductance of circuits in parallel is the sum of reciprocals of individual inductances.

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n} \qquad (15.9)$$

§15.4. The Mutual induction

Suppose that we have two coils placed side by side as shown in Fig.15.4 When the switch S is open, both coils have zero flux through them. We call the coil in the battery circuit the primary coil and other the secondary coil. If the switch is suddenly closed, a magnetic field is set up due to building up of current in the coil and magnetic flux is produced in its vicinity. Some portion of the flux will pass through the secondary coil. Thus the flux through the secondary coil will change when S is suddenly closed. According to Faraday′s law an induced emf will be produced in secondary for an instant as the current rises from zero to its maximum value in the primary. It can be shown that the direction of the current in primary circuit in Fig.15.4. will be from B to A when S is just closed. The current will flow in the opposite direction when the switch is opened. Since, the change of flux through the secondary will be proportional to the change of current in the primary, the induced emf in the secondary will be proportional to the rate of change of current in the primary $\frac{\Delta I_P}{\Delta t}$ The induced emf in the secondary is equal to

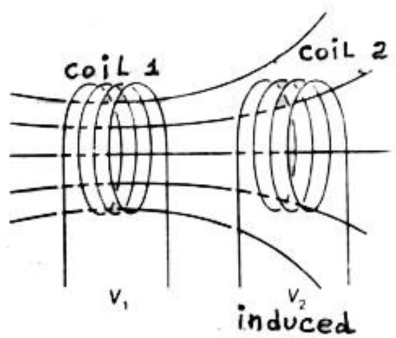


Fig.15.4

$$\varepsilon = -M\frac{\Delta I_P}{\Delta t}. \qquad (15.10)$$

The proportionality constant M depends on the geometry of the two coils. It is called the **mutual inductance** of two coils.

§15.5. Transformers

A simple transformer consists of two coils of insulated wire wound on the same iron core. When alternating current voltage is applied to the input coil, or primary coil, the alternating current gives rise to an alternating magnetic flux that is concentrated in the iron core. The changing flux passes through the output coil, or secondary coil, inducing an alternating voltage and current in it.

The induced secondary voltage differs from the primary voltage depending on the ratio of the numbers of turns in the two coils. By Faraday's law the secondary voltage is given by

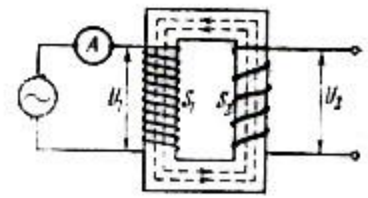


Fig.15.5

$$V_s = -N_s \frac{d\Phi}{dt} \quad , \qquad (15.11)$$

where N_s is the number of turns in the secondary coil. The changing flux in the primary coil produces a back emf equal to

$$V_p = -N_p \frac{d\Phi}{dt} \quad , \qquad (15.12)$$

where N_p is the number of turns in the primary coil.
From above equations we get

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad . \qquad (15.13)$$

If the transformer is assumed to be 100% efficient the power input Is equal to the power output and since P=IV

Using Eq. (15.13) the currents and voltages are related to the turn ratio by the relationship

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} \quad . \qquad (15.14)$$

Hence it is easy to see how a transformer affects the voltage and current. In terms of the output

$$V_s = \left(\frac{N_s}{N_p}\right) V_p \quad \text{and} \quad I_s = \left(\frac{N_p}{N_s}\right) I_p \quad . \qquad (15.5)$$

If the secondary coil has a greater number of windings $(N_s \succ N_p)$than the primary does, the voltage is stepped up $(V_s \succ V_p)$. However, less current flows in the secondary than the in the primary $(I_s \prec I_p)$ This type of arrangement is called a **step-up transformer.**

The opposite situation, where the secondary coil has fewer turns than the primary, characterizes a **step-down transformer**.

§15.6. Energy of magnetic field

In order to build the current I in the loop with inductance L the work for prevaluing the e.m.f of self -induction is needed. Energy of magnetic field of current carrying loop is equal to this work.

$$W = \frac{1}{2} L I^2 = \frac{\mathbf{I}\Phi}{\mathbf{2}} = \frac{\Phi^2}{\mathbf{2\,L}} \quad . \qquad (15.16)$$

The region of space in which a magnetic field exists, contains stored energy. The energy density of a homogeneous magnetic field can be computed by the formula:

$$w=\frac{1}{2}\frac{B^2}{\mu_0\mu} \quad . \qquad (15.17)$$

The energy stored in the long solenoid is equal to

$$W=\frac{1}{2}\mu_0\mu\, n^2 l^2 V \qquad (15.18)$$

where n-is the number of turns per length of solenoid, V- refers to the volume of solenoid.

The volume density of the electromagnetic field is calculated as

$$w=\frac{\varepsilon_0\varepsilon}{2}E^2 + \frac{1}{2\mu_0\mu}B^2 \quad . \qquad (15.19)$$

CHAPTER 16

Electric oscillations and waves

§16.1. Free electric oscillations. Thomson′s formula

The electric circuit consisted of capacitor C inductor L and resistor R in series is called the oscillation circuit. (Fig. 16.1)

In oscillating circuit can occur the periodical variation of charge *q*, potential difference V in capacitor and electric current I in inductance coil.. If at the initial moment of time (t = 0) the switch P (Fig.16.2) is at the position a), the capacitor is charged receiving the charge q_0. The energy W= $q_0^2/2C$ is given to the capacitor and difference in potentials between the plates of capacitor becomes equal the maximum value ΔV_0 . At this moment the current in the circuit is equal to zero. The discharge of capacitor begins when the switch is turned to the position b.

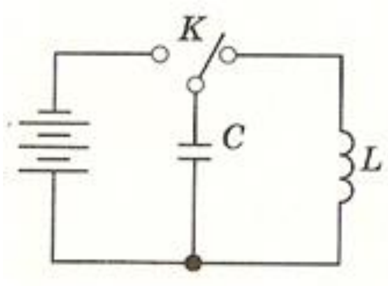


Fig.16.1

The current increases slowly due to self induction and reaches its maximum value I = I_0 at moment t = T/4 when q and ΔV changing its direction and becomes equal to zero at t = T/2 Then the charge of capacitor and the potential difference between its plates again reach maximum values, but signs of plates′ charges and direction of electric field are opposed to those, that were at the moment t=0 Finally, the capacitor is recharged from T/2 up to (3/4)T and from (3/4)T up to T . Then processes occur in reverse direction.

If the oscillatory circuit is considered to be ideal (in which the energy losses can be neglected) energy of electric field is totally converted to the magnetic field and inverse during oscillations. One can accept equality of these energies:

$$\frac{LI_m^2}{2} = \frac{CU_m^2}{2} \qquad (16.1)$$

Dependence of I_m versus U_m in a capacitor is given by

$$I_m = U_m \omega C \qquad (16.2)$$

Having put this expression into the (16.1) for cyclic frequency of free oscillations gives

$$\omega = \sqrt{\frac{1}{LC}} \quad . \qquad (16.3\ )$$

Hence for the period of oscillations we get

$$T = 2\pi\sqrt{LC} \quad . \qquad (16.4\ )$$

This equation is known as **Thomson′s formula.**

Solving (16.1) for I_m gives

$$I_m = \frac{U_m}{\sqrt{\frac{L}{C}}} = \frac{U_m}{\rho} \qquad (16.5)$$

Where $\rho = \sqrt{\frac{L}{C}}$ - is the *wave resistance*.

The free electric oscillations in real ($R \neq 0$) circuit are damping. For example, the change of charge q in capacitor plates is described by the law

$$q = q_0\, e^{-\beta t} \sin(\omega_{damp.} t + \varphi_0)$$

where q_0 – the amplitude value of charge at moment t=0, β=R/2L – is the damping factor , R – resistance, L – inductance, φ_0 – phase constant of oscillations.

The quantity ω_{damped} is called the cyclic frequency of the electric oscillations.:

$$\omega_{damped} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad . \qquad (16.6)$$

Damping of oscillation is estimated by the quantity so called ***decrement of damping.*** It indicates the portion of energy loss during a half of period of oscillations.

A quantity of energy lost during half period is

$$W_R = \frac{RI_m^2}{2}\frac{T}{2} \qquad (16.7)$$

A quantity of energy stored in the circuit is

$$W_L = \frac{LI_m^2}{2} \qquad (16.8)$$

Decrement of damping is

$$\Lambda = \frac{W_R}{W_L} = \frac{\pi R}{\rho} \qquad (16.9)$$

Quantity equal to the reciprocal of damping decrement reduced as π times is called ***quality*** of circuit.

$$Q = \frac{\pi}{\Lambda} = \frac{\rho}{R} \qquad (16.10)$$

§16.2. Forced electric oscillations

Forced oscillations are called nondamping oscillations of the charge, potential differences and current due to periodically changing e.m.f. in oscillating circuit

$$\varepsilon = \varepsilon_0 \sin\omega t \qquad (16.11)$$

where ε_0 – is the amplitude value of e.m.f., ω - the cyclic frequency of alternating e.m.f.

Sinusoidally e.m.f. appears in the loop, which rotates (revolves) with angular velocity ω in a stationary uniform magnetic field of induction B

The magnitude of magnetic flux is determined by the formula (14.4). The harmonically changing magnetic flux leads to e.m.f. of induction .

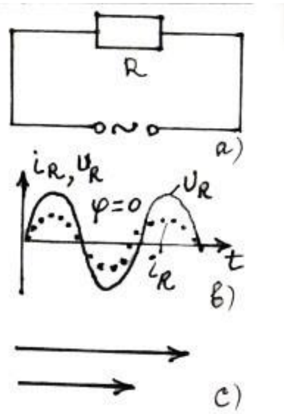
§16.3. Alternating current. Active, inductive and capacitive resistances. Total reactance

The current changing according to the harmonic (sine) law is known as **alternating current**. The frequency of alternating current is equal to the frequency of imposed e.m.f.

The effective value of an alternating current is defined as the direct current which would develop the same power in an active resistance as the given alternating current.

Effective values of an alternating current and voltage are given by

$$I_{eff.} = \frac{I_{\mathbf{m}}}{\sqrt{\mathbf{2}}} = \boldsymbol{0.707}\ \mathbf{I}_{\mathbf{m}}. \quad , \qquad V_{eff.} = \frac{U_m}{\sqrt{2}} = 0.707\,\mathrm{V}_{\mathrm{m}} \quad , \qquad (16.12)$$

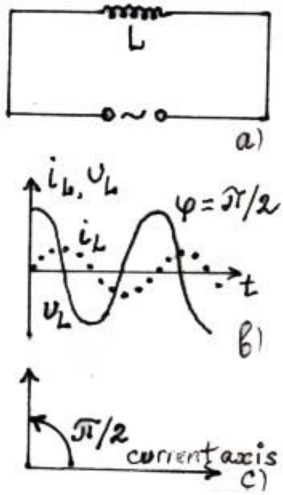


where I_m and U_m are the amplitude values of the current and voltage respectively.

The total resistance in a circuit of alternating current is equal to the vector sum of active and reactive resistances.

Active resistance. In the circuit with active (R) resistance (Fig.16.2a) the harmonic oscillations of current occur with the frequency and phase of imposed e.m.f.

$$I = I_0 \sin \omega t \quad , \qquad (16.13)$$



where $I_O = \varepsilon_0 / R$ the amplitude value of current. The graphs of both alternating e.m.f. and current and are shown in Fig.16.2b. Fig.16.2c demonstrates the vector diagrams.

Inductive resistance. An alternating current in an inductor lags behind the voltage by 90° (Fig..16.3 a-c) Relationship between amplitude values of imposed e.m.f. and current is as

$$\varepsilon_0 = I X_L \quad , \qquad (16.14)$$

where X_L is the inductive reactance and equal to

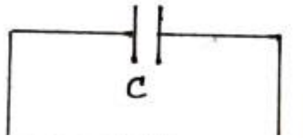
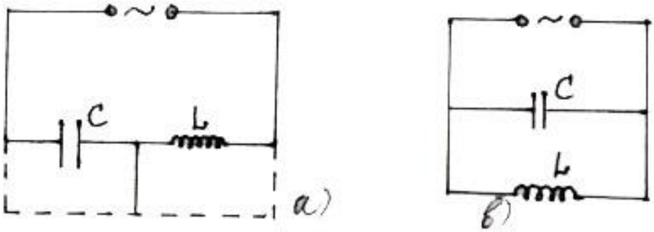
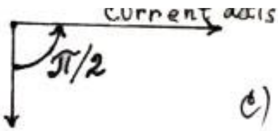


Fig.16.3

$$X_L = \omega L \quad . \qquad (16.15)$$



Capacitive resistance. When $U_L=0$ and $U_C>>U_R$, the current in capacitor leads the voltage by 90° (Fig.16.4. a-c) Capacitive resistance is determined by the equation



$$X_C = \frac{1}{\omega C} \quad . \qquad (16.16)$$

Total reactance.. When inductor L and capacitor C are connected in series Fig.16.5 the total reactance X is given by

$$X_{ser.} = X_L - X_C = \omega L - \frac{1}{\omega C} \quad . \qquad (16.17)$$

Fig.16.4

Fig.16.5

If the inductor and capacitor are connected in parallel the reactance is

$$X_{par.} = \frac{1}{X_L} - \frac{1}{X_C} = \frac{1}{\omega L} - \omega C \quad . \qquad (16.18)$$

§16.4. Ohm′ s law for a circuit of alternating current. Resonance

Ohm ′s law for a circuit of alternating current is given as

$$I = \frac{\Delta V}{Z} \quad . \qquad (16.19)$$

Fig.16.6

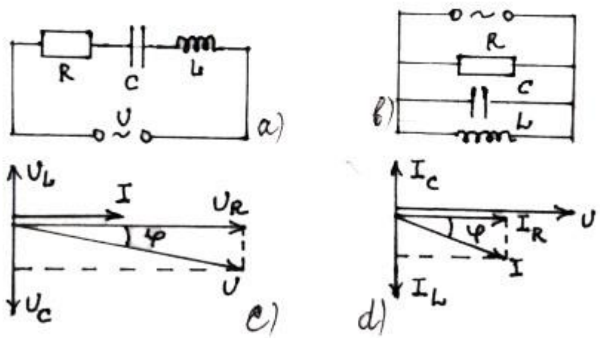
Here Z can be determined by the $Z = \sqrt{R^2 + X^2}$ when the active R and reactive resistances X are connected in series..

If the R and X are connected in parallel the expression for Z is

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X^2}} \quad . \qquad (16.20)$$

Phase difference. In case when R , C and L are connected in series (Fig.16.7) the phase difference φ between an active and reactive resistances is determined by the equation.

tg φ =



$$\frac{\omega L - \frac{1}{\omega C}}{R} \quad \text{and} \quad \cos\varphi = \frac{R}{Z} \quad . \qquad (16.21)$$

If the R, C and L are connected in parallel the equation (16.21) is given as

$$\text{tg}\,\varphi = R\left(\omega C - \frac{1}{\omega L}\right) \quad . \qquad (16.22)$$

Resonance.

The two types of resonance one can distinguished in a circuit of alternating current.

1. **Resonance in series circuit.** When capacitor and inductor are connected in series $X_L = X_C$ and the impedance is a minimum and the current has its maximum value. The voltages in capacitor and inductance coil exceed for several times the voltage applied to source of e.m.f. Ratios of voltages in L and C to the e.m.f. of generator is equal to quality Q. *Thus upon the resonance in a series circuit voltage in each reactance exceeds e.m.f. applied to the circuit as Q times.* Therefore resonance in series circuit refers to resonance of voltages.

2. **Resonance in parallel circuit.** In this case both junctions of circuit possess same voltage. Total current coming from generator is divided into two currents as I_L and I_C . The current in inductive junction lags the voltage by a definite angle given by

$$cos\varphi_L = \frac{R}{\sqrt{R^2+(\omega L)^2}} \qquad (16.23)$$

Resonance resistance of the circuit in parallel is the function of quality of circuit:

$$R_{res} = \rho Q = RQ^2 \qquad (16.24)$$

Thus upon resonance current in each of reactance exceeds the current coming from generator (minimum current) as Q times.

Therefore resonance in parallel circuit is the resonance of currents..

§16.5. Power of alternating current

Instantaneous power of alternating current is determined by the production of instantaneous values of current and e.m.f.

$$P=I\,\varepsilon= P_0 \sin^2 \omega t \;, \qquad (16.25)$$

where $P_0 = I_0^2 R$ is the amplitude value of power.

Average power. Average power is defined with the work done by alternating circuit during time interval equal to the period *T* **:**

$$P = \frac{A}{T} = \frac{1}{2} I_0^2 R \quad . \qquad (16.26)$$

Active power: The power developed by an alternating current in the circuit is

$$P = U_{eff.} I_{eff.} \cos \varphi \, . \qquad (16.27)$$

The factor **cos φ** is called the **power factor**.

§16.6. Production of electromagnetic waves

A transmitting antenna is a good example of a system generating electromagnetic waves through an oscillating charge. An antenna is a long loop of wire charged by an alternating source of potential, say, of frequency *f* and period *T*.

As the charging potential varies, so does the charge on antenna. The dimensions of the antenna are adjusted for this purpose. If the charge on the top of the antenna is + q at any time, the charge at this point will be – q after time T/2. This reversal of charges occurs due to accelerated motion of charges which creates electric flux changing with frequency .. The changing electric flux gives rise to electromagnetic waves that propagate away from the antenna. The frequencies of such electromagnetic waves are equal to the frequencies of the sources generating them. The oscillating magnetic field produced by changing electric field is at right angle to it. The direction of propagation of the waves is perpendicular to the electric and magnetic fields . If a such wave is intercepted by a conducting wire, the oscillating electric field will develop an oscillating voltage in it .

The frequency of the voltage so developed will be equal to the frequency of the wave. There can be many transmitting stations producing electromagnetic waves of different frequencies simultaneously. We can make arrangements to receive one of these stations by connecting inductance L and a variable capacitor of capacitance C in parallel . We adjust the value of *C* such that the natural frequency $\frac{1}{2\pi\sqrt{LC}}$ of this *LC* circuit is equal to the frequency of the incoming electromagnetic wave. At this stage, resonance is said to be achieved between the frequency of the approaching wave and the *LC* circuit. The response to a particular frequency given by $\frac{1}{2\pi\sqrt{LC}}$ is maximum at resonance. By changing the value of *C*, we can get resonance with other frequencies and receive electromagnetic waves from other transmitting antenna.

Maxwell had proved that radiation of energy in space occurs in the form of electromagnetic waves. He showed that the velocity of these waves is given by $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ in space.

§16.7. The transmission of information and its reception

The electromagnetic waves emitted by the LC circuit of aerial of a transmitting station are of constant amplitude . These waves have frequency of the range of 10^6 Hz, traveling with the velocity of light i.e., 3×10^8 m s^{-1} . Sound waves travel with a speed 330 m s^{-1} . The sound waves are superimposed on electromagnetic waves called carrier waves of an aerial. The sound waves are converted into electrical oscillations. This is done by a microphone. The amplitude of the electro magnetic waves begin to fluctuate with the frequency of the sound waves. The louder the sound wave, the greater the fluctuation in the amplitude of the electro magnetic wave which is called modulated wave.

The modulated electromagnetic waves falling on an aerial, induce a current of frequency equal to the frequency of the electromagnetic wave. If this current is passed through a headphone, the diaphragm of the headphone will not vibrate with this high frequency. Even if it does, normal human ear will not respond to it because only the frequency of the range 20 to 20×10^3 Hz is audible. In order that these frequencies can be audible, the electromagnetic waves are rectified. The current now fluctuates but it flows in one direction.

A crystal diode is used for rectification of the electromagnetic waves. The electromagnetic waves are received by an aerial of inductance L_1 coupled to an another tuning circuit containing inductance L_2 and a variable capacitor C_2 . The capacitance C_2 is varied so that the frequency of the tuning circuit L_2C_2 is equal to the frequency of the incoming electromagnetic waves. The amplitude of this current varies with the frequency of the sound wave. Thus the original sound is reproduced and heard.

CHAPTER 17

Reflection and refraction of light

§17.1. Photometrical quantities

Luminous intensity. The luminous intensity **I** of a light source is measured by comparing it with the international unit, the **candela**. At present, the standard source of luminous intensity is one sixtieth of a square centimeter of fused thoria. At the temperature of freezing platinum, thoria is incandescent and emits a steady flow of light energy. One candela is defined to be the **luminous intensity** of this source.

If the two sources of light produce the same illuminance at the same surfaces then the luminous intensity is determined by a comparison:

$$E = \frac{I_1}{r_1^2} = \frac{I_2}{r_2^2} \quad . \tag{17.1}$$

Hence for unknown I_2 we get:

$$I_2 = I_1 \frac{r_2^2}{r_1^2} \quad , \tag{17.2}$$

where r_1 and r_2 are the distances between point of interest and light sources.

Luminous flux. The flow of light energy from a source is called the **luminous flux**: the unit of luminous flux is called the **lumen**. One lumen is equal to the luminous flux contained in a solid angle of one steradian, when the luminous intensity is one candela:

$$\Phi = I\Omega \quad , \qquad (17.3)$$

where Ω - is the solid angle, which is determined as a ratio of surface area of a sphere to the square of radius.

$$\Omega = \frac{\Delta A}{r^2} \qquad (17.4)$$

Since the sphere of one meter radius has a total surface area of $4\pi\,(1\text{m})^2$ or $4\pi\text{m}^2$, the solid angle will be equal to 4π for isotropic point source.

$$\Phi = 4\pi I.$$

Illuminance. Illuminance is the rate at which light energy falls on a unit area some distance from a light source (Fig. 34). It is determined by the formulas

$$E = \frac{\Delta \Phi}{\Delta A} \quad \text{or} \quad E = \frac{I\cos\alpha}{r^2}, \qquad (17.5)$$

where ΔA is the surface area receiving the luminous flux.

Thus, illuminance is varyed directly with the source intensity and varies inversely with the square of distance away from the source. The SI unit of illuminance is the **lux.**

Luminous emittance is expressed by the formula

$$R = \frac{\Delta \Phi}{\Delta A} \quad , \qquad (17.6)$$

where ΔΦ -is the luminous flux emitted out in all directions by surface element of the source. The unit of measurement of luminous emittance is the **lux**.

Brightness. To characterize the light irradiation in the fixed direction the brightness is usually used. Brightness is defined as

$$B = \frac{\Delta \Phi}{\Delta \Omega \ \Delta A\cos\alpha} \quad , \qquad (17.7)$$

where **α** - is the angle between axis of solid angle and the normal, drown to the surface ΔA.

SI unit of brightness is cd/m^2

For lambert sources (for which B=const. and same in all directions) relation between luminous emissivity and brightness has form:

$$R = \pi \mathbf{B} \, .$$

Geometrical optics deals with the ***rectilinear propagation*** of light.

§17.2. Rectilinear propagation of light. The speed of light

In homogeneous medium light travels in straight lines and does not bend around objects.

The appearance of a shadow is explained by the rectilinear nature of light propagation in the homogeneous medium. From a single point source of light arises the whole shadow (Fig.17.1a).

Two and more sources of light give the whole shadow and semi-shadow (Fig.17.1b).

Among significant methods for the measurement of speed of light may be mentioned the following:

a) Roemer′s method, which rested on the observation of the time between eclipses of Jupiter′s moons.

b) Bradley′s aberration method, which depends on the apparent change in position of ′fixed′ starts as the earth makes its circuit around the sun.

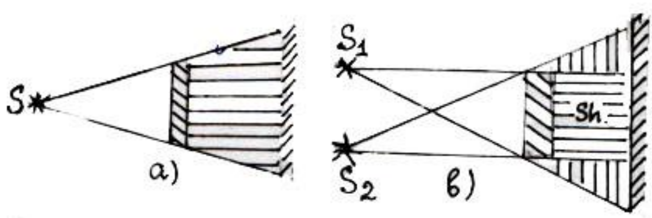


Fig.17.1

c) The method of Fizeau and Foucault in which an interrupted light beam was made to travel several miles, to and from a mirror placed at the greatest feasible distance.

d) Michelson′s method amounted to an improvement of Foucault′s method.

Numerous measurements made afterwards by various methods indicated that the speed of light in vacuum is the universial constant and is not dependent on the frequency and equals to **c=3x10^{8}m/sec**.

§17.3. Laws of Reflection and Refraction of light

A ***light ray*** is the direction along which light propagates. **Incident angle.** Angle α_i, formed by the incident ray and the perpendicular to surface drown from point of incidence O is called the ***incident angle***. (Fig. 17.2.a).

Reflection angle. Angle α_r formed by the same perpendicular and the reflected ray is called the ***reflection angle***.

The reflection of light obeys the following laws:

1.**The incident and reflected rays lie in the same plane as the perpendicular to the reflecting surface at the point of incidence**.

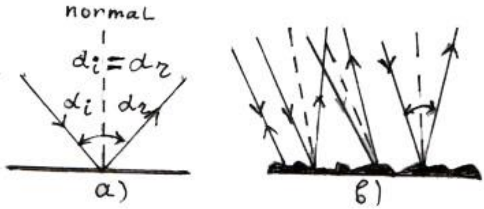


Fig.17.2

2.**The angle of reflection is equal to the angle of incidence**.

Reflection may be mirrored (regular, Fig. 17. 2.a) and diffused (irregular, Fig.17. 2.b).

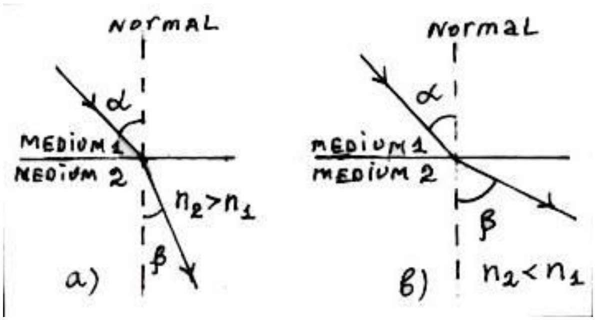


Fig.17.3

Refraction occurs because of dependence the speed of light on the medium in which the light is traveling. The relation between the speed of light in a medium υ and the index of refraction n is $n = c/\upsilon$, i.e. the index of refraction of a medium is the ratio of the speed of light in a vacuum to the speed of light in the medium.

Optical density is the property of a medium, that determines the speed of light in that medium. If a medium is optically dense, it slows light more than a medium which is less optically dense.

At the borderline of two media of different optical densities light ray changes its direction when it passes from one the medium into another

When the ray of light passes into the medium with greater optical density the angle of refraction is smaller than the angle of incidence; (Fig.17.3a) when the ray of light passes from an optically denser medium into a rarer medium, the angle of refraction is greater than the angle of incidence (Fig.17.3b)

Law of refraction of light.

Snell′s law states that the ratio of the sine of the incident angle to the sine of the refracted angle is constant:

$$n=\frac{\sin\alpha}{\sin\beta}=\frac{n_2}{n_1}\ . \qquad (17.8)$$

§17.4.Total internal reflection. Critical angle

Total internal reflection occurs when a ray of light passes from a medium with greater optical density into medium with a less optical density. By other words, in the medium in which light falls from, a speed is lower than in the adjacent medium or the angle of refraction is larger than the angle of incidence.

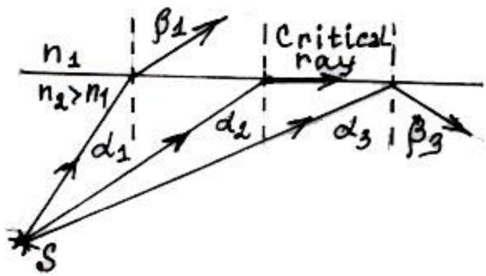


Fig.17.4

The fact that the angle of refraction must be larger than the angle of incidence leads to an interesting phenomenon known as a ***total internal reflection***.

The angle of incidence α for which the angle of refraction β from any medium into air is 90° is called the ***critical angle*** for the medium. If $\angle\alpha > \alpha_{cr}$ total internal reflection occurs and no light rays are transmitted into second medium. From the law of refraction a ***critical angle*** is determined as

$$\sin\alpha_{cr.} = \frac{n_2}{n_1}\sin 90° = \frac{n_2}{n_1}$$

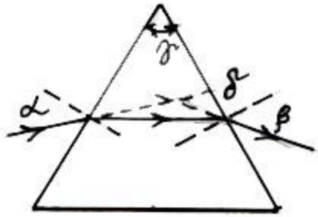


Fig. 17.4

For light refracted from the borderline between of water and air $\sin\alpha_{cr.} = 1/1.33 = 0.75$ i.e. $\alpha_{cr.} = 49°$.
shows an performance of total internal reflection. Ray 1 is refracted. Ray 2 is refracted along the boundary of the medium showing the **critical angle**. An angle of refraction grater than the critical angle results in the ***total internal reflection*** of ray 3.

§17.5. Plane Parallel Plate. Prism

When a light passes through the plane parallel plate (Fig.17.5) a refraction occurs at two parallel (upper and lower) surfaces. Therefore light ray after passing the plate does not change its direction, but displaces parallel to itself. The displacement X of the ray is expressed by the formula

$$X = \frac{d\sin(\alpha - \beta)}{\cos\beta}, \quad (17.9)$$

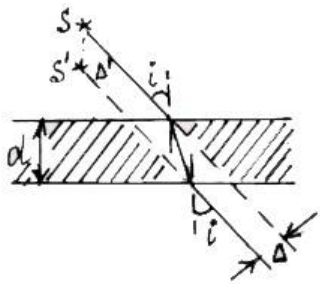


Fig.17.5

Here d-the thickness of the plate , α -the angle of incidence at the first surface, β -the angle of refraction at the first surface that equals to the angle of incidence to the second surface.

When light ray falls to the surface of **prism** (Fig.17.6) the double refraction occurs.

Total deviation δ from initial direction depends on the angle of incidence α, angle of refraction β on the second
Fig.17.6
surface and the refracting angle γ of prism:

$$\delta = \alpha + \beta - \gamma \ .$$

If the angle γ is very small we can use

$$\delta = (n-1)\,\gamma \ , \qquad (17.10)$$

where **n** –is the refracting index of prism.

CHAPTER 18

Mirrors and lenses

§18.1. Formation of an image by plane mirror

A plane mirror reflects light rays in the same order that they approach it. The image is of the same size as an object and is the same distance behind the mirror as the object is in front of it. The image is also erect but reversed right for left (18.1)

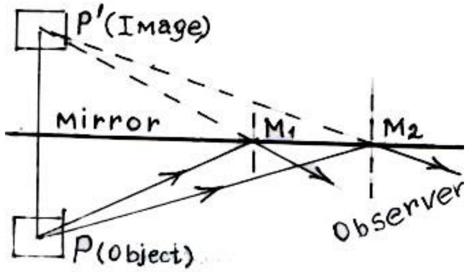


Fig.18.1

§18. 2. Concave mirror

The ***principal axis*** is an imaginary line extending from the geometric center of a spherical mirror to its centre of curvature .

A concave mirror is a converging mirror. The ***focal point*** F of a converging mirror is the point where parallel rays of light meet after being reflected from the mirror.(Fig.18.2)The distance from the vertex to the focal point of a spherical mirror is called the focal length (F) The is related to the radius of curvature (R) by this simple equation: F=R/2

It is important to remember two rules concerning concave mirrors:

1. Any light ray, approaching the mirror parallel to the principal axis is reflected through the focal point.

2. Any ray that approaches the mirror through the focal points is reflected parallel to the principal axis.

Let us consider some cases of formation of image from a converging mirror. In Fig.18.2 different locations of image are shown:

a) An object is placed at a distance greater than 2F. The image location is between F and 2F, inverted, real and smaller than object

b) Object is at 2F. Image is at 2F, inverted, real and the same size as the object.

c) Object is between F and 2F. Image is beyond 2F, inverted, real and larger than the object.

d) Object is between F and mirror. Image is behind mirror, erect, virtual and larger than the object.

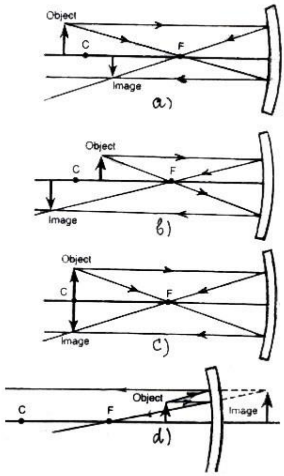


Fig.18.2

§18.3. Convex mirror

It is a spherical mirror which is reflective on its outer surface. Convex mirrors never form real images. The focal length F of a convex mirror is negative.Fig.18.3 shows that an object can be anywhere image is behind **mirror, erect, virtual** and **smaller** than the object.

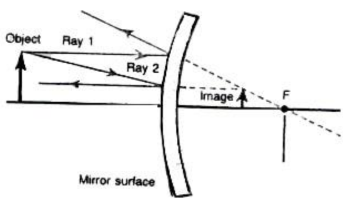


Fig.18.3

Spherical mirror equation:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{F}$$

where d_o is the object distance, d_i refers to image distance and F indicates the focal length.

The ratio of image size (H) to the object size (h) is called the lateral magnification: γ=H/h that is can be expressed in terms of image and object distances : $\gamma=-\frac{d_i}{d_o}$

Sign conventions for spherical mirrors:

■ The focal length F (or R) is positive for a concave mirror and negative for a convex mirror.

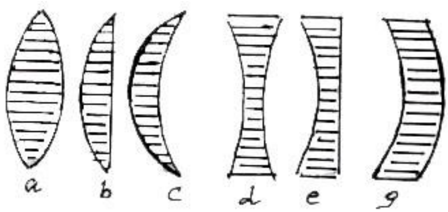
■ The object distance d_o is always taken to be positive.

■The image distance d_i is positive for a real image(formed on the same side of the mirror as the object) and negative for a virtual image (formed behind the mirror).

■The magnification γ is positive for an upright image and negative for an inverted image.

§18.4. Lenses

A *lens* is a transparent curved piece of polished or moulded material, usually glass restricted on two sides by curved surfaces.



The following shapes of lenses are known (Fig.18.4)

For *converging* lenses: a) double-convex; b) plano-convex; c)concave-convex.

For **diverging** lenses: e) double-concave;
d) plano-concave;
g) convex-concave.

A straight line passing through the centers of surfaces of two lenses is Fig.18.4
called the ***principal optical axis***.

The **focal point** of a lens is the point where rays that approach the lens parallel to the principal axis meet after being refracted by the lens.

The distance from the lens to its focal point (focus) is called **focal distance** *or* **focal length**.

The lens is called **converging** if its focus distance F > 0, and **divergin*g*** if focus distance F< 0.

The focal point of diverging lens is called the **virtual focus.** **(** Fig.18.6)

The focal length of a lens depends on its shape and its index of refraction.

In general, the focal length of a thin lens in air is given by the so called lens maker's equation

$$\frac{1}{F} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \qquad (18.1)$$

where n-is the index of refraction of the material , R_1 and R_2 radii of curvature of the first and second lens surfaces respectively. In this equation the radius of curvature is positive **(+R)** when center of curvature is on side of lens from which light emerges, is negative **(-R)** when center of curvature is on side of lens on which light is incident and becomes infinite **(** ∞R **)** for a plane (flat) surface. The focal length is positive **(+F)** for converging lens and negative **(-F)** for diverging lens.

	F	R_1	R_2
1. Double-convex	+	+	+
2. Plano -convex.	+	+	∞
3. Concave convex	+	+	–
4. Double-concave	–	–	–
5. Plano-concave	–	–	∞
6. Convex-concave	–	–	+

§18.5. Lens equation. Combination of lenses. Linear magnification.

Lens equation has the form

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{F} \qquad (18.2)$$

where d_o is the object distance, d_i refers to image distance and F indicates the focal length.
If d< F the equation (18.2) takes the forms

$$\frac{1}{d_o} - \frac{1}{d_i} = \frac{1}{F} \qquad \text{for convex lens}$$

$$\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{F} \qquad \text{for concave lens}$$

The quantity $D = 1/F$ is called the ***optical power*** of lens. D>0 for convex and D< 0 for concave lenses. The unit of measurement of D is diopter (m^{-1})

Optical power for a system of two lenses is determined by the formulas

$$D = -LD_1 D_2 \quad \text{and} \quad D = D_1 + D_2 - dD_1 D_2 , \qquad (18.3.)$$

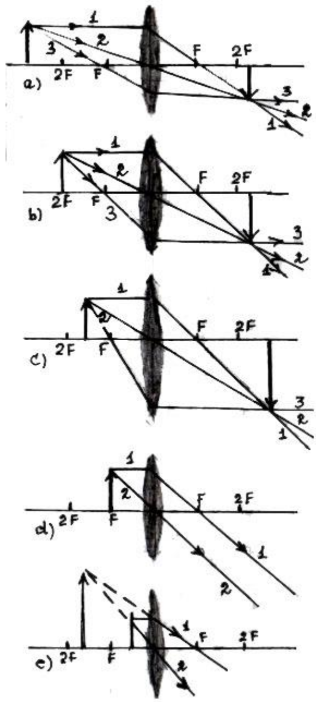
where L-is the distance between the rear focus of the first lens and the front focus of the second lens, d-the distance between lenses, D_1 and D_2 the optical powers of the first and second lenses respectively.

Linear magnification of the lens is defined as the ratio of image to the object size given by

$$\gamma = \frac{H}{h} \quad ,$$

where H-is the height of image and h-is the height of an object. Magnification is also given by $\gamma = \frac{d_i}{d_o}$ where d_i and d_o are the image and object distances from the lens respectively. Thus, from those expressions we get

$$\frac{H}{h} = \frac{d_i}{d_o} \qquad (18.4.)$$



§18.6. Constructing the image of an object by the lenses

Usually for the construction the image two from three rays are used:

1.) Ray, which passes from center of the lens without refraction;

2) Ray, which propagates parallel to the principal axis; this ray after refracting by the lens passes from second focus.

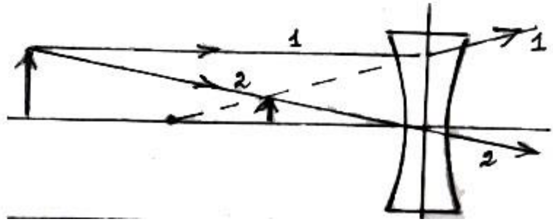
3) Ray (or its continue) which passes from first focus and after refracting by the lens goes parallel to the principal axis.

(Fig.18.5) Nature and position of image for both converging and diverging lenses are shown below:

For converging lenses**.**

Object Image Size Distance of image Magnification, nature of image

a) d>2F 2F > f>F
H< h $\gamma<1$, real, inverted, smaller
b) d=2F f=2F
H=h $\gamma = 1$ real,



inverted, the same size as the object
c) 2F> f >F f> 2F H> h $\gamma >1$, real, inverted, magnified
d) d=F f=∞ H=∞ $\gamma =\infty$ image is in endlessness
e) d<F f < 0 H> h $\gamma > 1$. virtual, erect, magnified and further
from the lens than the object

For diverging lenses.

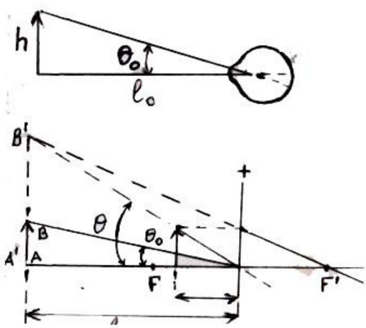
1. If a real object is placed in any position its image will be virtual, erect, Fig.18.5
diminished and placed inside the focus (Fig.18.6)
2. In case of virtual object placed outside the focus the image is virtual, inverted and outside the focus.
3. The image will be real, erect, magnified and farther from the lens than the object if object is real and placed inside the focus.

§18.7. Lens aberrations (defects)

Chromatic aberration.

A lens may be regarded as made up of prisms. It is evident that when a ray of white light passes through it, it will be dispersed into its constitue colors. All the red rays are brought to focus at F_r whereas all the blue rays are brought to focus at F_b as shown in Fig.18.7a. A complete image will consist of all the colored of the spectrum. It is the inability of the lens to focus all the dispersed colors at one point. Therefore the image will be colored and not well defined. This defect Fig.18.6
of the lens is called **chromatic aberration**.

This defect in the lens can be removed by using a combination of a convex lens and a concave lens made of two different materials having unequal dispersive powers. These lenses are given such suitable shapes that the dispersion produced by one lens is exactly equal and opposite to that produced by the other. The focal lengths of the lenses are, of course, unequal in numerical values; so that the focal length of the combination has a finite value. Such a combination is called an achromatic omination of lenses and is shown in Fig.18.7a



Spherical aberration.

A beam of parallel rays is focused at point by a lens only if the aperture of the lens is small, otherwise the lens will refract outer rays slightly more than the inner

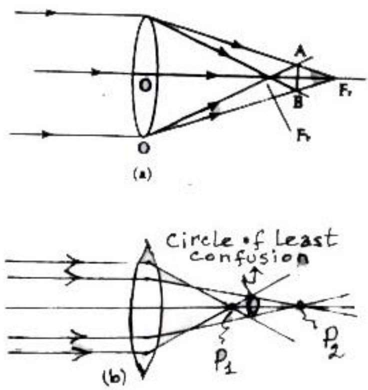
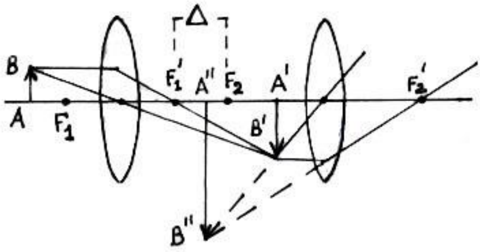


Fig.18.7

rays Fig.18.7b. The image produced will not be well defined and sharp. This defect in a lens is called spherical aberration. To refuse this defect, optical instruments using lenses are provided with a stop which allows only the central rays to pass through the lens. In this way, the effective aperture of the lens remains small and so the spherical aberration is almost removed.

The spherical aberration can also be reduced by taking suitable values of the radii of curvature of the surfaces of a lens of by using two lenses kept at a suitable distance apart. Now we shall discuss how the lenses are used in optical instruments.

Fig.18.8



§18.8. Angle of vision. Magnification of simple magnifier

Angle of vision is the angle, under which the object is seen. Fig.18.8 shows that $tg\theta_0 = h / l_0$, where h-size of object, l_0-the object′s distance.

Magnification is determined as $\Gamma = tg\theta / tg\theta_0$, where θ - is the angle of vision in presence of device and θ_0 -the angle in absence of device. (Fig.18.8)

Few eyes can see clearly an object whose distance is less than 25 cm, because of the insufficient *angle of vision* under which they are seen. To make angle greater various optical devices are used. Fig.18.8 shows one such device called a **magnifying glass**. In order to observe the magnified image, which can be obtained by means of a converging lens, the object should be placed some where between the lens and its focus so that its image be at a distance of the best vision from the eye. *Magnification* is determined as

$$\Gamma = \frac{s}{f} + 1 \,, \qquad (18.5)$$

where s-is the distance of best vision for normal eye equals to 25 cm., and f-is the focal distance of magnifier.

§18.9. Microscope

Larger magnifications are obtained by use the microscopes. The optical system of a microscope consists of the objective (facing an object) and the eyepiece (facing the eye). The path of the rays in a microscope is shown in Fig.18.9

Fig.18.9.The distance **Δ** between the rear focus of the objective and the front focus of the eyepiece is known as the optical length of the microscope tube. Calculations show that the magnification of a microscope is equal to the product of magnifications of the objective and eyepiece.

$$\Gamma = \frac{s \cdot \Delta}{f_1 \cdot f_2} , \qquad (18.6)$$

where Δ -the optical length of the microscope tube, f_1 and f_2 are focal distances of objective and eyepiece respectively.

§18.10. Telescope

Telescope are intended to enhance the angle of vision upon the observations of objects located at a great distances.
Telescope consists of two convex lenses: objective and eyepiece. The object is at a great distance from the objective. The intermediate image appears between the objective and eyepiece. The image distance from eyepiece is less than the focus of latter. The image becomes virtual, magnified and inverted. The magnification of telescope is calculated by the equation

$$\Gamma = \frac{f_1}{f_2} \qquad (18.7)$$

where f_1 and f_2 are the focal distances of objective and eyepiece respectively.

The telescope length ***l*** is equal to $l = f_1 + f_2$.

§18.11. The eye and vision

Fig.18.10 shows schematically how an image is obtained on the eye retina. The closer is the object to the eye, the bigger is its image on the retina and the greater is the angle θ at which the object is viewed (Look at Fig.18.10). This is the so-called **visual angle**. When the object is moved away from the eye, the visual angle at which the object is viewed will diminish and the object will appear smaller.

If the visual angle θ is small our eye will see two points situated side by side as one.

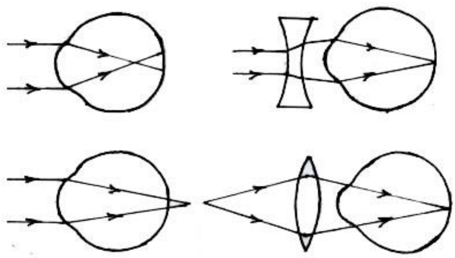


Fig.18.10

The distance which is the most convenient for observation an object is called the ***distance of the best vision*** **s** and equals to 25 cm.

Defects of eyesight: **a**) Near-sighted eye, **b**) Far-sighted eye.

To correct near-sightedness eyeglasses with concave lenses are worn.(Fig.18.10)

To correct far-sightedness eyeglasses with convex lenses are used.(Fig.18.10)

CHAPTER 19

Physical optics

§19.1. Interference of light. Huyghens′ principle

An interference pattern is set up when light falls on two narrow slits that are close together. In Fig.19 are shown two waves of wavelength λ traveling from slits, between which distance is **d**. In front of slits waves propagate in all directions, however, here are given waves under three different angles, In a) waves pass the same ways from slits and reach screen in the same phase. In this case the enhancement occurs and a bright spot appears at the centre of the pattern. b) Constructive interference appears then, when optical path difference equals to one wave length λ (or any whole number of wavelength) as is shown in Fig.19b Note, that in general case the **optical path** is equal to **n×s,** where **n**-the refracting index and **s** -is the geometrical path. It can be seen that

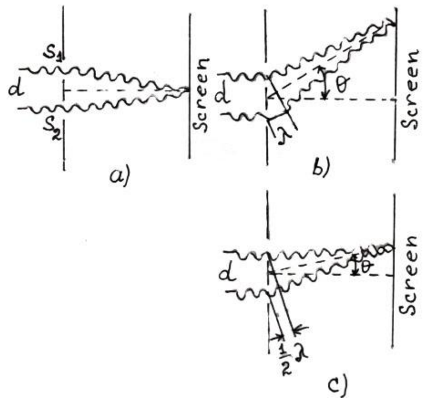


Fig.19.1

for constructive interference the optical path-difference $\Delta = d\sin\theta$ is determined as

$$d\sin\theta = 2m\frac{\lambda}{2} \quad , \qquad (19.1)$$

where m=0,1,2,... and is called the **order of interference**.
Destructively interference (Fig.19c) is observed ,when the path difference $d\sin\theta$ is equal to $\frac{1}{2}\lambda,, \frac{3}{2}\lambda,, \frac{5}{2}\lambda.....and$

$$d\sin\theta = (2m+1)\frac{\lambda}{2} \quad ,m=0,1,2,... \qquad (19.2)$$

.

Phase shift δ is related to the optical path difference by equation

$$\delta = \frac{2\pi}{\lambda} d\sin\theta \quad . \qquad (19.3)$$

Amplitude of resultant oscillations at the given point is determined as

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\theta_2 - \theta_1) \qquad (19.4)$$

If the phase difference θ_2 - θ_1 is constant, then oscillations are called-**coherent**. Then the resultant intensity is expressed by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\theta_2 - \theta_1) \quad . \qquad (19.5)$$

Resultant intensity I will be exceed I_1+I_2 at the points , for which $\cos(\theta_2-\theta_1) > 0$, and will be less than I_1+I_2 when $\cos(\theta_2-\theta_1)<0$.

Dependence of intensity at any point of screen on the phase shift $\delta = \theta_2-\theta_1$ is given by

$$I_\theta = I_0 \cos^2 \delta / 2 \quad , \qquad (19.6)$$

where I_0-is the intensity at the center of pattern. An interference pattern can be used to measure the wavelength of light waves. Fig.19.2 gives schematic diagram for analysis the double-slit interference (Young′s double-slit experiment). Consider a point P on the screen. The path difference between the two rays arriving at P is

$$S_2P - S_1P = S_2Q = d\sin\theta,$$

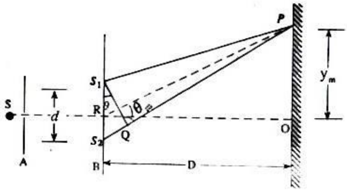


Fig.19.2

thus for P to be a bright fringe, the path difference should be an integral multiple of the wavelength as explained above; that is

$$d\sin\theta = m\lambda \ , \qquad (19.7)$$

where m=0,1,2,... for bright fringe. At point O, $S_2P - S_1P = 0$. the path difference is zero; there will be a bright at O corresponding to m=0, and is called zeroth - order fringe or zero order fringe. According to Fig. 20 a calculation y the position of point P from the central point O gives

$$\mathbf{y_m} = \mathbf{m}\lambda \frac{\mathbf{D}}{\mathbf{d}} \ .$$

Hence for bright fringes

$$\lambda = \frac{\mathbf{y_m}}{\mathbf{m}} \frac{\mathbf{d}}{\mathbf{D}} \ . \qquad (19.8)$$

By knowing d and D, measuring y and counting the number of fringes m from O, we can calculate the wave length. Distance between the centers of two consecutive bright fringes or dark fringes is called *width* or fringe *spacing*. In order to obtain it we first find the position of the m-th dark or bright fringe and subtract from it the position of the (m−1)- th dark or bright fringe. This will the fringe width:

$$\Delta y_m=\lambda(D/d) \quad . \qquad (19.9)$$

Huyghens′ Principle. Huygens’ principle states: Every point on a wave front may be considered as a source that produces secondary wavelets. The wavelets propagate in the forward direction with a speed equal to the speed of the wave motion. Let us consider a wave front A produced by a source of light S (Fig.19.3). Consider the dots on this wave front as secondary sources, each producing a spherical wave. This is done by drawing hemisphere of radii $c\Delta t$, where c is the speed of light and Δt is the time during which the wave propagates from one wave front to the other. If we join the points on these spherical surfaces which are in phase, the result is a new wave front B. Similarly we can construct a wave front C from wave front B. The process can be repeated. There are an infinite number of wave fronts but we have just shown three of them.

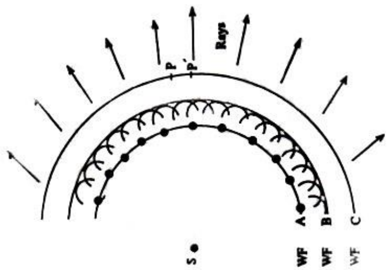


Fig.19.3

Dutch philosopher Huyghens in his attempt to fit a wave theory to the known facts of straight line propagation, reflection and refraction proposed to consider that every point of any wave front W_1 acts as a center from which elementary spherical waves spread out front W_2 is continually formed along the line that touches all of the secondary wave fronts.(Fig. 19.3)

§19.2. Interference in thin films

Light reflected from a thin films shows interference. There are two important cases. In first case the two surfaces of the thin film are parallel and the incident light is inclined to the surfaces. A part of falling light reflects at point A from upper surface, other part passes into film and reflects at point D from its lower surface (Fig.19.4) Light reflected from the lower surface passes additional distance ADC. If the distance ADC equals to wavelength or integral number wavelength, then both of waves(they are coherent, since appears in the same point of source) show interference enhancing one another: **Δ=mλ** ,where m=0,1,2,... etc. If the way ADC equals to $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{3}\lambda$ and etc. waves appear in opposite phases and damping one another. (Δ=(m+$\frac{1}{2}$)λ), here **Δ**=(AD+DC)n - BC is the **optical path difference.** This quantity may be expressed as

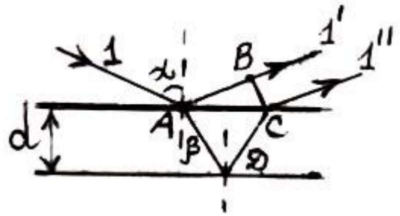


Fig.19.4

$$\Delta = 2d\sqrt{n^2 - \sin^2\alpha} \quad , \qquad (19.10)$$

where **d**-the thickness of the thin film, n-the refracting index and **α** -the angle of incidence.

Finally, for interference maximum of reflected ray

$$2d\sqrt{n^2 - \sin^2\alpha} = (2m+1)(\lambda/2) \quad , \qquad (19.11)$$

and for interference minimum of reflected ray we get

$$2d\sqrt{n^2 - \sin^2\alpha} = 2\,m(\lambda/2) \quad . \qquad (19.12)$$

Note, that relations (2.11) and (2.12) are conditions for minimum and maximum of passed ray respectively.

The second case is of a wedge-shaped film such as the air film between two smooth glass surfaces. In this case the reflected light from the upper and lower surfaces of the air film have a path difference. When emerging from the top they are superimposed to produce interference fringes. The fringes in this case are straight lines and parallel to the edge of wedge. The fringe at the point where the thickness of film is zero is a dark fringe even though the optical path lengths are the same. It is due to the rule given below.

When a light wave traveling in one medium is reflected from the surface of a second medium which has a greater refractive index, the reflected ray suffers an additional phase change of 180° or a path difference of $\lambda / 2$. No phase change takes place if the second medium has a lower refractive index. The conditions of refractive and destructive interference will interchange if there is an additional phase difference of 180 ° .

§19. 3. Newton′s rings

Newton devised an experiment to find the wavelength of light. The interference pattern is formed in plane-convex lens placed in contact at the point O with a plane-glass surface. When this arrangement is illuminated from above by light a series of concentric rings can be observed. These are called Newton’s rings. Circular or ring like fringes are formed due to the interference between the rays 1 and 2 reflected by the bottom and top surfaces of the air gap between the convex lens and the plane glass.

Because this air gap increases in width from the central contact point out to the edges, the extra path length for the lower ray varies, where it equals 0, $\frac{\lambda}{2}$, λ, $\frac{3}{2}\lambda$, 2λ and so on, and results in series of bright and dark rings. The interference pattern which is formed in plane-convex lens, coinciding at the point O with the plane parallel glass is called ***Newton's rings***. Ray 1 passes the air gap twice and shows an interference with the ray at the point C. Interference pattern consists of bright and dark circles with radius r_N .

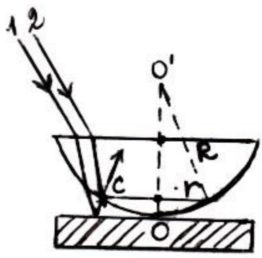


Fig.19.5

(Fig.19.5). Newton′s rings
allows to calculation the wavelength by the following relation:

$$\lambda = \frac{2r_N^2}{(2N-1)R} \quad , \qquad (19.13)$$

where r_N is the radius of the N-th light ring, R-is the curvature radius of the lens.

§19.4. Diffraction of light . Single slit Diffraction

A phenomenon of bending of light around the edges of an obstacle is called diffraction of light. This phenomenon is exhibited when light passes through a narrow slit or aperture. When light shines on a slit placed in front of a screen the interference pattern consisting of the seriesly mutual images of source, separated with the distances is observed. All parallel rays (Fig. 19.6), falling under angle **φ** to the optical axis meet at the point F_φ. For constructive interference the angles φ obey condition

$$b\sin\varphi = (2m+1)\frac{\lambda}{2} \quad m=0,\ 1,\ 2..... \ , \qquad (19.14)$$

here b-is the width of slit and number m-is called the **order of diffraction maxima or minima.**
Condition of destructive interference (diffraction minima) is

$$b\sin\varphi = 2m\frac{\lambda}{2}$$ **m=1, 2,...** (19.15)

The quantity Δ= CD=bsinφ is the optical path difference between edge rays CN and BM going from slit under the angle φ.

In the direction φ =0 the zero order most intensively central maximum is observed. The light amplification always is observed at the point F_0 independently of the wavelength λ.

There is no significant difference between an interference and diffraction phenomenon. Interference is usually referred to as the superposition of only a few no of very large number of secondary waves.

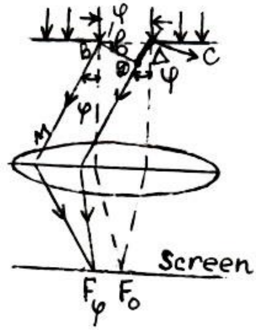


Fig.19.6

§19.5. Diffraction grating

The most interesting case of diffraction can be realized by means of a **diffraction grating**. An array of large number of parallel slits all of the same width and spaced equally is said to form a diffraction grating. It is formed by drawing very fine equidistant parallel lines on the glass plate. The light cannot pass through the lines while the spacing between the lines is transparent to light. A simple diffraction grating is a plate with alternating narrow transparent and oraque parallel strips. The sum of the widths of a transparent(reflecting) and oraque (scattering) strips **d** is called the **grating period** and is given by **d=1/ n** where

n is the number of lines in one unit length of a diffraction grating respectively. In the best of modern gratings there are up to 1800 slits per millimeter so that the grating period is about **0.8μm**.

If the general width of grating is L, the number of slits is N, then grating interval d may be computed as

$$d = \frac{L}{N}$$

Let us consider the wave propagating from the grating in a direction forming angle **φ** with the normal to the grating plane (Fig.25). The path differences for the rays emerging from similar points have the same value**: Δ=dsinφ.** In order that all beams enhance one another, it is necessary that **dsinφ** be equal to an integral number of wavelength **λ** i.e.

$$\mathbf{d\sin\varphi = n\lambda,} \qquad n=0;\ 1;\ 2;\ \ldots \qquad (19.16)$$

Expression (19.16) is the condition of constructive interference. n=0 is the central bright line or central maxima; n=1 is the first order maximum; n=2 is the second order maximum and so on. The direction with φ = 0 (or n=0) is the direction of the initial beam.

If white light is used in a grating the diffraction produces several color lines on either side of the central maximum for each n.

A grating is used for determination the wave length of light by measuring angle φ which shows the position of the maximum of a given order. From the measured value of φ and from the knowledge of the grating interval, the wavelength can be determined. From the relation (2.14) we get

$$\lambda = \frac{d\sin\varphi}{n} \quad . \qquad (19.17)$$

For small angles φ formula (19.8) has the form

$$\lambda = \frac{\mathbf{x}}{\mathbf{y}} \frac{\mathbf{d}}{\mathbf{n}}, \tag{19.18}$$

where **x** – the distance between **n** – th and 0 – th (central) fringes of maximum intensity, **y** is the distance between grating and screen.

§19.6. Polarization of light

Consider a beam of ordinary light. It consists of a large number of waves, each with its own plane of vibration. All directions of vibration are equally probable and are always perpendicular to the direction of propagation. Now if we could have some such arrangement that allows only those vibrations that are parallel to slit to pass through it. The resulting beam is said to be polarized. This process of confining the beam of light to one plane of vibration is called **polarization.**

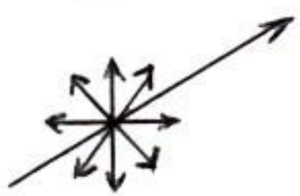


Fig.19.7

Plane of polarization is called the plane in which the vectors E are placed. If the plane of oscillation does not change its orientation in the space, then wave is called **plane (linearly) polarized**. Light, in which the orientation of polarization plane is changed irregularly is called ***natural light .*** (Fig.19.7).

Plane-polarized light can be obtained by help of a filter (**polarizer**), such as polaroid sunglasses, is placed in front of the beam of light, only those waves that vibrate parallel to the permitted plane pass through(Fig.19.8)

Suppose a second sheet of polaroid material(**analyzer**) is placed in the path of the polarized light. If its permitted plane is perpendicular to the light that passed through the polarizer, almost no light will pass through. Intensity of the light passed through polarizer is determined by the **Maluce′s law:**

$$I=I_0 cos^2\theta \quad , \qquad (19.19)$$

where θ-the angle between polarizer axis and polarization plane of incident wave, I_0-the intensity of incident wave.

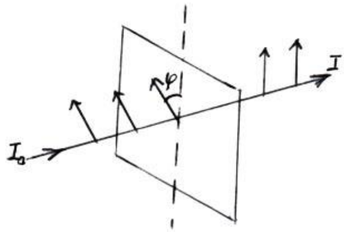


Fig.19.8

Polarization by reflection. A natural light can be partially polarized by reflection from the surface of dielectric . The light which is reflected from the glass becames completely polarized at the angle of incidence, determined from a relation called **Brewster′s law:**

$$tg\theta_B = n \quad , \qquad (19.20)$$

where angle θ_B – is called the Brewster′s or angle of total polarization , n-the refractive index.

Under perfect polarization by reflection the angle between reflected and refracted rays is equal to 90°.(Look at Fig.19.9)

Polarization by double refraction. When monochromatic light travels in glass, its speed is the same in all directions and is characterized by a single index of refraction. Any material like this is said to be isotropic, meaning that it has the same characteristics in all directions. Some crystalline materials, such as quartz, calcite, and ice are anisotropic with respect to the speed of light; that is, the speed of light is different in different directions within the material. Anisotropizm gives rise to some unique optical properties, one of which is that of different indices of refraction in different directions. Such materials are said to be

double refracting or to exhibit **birefringence.** For example, a beam of unpolarized light incident on a double refracting crystal of calcite ($CaCO_3$, calcium carbonate) is shown in Fig. 19.10. When the beam is at an angle to a particular crystal axis, it is doubly refracted and separated into two components or rays. Also, the two rays are linearly polarized in mutually perpendicular directions. One ray, called the ordinary (**o**) ray, passes through the crystal in an undeflected path and is characterized by an index of refraction (n_o) that is the same in all directions. The second ray, called the extraordinary (**e**) ray, is refracted and is characterized by an index of refraction (n_e) that varies with direction. The direction along of which $n_o = n_e$ is called **optical axis**. Some double refracting crystals, such as tourmaline, exhibit the interesting property of absorbing one of the polarized components more than the other. This property is called **dichroism**. Double refraction is due to features of propagation of electromagnetic waves in anisotropic medium: amplitudes of forced oscillations of electrons depend on the directions of these oscillations. Propagation of ordinary and extraordinary rays can visually be described by the wave surfaces. Only in the direction of optic axes speeds of ordinary and extraordinary rays are the same and equal to $\upsilon_0 = c/n_0$,where n_0 –is the index of refraction of ordinary ray, and is different for different crystals. For positive crystals $\upsilon_e \le \upsilon_0$, but for negative crystals $\upsilon_e \ge \upsilon_0$. Greatest difference

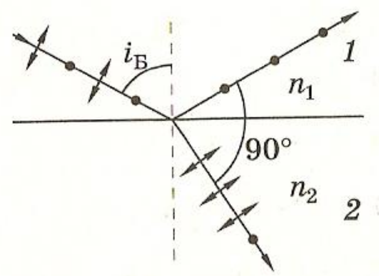


Fig.19.9

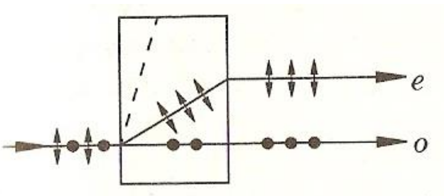


Fig.19.10

between speeds of ordinary and extraordinary rays is in the directions perpendicular to optic axis.

Double refracting crystals are used for preparation the polarizing prisms. In order to obtain two linearly polarized light by double refraction the widely used crystal the Iceland spar. with different refractive indices (n_o)

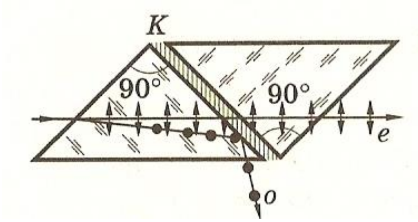


Fig.19.11

and (n_e). At wavelength $\lambda = 589.4 nm$ the refractive indices of Iceland spar are $n_o = 1.658$ and (Fig.19.11)

§19.7. Dispersion of light

A dependence of light speed (or refracting index) on the wavelength(or frequency) in a media is called a ***dispersion of light.***

$$\upsilon = f(\lambda) = \varphi(f)$$

White light is a combination of many colors of light. Each color corresponds to a different wavelength. When white light passes through a glass prism, these wavelengths are separated. Waves with different wavelengths due to dispersion are propagated in various directions: waves with shortest lengths are bended to the base side of prism under greater angle in comparison to waves with longest lengths. Light spectrum consist of seven colors (violet, orange, navy blue, dark blue, green, yellow, red): Red and violet lights are at opposite ends of the spectrum .

§19.8. The photoelectric effect

According to the quantum optics light is considered to be the stream of particles called - ***photons,*** having no mass of rest, and moving with velocity **c** , equal to speed of light in vacuum. The main characteristics of photons are

Energy $$E = hf = \frac{hc}{\lambda_0} \quad . \qquad (19.21)$$

Linear momentum $$p = \frac{hf}{c} = \frac{h}{\lambda_0} \quad . \qquad (19.22)$$

Mass $$m = \frac{E}{c^2} = \frac{hf}{c^2} \quad , \qquad (19.23)$$

where h-is the Planck's constant, h=6.62$\bullet$ 10^{-34} J$\bullet$sec.
Note, that wavelengths and frequencies of light have the following boundaries respectively:

$$\lambda = (\mathbf{0.40 - 0.75})\ \mu m$$
$$f = \mathbf{(0.75 - 0.40)\ 10^{15}}\ Hz$$

The emission of electrons from a metal plate exposed to light of certain frequencies is called the ***photoelectric effect*** (Fig.19.12).

The electrons ejected from the metal plate by the light are called ***photo-electrons.*** Photoelectrons accelerated by electric field between electrodes create a photocurrent **I**. Dependence of **I** versus potential difference is shown in Fig.19.13.

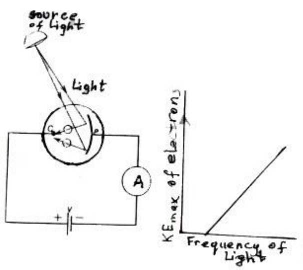


Fig.19.12

Curves in figure correspond to different illuminations **E** of cathode.

At a certain $V = V_{st.}$ the photocurrent reaches maximum value $I = I_{s.}$, which is called ***saturated photocurrent***. At $V = V_{st.}$ all electrons ejected from a metal plate reach anode: $I_{s.}=en.$, where n- is the total number of electrons ejected per unit time.,e-the electron′s charge. Existence of photocurrent at negative values of potential difference is explained so, that photoelectrons have initially kinetic energy with, maximum $m\upsilon_{max}^2/2$. The work done, required for stopping the electrons is equal to the maximum kinetic energy and is given by the equation

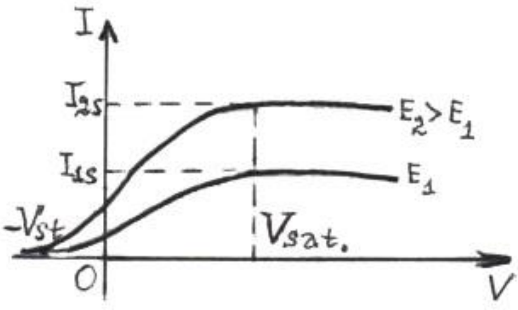


Fig.19.13

$$\frac{m\upsilon_{max.}^2}{2}=eV_{St}. \quad . \qquad (19.24)$$

Here $V_{st.}$ is the **stopping potential**.

§19.9. Laws of external photo-effect. Einstein′s equation

1. The maximum initial velocity $\upsilon_{max.}$ of photoelectrons depends on the light frequency and properties of metals surface.

2. Total number **n** of photoelectrons and power of saturated photocurrent is direct proportional to the illumination E of cathode.

3. For each substance there is a red boundary of photo-effect (threshold value), which is the minimum frequency $f_{min.}$ of light

needed to eject electrons from a plate. Light of frequency *(* $f \leq f_{min.}$*)* or *(*$\lambda \geq \lambda_{max.}$*)* does not eject electrons from metal.

Suppose that an electron in the metal surface absorbs this photon of energy .., before the electron can leave the surface, part of the incident energy must be used to overcome the binding energy of the electron in metal. This is called the work function. Thus in order to eject an electron out of the metal surface a photon must have an energy .. called the threshold energy equal to the work function of the metal.

Energy of photons *hf* falling on the metal represents the sum of the work done to stop the electrons and the work done A to free the electrons from the metal surface:

$$hf = \frac{m\upsilon_{\max.}^2}{2} + W. \qquad (19.25)$$

This expression is known as Einstein′s equation. The maximum kinetic energy of the emitted electron can be calculated from the ***photoelectric equation****:*

$$\frac{m\upsilon_{\max.}^2}{2} = eV_{st.} = h(f - f_{min}). \qquad (19.26)$$

In Fig.19.12 the graph shows the kinetic energy of ejected electrons as a function of frequency of incident light.

§19.10. Pressure of light

A pressure which is created by electromagnetic waves, falling on the surface of any object is called ***pressure of light***. Existence of light′s pressure is predicted in electromagnetic theory of light. Under action of electric field electrons move in opposite direction of E with velocity υ. Magnetic field acts on moving electrons with the Lorentz' force $F_{L.}$ in the sense perpendicular to metal's surface. Thus, light wave exerts a pressure to the metal′s surface, given as

$$p = (1+ r) w \, , \qquad (19.27)$$

where r-the reflection coefficient of medium, w-the density of energy flux.

CHAPTER 20

Relativistic mechanics

§20.1. Inertial frames of reference

In all frames of reference that move with uniform velocity with respect to one another, Newton's laws are valid in inertial reference frames. Such reference frames are called inertial reference frames. All inertial systems of reference are thus equivalent.

It is necessary to distinguish:

a) the motion of systems with lower relative speed ($\upsilon << c$) . In this case the laws of classical mechanics are true.

b) the motion of systems with higher relative speed ($\upsilon >> c$) . In this case the laws of relativistic mechanics are valid.

§20.2. Galilean transformations

Galilean transformation of space time coordinates relate the motions in an two inertial systems of reference that move with small relative velocity.

If (x , y , z , t) are coordinates of point with respect to the unprimed frame of reference in stationary and (x' , y' , z', t')

are coordinates and time with respect to the primed frame of reference

$$x' = x - \upsilon t \; ; \quad y' = y \; ; \quad z' = z \; .$$
$$x = x' + \upsilon t \; ; \quad t' = t \qquad (20.1)$$

§20.3. Velocity transformations

From the transformation of the space time coordinates we can find that

$$\upsilon_x' = \upsilon_x - \upsilon; \quad \upsilon_y' = \upsilon_y; \quad \upsilon_z' = \upsilon_z; \quad \upsilon_x = \upsilon_x' + \upsilon , \qquad (20.2)$$

where υ_x' is the velocity measured in K′ frame of reference, and υ_x is the velocity measured in K frame of reference .

§20.4. Acceleration and force transformations

An acceleration of an point in an inertial frame of reference we find as the ratio of the difference of velocities to the time interval.

$$a = \frac{\upsilon_2 - \upsilon_1}{t_2 - t_1} \; ,$$

where υ_1 and υ_2 are the velocities in the unprimed frame of reference. According to the law of addition of velocities

$$\upsilon_1' = \upsilon_1 - \upsilon \; ; \qquad \upsilon_2' = \upsilon_2 - \upsilon \; ,$$

where υ_1' and υ_2' are the velocities measured in the primed frame of reference. Therefore the new acceleration a′ in the primed frame K′ is

$$a' = \frac{\upsilon_2' - \upsilon_1'}{t_2' - t_1'} \quad . \qquad (20.3)$$

Having put (20.2) into (20.3) we get :

$$a' = a . \qquad (20.4)$$

Thus the acceleration measured in the moving frame of reference is exactly equal to that measured in the stationary frame.

Since the mass is independent of velocity and the assumption that the laws of physics are the same in the primed as well as in the unprimed frame of reference means that

$$F = F' . \qquad (20.5)$$

Thus in all inertial frames of reference would agree in the magnitude and direction of the force F independent of the relative velocities of the reference frames.

§20.5. Lorenz′s transformation

Lorenz′ s transformation give a relation between the space time coordinates of two frames of reference at higher velocities of their relative motion.

Thus

$$x' = \frac{x - \upsilon \mathbf{t}}{\sqrt{\mathbf{1} - \beta^2}}; \quad y' = y; \quad z' = z ; \quad x = \frac{x' + \upsilon \mathbf{t}'}{\sqrt{1 - \beta^2}} , \qquad (20.6)$$

$$t' = \frac{t - \beta x / c}{\sqrt{1 - \beta^2}} \quad ; \qquad t = \frac{t' + \beta x'/c}{\sqrt{1 - \beta^2}} , \qquad (20.7)$$

where $\beta = \upsilon / c$ and c is the speed of light in a vacuum.

A linear momentum and energy in the moving frame of reference are related to the correspondingly quantities in the stationary frame of reference as follows

$$p'_x = \frac{\mathbf{p}_x - \upsilon E / c^2}{\sqrt{1-\beta^2}} \quad ; \qquad p'_y = p_y \; ; \qquad p'_z = p_z \; , \qquad (20.8)$$

$$E' = \frac{E - p_x \upsilon}{\sqrt{1-\beta^2}} \quad , \qquad (20.9)$$

§20.6. Einstein′s postulates of special theory of relativity

A. Einstein proposed a new theory of relativity based upon two postulates which are follows :

1. All laws of physics are the same in every inertial reference frame .

2. The speed of light in a vacuum, measured in all inertial reference frames always has the same value of c , no matter how fast the source of light and the observer are moving relative to each other.

Some interesting results of the special theory of relativity can be summarized in the following: time dilation, length contraction , mass and moment increasing .

§20.7. Time dilation

If t_0 is the instant of time of an event measured by an observer in a moving frame of reference, the time instant t appears to a stationary observer with respect to the moving one is given as

$$t = \frac{t_0}{\sqrt{1-\beta^2}} \quad . \qquad (20.10)$$

The time interval of the moving clock as viewed by the non-stationary observer is longer than the interval of the clock in the stationary frame of reference. Thus, a moving clock runs slower than an identical clock that is stationary in his of reference. This is called the time dilation.

§20.8. Length contraction

If l_0 is the length of an object in a stationary frame of reference, then the length l of the same object as vowed by the observer in frame of reference moving with a velocity υ relative to the object, is given by

$$L = L_0 \sqrt{1-\beta^2} \quad , \qquad (20.11)$$

since $\beta < 1$, therefore $L < L_0$. There is a shortening of length. This is known as the Lorenz contraction of length.

In accordance with the equation (20.7) we can write a relation for a volume of the body as

$$V = V_0 \sqrt{1-\beta^2} \qquad (20.12)$$

where V_0 and V are the volumes in a stationary and moving frames of reference respectively.

§20.9. Addition of velocities

In result of time dilation and the length construction the velocity in an frame of reference that move relatively to the given system changes in magnitude as well in direction:

$$\upsilon_x = \frac{\upsilon_x' + \upsilon}{1 + \beta\,\upsilon_x'/c};\quad \upsilon_y = \frac{\upsilon_y'\sqrt{1-\beta^2}}{1+\beta\,\upsilon_y'/c};\quad \upsilon_z = \frac{\upsilon_z'\sqrt{1-\beta^2}}{1+\beta\,\upsilon_z'/c} \qquad (20.13)$$

or

$$\upsilon_x' = \frac{\upsilon_x - \upsilon}{1-\beta\,\upsilon_x/c};\quad \upsilon_y' = \frac{\upsilon_y\sqrt{1-\beta^2}}{1-\beta\,\upsilon_y/c};\quad \upsilon_z' = \frac{\upsilon_z\sqrt{1-\beta^2}}{1-\beta\,\upsilon_z/c} \qquad (20.14)$$

§20.10. Relativity of mass and linear momentum

If m_0 is the rest mass of an object in a frame of reference, then, the mass m as measured by an observer moving with a velocity v relative to the object is

$$m = \frac{m_0}{\sqrt{1-\beta^2}}\ . \qquad (20.15)$$

This equation shows that mass m changes with velocity. The mass m here is known as relativistic mass to distinguish it from the rest mass since β is less than 1, and the relativistic mass m is greater than the rest mass m_0.

From relationships (20.12) and (20.15) it follows that the density of a body varies in accordance with the following relation

$$\rho = \frac{\rho_0}{1-\beta^2}\ . \qquad (20.16)$$

From equation (20.11) it also follows that relativistic momentum is expressed as

$$p = \frac{m_0 \upsilon}{\sqrt{1-\beta^2}} \quad . \qquad (20.17)$$

§20.11. Mass - energy equivalence

The most remarkable achievement of Einstein's special theory of relativity is the revelation of the equivalence of mass and energy. The work done on an object not only increases its kinetic energy, but its mass too. Thus , energy converted to mass, and mass and energy are equivalent. The total energy of moving object is mc^2 which is the product of its relativistic mass and the squared speed of light . The rest energy of an object is m_0c^2 or the product of its mass at rest and the squared speed of light . The difference between some object's total energy and its rest one is just the kinetic energy of the object. Because c^2 is very larger quantity it can be recognized that even a small amount of mass corresponds to a large amount of energy. Thus

The total energy $$E_t = mc^2 = \frac{m_0 c^2}{\sqrt{1-\beta^2}} \qquad (20.18)$$

Resting energy $$E_r = m_0 c^2 \qquad (20.19)$$

The kinetic energy $E_k = E_t - E_r = \Delta m c^2$, where $\Delta m = m - m_0$ is the relativistic mass increment.

Finally kinetic energy is given by

$$E_k = \frac{m_0 c^2}{\sqrt{1-\beta^2}} - m_0 c^2 \ . \qquad (20.20)$$

Relationship between the total energy and linear momentum has the form

$$E^2 = p^2 c^2 + m_0^2 c^4 \quad . \qquad (20.21)$$

Now consider some interesting cases:

1. If a body of a mass m_0 is heated by Δt its mass also increases by a quantity

$$\Delta m = \frac{c_{s.h.c.} m_0 \Delta t}{c^2} \quad , \qquad (20.22)$$

where $c_{s.h.c.}$ is the specific heat capacity.

2. A mass m_0 of a body increases as a result of melting by a quantity

$$\Delta m = \frac{\lambda m_0}{c^2} \quad , \qquad (20.23)$$

where λ- is the latent heat of fusion.

CHAPTER 21

The Atomic Spectra

§21.1. Spectrum of hydrogen atom

The spectrum of hydrogen atom was experimentally observed for the first time by J.Balmer.

J.Balmer explained the wavelength of these lines on the background of an empirical formula

$$\text{(visible region)}\ \frac{1}{\lambda_n} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right),\quad n = 3, 4, 5 \quad (21.1).$$

where λ_n is the wavelength corresponding to n , which may have the integral values 3, 4, 5, . . The coefficient R is known as Rydberg constant. If λ_n is in meters, then R= $1.0974 \times 10^7 m^{-1}$. This series of lines were called *Balmer series*.

Balmer empirical formula not only fitted the visible lines but also predicted several other series of lines in the hydrogen spectrum. These lines had not been seen before because they were not in the visible part of the spectrum. The series are now known as the Lyman, Paschen and Brackett series after the names of their discoverers. These are described by the formulae as under

Lyman series

(Ultra violet region) $\frac{1}{\lambda_n} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, n = 2,3,4 , . .(21.2)

.

Paschen series

(Infra red region) $\frac{1}{\lambda_n} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, n = 4, 5, 6, ..(21.3) .

Brackett series

(Infra red region) $\frac{1}{\lambda_n} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, n = 5 , 6, 7,. .(21.4) .

All the above formulae may be written in a general form as

$$\frac{1}{\lambda_n} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right) , \qquad (21.5)$$

where p = 1 , 2 , 3 , ... and n = p+1 , p+2 , p+3 , . .

§21.2. Rutherford and Bohr models of an atom

On the basis of alpha particle scattering experiments, Rutherford concluded that the atom consisted of a tiny but massive positively charged central part, called the nucleus, surrounded by orbiting electrons at some distance away. Now an electron in an orbit is an accelerating charge and according to electromagnetic theory it should give off energy in the form of radiation continuously. An electron moving in a circular orbit as proposed by Rutherford, should continuously lose energy, and thus spiral inward until it is swallowed up by the nucleus. A successful theory was proposed by Niels Bohr. One aspect of this theory was that Bohr s formula for the allowed frequencies of emitted radiation exactly fitted the experimentally observed atomic spectrum of hydrogen atom.

A model of atom suggested by Bohr is known as a planetary model. According to this model electrons orbited the nucleus much like planet orbits the sun. Bohr suggested that an electron has a certain energy in each orbit and moves without the emission of radiation. These allowed orbits are known as stationary states. Bohr suggested also, that electron could radiate the energy, when it passes from one state to another one with lower energy. On the contrary, when electron passes from any orbit to another with greater value of energy it absorbs a certain energy, equal the difference of two levels. Upon each transition only one particle of light- photon is radiated, which energy is determined from the equation.

Generally this model is based upon the following postulates:

1. An electron can not revolve in an circular orbit. Only those orbits are possible for which the angular momentum of the electron about the nucleus is an integral multiple of $\frac{h}{2\pi}$ where h is Planck s constant. These orbits are called allowed orbits. The angular momentum of an electron of mass m, moving with velocity υ_n in an orbit of radius r_n is given by m $\upsilon_n r_n$. Thus according to this postulate, we have

$$m \upsilon_n r_n = n \frac{h}{2\pi} , n= 1, 2, 3, . . . \qquad (21.6)$$

2. The total energy of the electron in one of its allowed orbit remains constant as long as it remains in the same orbit.

3. An atom radiates energy only when an electron jumps from an allowed orbit of higher energy E_n to one of lower energy E_p . The difference of energy appears as photon or quanta of energy, h , where f is the frequency of the photon.

$$h f = E_{initial} - E_{final} \quad . \qquad (21.7)$$

§21.3. The radii of quantized orbits

According to Coulomb's law the force of attraction between negatively charged electron and positively charged nucleus is

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} \quad . \qquad (21.8)$$

In a circular motion around the nucleus an electron is also acted by the centripetal acceleration. Then in agreement with the Newton′ s second law we can write:

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{m\upsilon^2}{r} \quad , \qquad (21.9)$$

Solving for radius of n– th orbit gives

$$r_n = \frac{Ze^2}{4\pi\varepsilon_0 m\upsilon^2} \quad . \qquad (21.10)$$

The speed of electron υ can be calculated from the Bohr′s quantization conditions (21.6).

Having put the value of speed from (21.6) into (21.10) we obtain

$$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2} \quad . \qquad (21.11)$$

For nearest orbit (n=1) of hydrogen atom (Z=1) we get

$$r_1 = 0.529 \times 10^{-14}\ \mathrm{m}$$

This smallest radius is known as Bohr′ s radius.

Note, from relation (21.11) it follows, that radii of another orbits increase by n^2:

$$r_2 = 4r_1 \;, \quad r_3 = 9r_1 \;, \quad r_4 = 16\, r_1 \quad \text{and etc.}$$

§21.4. Energy of electron in an orbit

The orbital energy of electrons in the hydrogen atom can be calculated using the Bohr equation. Total energy of electron is equal to the sum of kinetic and potential energies. Since the electron is negatively charged, its potential energy is also negative .

$$\mathbf{E_n} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m}\upsilon^{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{4}\,\pi\,\varepsilon_{\mathbf{0}}}\frac{\mathbf{Ze}^{\mathbf{2}}}{\mathbf{r}_{\mathbf{n}}} \quad . \qquad (21.12)$$

Substituting $\upsilon_n = n\,h / 2\,\pi\, m\, r_n$ from (21.6) and r_n from (21.11) into (21.12) gives

$$E_n = -\frac{1}{n^2}\left(\frac{Z^2\, e^4 m}{8\,\varepsilon_0^2\, h^2}\right). \qquad (21.13)$$

Having put Z=1 and n=1, for hydrogen atom yields to

$$E_1 = -\,13.6 \text{ eV. ,}$$

where 1 eV=1.6x10 $^{-19}$ J

More general expression for energies of other levels is given by

$$\mathbf{E}_{\mathbf{n}} = -\frac{\mathbf{13.6\ eV}}{\mathbf{n^2}}\,, \qquad (21.14)$$

Hence

$E_2 = -\ 3.4$ eV , $E_3 = -\ 1.51$ eV and etc.

§21. 5. Emission of Spectral lines (Energy level diagram)

Using equations (21.7) and (21.13) gives

$$\frac{1}{\lambda} = \frac{E_1}{hc}\left(\frac{1}{p^2} - \frac{1}{n^2}\right) \quad \text{or} \quad \frac{1}{\lambda} = R_n\left(\frac{1}{p^2} - \frac{1}{n^2}\right) \qquad (21.15)$$

Compare the theoretical equation (21.15) of the Bohr theory with the empirically formulated equation (21.1) that fits in the observed spectrum of hydrogen atom. The form of equation is the same. The value of R_H in Bohr's model was also found in excellent agreement with the value of the Rydberg′s constant determined empirically in the Balmer's formula, i. e. $R_H = 1.0974 \times 10^7\ m^{-1}$.

It is convenient to represent the energy of the quantized states of the atoms on an energy level diagram. The energy level diagram for hydrogen atom is shown in Fig. 21.1. The energy levels of the atom are represented as a series of horizontal lines and the transitions between the levels are represented by vertical arrows.

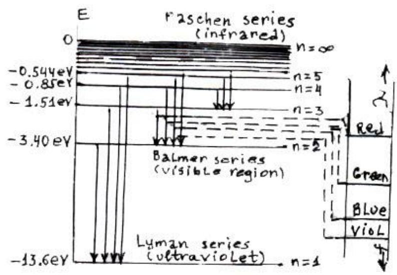


Fig.21.1

The transitions from various energy levels terminating on the lowerest level (n=1) give rise to Lyman series. Balmer series occur for transitions ending at second energy level (n=2). The Paschen, Brackett and Phund series occur for transitions from various energy levels to third (n=3), fourth (n=4) and fifth (n=5) energy levels, respectively.

Only the Balmer series is in the visible part of the spectrum. The Lyman series is in the ultraviolet and the others are in the infrared region. Two of the possible transitions are also shown in the Fig.21.1 that result in spectral lines in the hydrogen spectrum. The transition from state with n=3 to the state with n=2 produce a photon of wavelength 656.3 nm, the resulting spectral line occurs in the Balmer series. Similarly the spectral line wavelength 121. 6 nm is given off due to transition from n=2 state to n=1 state. The wavelength calculated in this way are in excellent agreement with those observed experimentally in the line spectrum of hydrogen. Bohr theory successfully explains Balmer formula and predicts

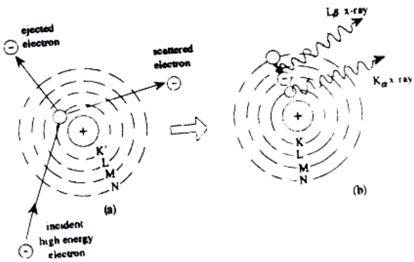


Fig.21.2

the wavelength of all observed spectral lines in hydrogen spectrum. However, Bohr theory is unable to predict or explain energy levels in complex atom, which has many electrons.

.

§21.6. Inner – shell transitions and X – rays

In the heavy atoms, the electrons are assumed to be arranged in concentric shells at increasing distance from the nucleus. These shells are labeled as K, L, M, N etc. the K shell being closest to the nucleus, the L shell next, and so on. The outer shell electrons are loosely bound and relatively small amount of energy is sufficient to raise them to excited state or even for ionization. On their return to normal states, spectral lines of wavelength in or near the visible region are emitted. However, the situation is quite different for transitions involving inner shell electrons. They are closer to the nucleus and hence more tightly bound. Large amount of energy is required for their displacement from their normal energy levels. Consequently, photons of larger energy are emitted when the atoms return to their normal states. Thus the transitions of inner shell electrons in heavy atoms give rise to the emission of high energy photons or X – **rays.** Let K – shell electron is removed from an atom creating a vacancy in the K – shell (Fig.21.2a) then an electron from either the L, M, or N– shell will quickly jump down to fill the vacancy in K – shell emitting the excess energy as X – ray photon (Fig.21.2b).

An X – ray photon due to transition from L – shell to the vacancy in the K shell is called K_α characteristic X –ray . The transitions from M and N– shells to the K shell give rise to K_β and K_γ X –rays , respectively. Similarly L, M, and N series X –rays, relatively of lower energy, are produced due to the ejection of electrons from L, M and N– shells, respectively.

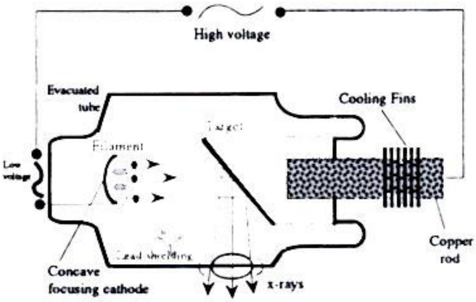


Fig.21.3

A schematic diagram of a modern X –ray tube is shown in Fig.21.3. The tube is evacuated so that the focused beam of electrons from the filament is pulled on to the anode at very high speed because of very high potential difference applied between the filament and the target. The high potential difference is provided by a transformer from a.c. supply. The accelerated electrons are suddenly decelerated on impact with the target and some of the kinetic energy is converted into electromagnetic energy, as X –rays .

In case when electrons lose all their K.E. in the first collision, the entire K.E. appears as a photon of maximum energy. The other electrons which suffer a no of collision before coming to rest will give rise to photons of smaller energy.

The X –rays spread out from the target and pass easily through the glass tube walls, X –rays are only produced on the half cycle of a.c. when target is positive with respect to the filament. Most of the kinetic energy of the electron goes into producing heat. The heat is conducted away from the target by a copper rod in which the target material is embedded. The rod is cooled by circulating oil / water through it or by the use of cooling fins do dissipate the thermal energy.

The intensity of X –rays increases with the number of electrons hitting the target and, therefore , depends on the filament current. The energy of the X –rays increases with the operating voltage of the tube.

§21.7. The Laser Principle and its operation

The device can produce a very narrow, intense beam of coherent light by stimulated emission. This is a process in which de - excitation of an atom is caused by an incident photon with the emission of a second photon of the same energy which is coherent with the original photon as shown in Fig.21.4. Suppose an atom jumps from its ground state E_1 to an excited state E_2 and a photon of energy exactly equal to the difference of the two states ($hf=E_2-E_1$) is incident on it (Fig.21.5a) When the excited atom is the presence of an electromagnetic field that oscillates exactly with the frequency ν, the field will stimulate the atom to di-excite by emitting a photon in phase with external field. The incident photon thus increases the probability that the electron will return to the ground state (E_1) by emitting a second photon having the same energy and exactly in phase with incident

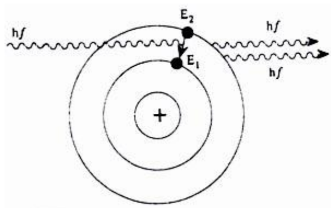


Fig.21.4

photon (Fig.21.5b) These photons can , in turn, stimulate other atoms to emit photons in a chain of similar events as shown in Fig. 21.6.

The operation of laser depends upon the existence of metastable state in the atoms of some substances . The metastable states are excited atomic states that can persist for unusually long periods of time. For example, the life time of most excited states is only about 10^{-8} second, but there exist in some substances such excited states in which the life is about 10^{-3} second or even more.

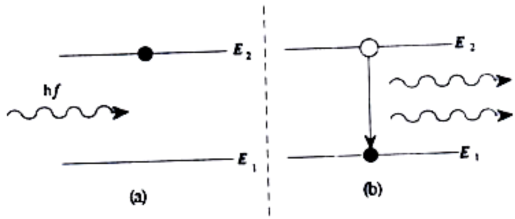


Fig.21.5

When light is made incident on such a substance usually called the active medium, there is a net absorption of photons due to more atoms in the ground state than in the excited state (Fig.21.6). It is only if the situation can be inverted, that is , there should be more number of atoms in the excited state than in the ground state, a net emission of photons can result. Such a situation is called “population inversion“.

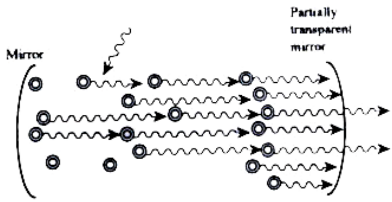


Fig.21.6

Let the incident photons of energy $h\nu_{13}$ equal to $E_3 - E_1$ pump up the atoms from ground state E_1 to excited state E_3 (Fig. 21.7) but the excited atoms do not decay back directly to state E_1. Instead , they decay spontaneously emitting photon of energy hf_{23} to the lower energy state E_2 which is a metastable state. This state has the special property of having a large stimulated emission probability because of relatively long life time compared with usually short life times of other excited states. As a result of its ability to retain electrons, state E_2 soon contains more atoms than ground state E_1.

Now suppose that light, whose photons have energy $h\nu_{12}$ equal $E_2 - E_1$ is incident on the substance. These photons can be absorbed and excite the few electrons in level E_1 and to E_2.

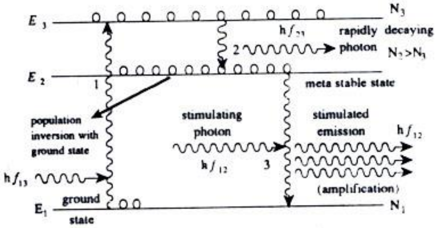


Fig.21.7

Also they can cause electrons to fall from E_2 to E_1 giving rise to stimulated emission of light waves that are identical to the incident light wave. Because of population inversion, the rate of energy radiation by stimulated emission exceeds the rate of absorption and the result is an amplification of the ordinary light.

§21.8. Atomic mass

From the Avogadro' s law we have that the mass of a nucleus is of the order of 10^{-24}kg. Since, it is very small number, we will express it in terms of unified mass scale. Thus one atomic mass unit (amu) is equal to 1 / 12 of the mass of the carbon atom.

1 amu = 1. 660 x 10^{-27} kg.

CHAPTER 22

The Atomic Nucleus

§22.1. A structure of nucleus

The central region of the atom is called nucleus with a nuclear diameter of the order of 10^{-14} m surrounded by a cloud of electrons giving an atomic radius of order of 10^{-10} m.

The nucleus contain the protons and neutrons generally called the **nucleons.**

The mass of proton on unified atomic mass scale is 1.007296 and that of neutron is 1.008665. The electron mass has only 0.000548 amu . The properties of particles in the atom are summarized in table.

Total number of nucleons is known as the **mass number**. This quantity can be used in order to calculation of radius of nucleus by empirical formula

$$r = 1.4 \times 10^{-15} A^{1/3} \text{ m.}$$

where A is the mass number.

The number of protons Z is the atomic number. The number of neutrons N is equal to $A - Z$.

The charge of the nucleus is equal to the number of the element in the Mendeleyev s periodic system . The number of electrons in a neutral atom is equal to the charge of the nucleus.

Isotopes are the nuclei with the same Z , but various number of neutrons. The isotope of chemical element with the Z and A is accepted to denote as ${}_{Z}^{A}\mathbf{X}$.

Nuclei of different elements with the same mass number A are called **isobars.** The isobars have different Z and different N, but the mass number A = Z + N is the same.

§22.2. Mass defect. Nuclear binding energy

The mass of nucleus always less than the masses of the constituent nucleons, together in the free state. The difference between the sum masses in rest of nucleons and the mass of nucleus is known as the **mass defect:**

$$\Delta \mathbf{M} = \mathbf{Z m_p} + (\mathbf{A} - \mathbf{Z})\, \mathbf{m_n} - \mathbf{M} \qquad (22.1)$$

where m_p , m_n and M are the masses of proton, neutron and nucleus respectively.

Knowing the mass defect the **binding energy** can be determined.

It is equal to the work need in order to escape the nucleon outer the nucleus. The binding energy of nucleus is calculated as

$$\Delta \mathbf{E} = \Delta \mathbf{M}\, \mathbf{c}^2 = \left[\mathbf{Z\, m_p} + (\mathbf{A} - Z)\mathbf{m_n} - \mathbf{M}\right] \mathbf{c}^2 \qquad (22.2)$$

According (22.2) for the binding energy per nucleon we get

$$\mathbf{E} = \frac{\Delta \mathbf{E}}{\mathbf{A}} \qquad (22.3)$$

where A – is the **mass number**.

A dependence of the binding energy per nucleon upon mass number for stable nuclei is shown in Fig. 22.1 The curve rises initially with increasing the A and reaches the saturated value for A ≅ 15. When A > 60 the curve slowly decreases, because of more heavy nuclei are less stable as compared to that elements occupied the middle of the periodic system. To support the stability of nuclei a great number of neutrons is required, because they have only the nuclear attraction. Under most greater Z the excess neutrons can not compensate the increasing Coulomb repulsion and therefore there are no stable nuclei with Z >82.

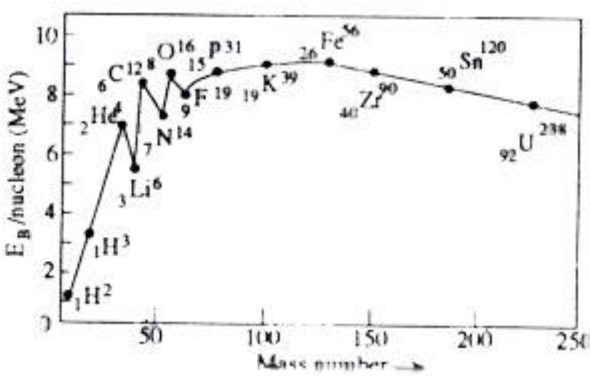


Fig.22.1

A value of energy, corresponding to the mass defect, equal 1a.m.u is 931.5 MeV. Thus,

$$\frac{\Delta \mathbf{E}}{\Delta \mathbf{m}} = \mathbf{c}^2 = \mathbf{931.5}\frac{\mathbf{MeV}}{\mathbf{a.m.u.}} \qquad (22.4)$$

§22.3. Radioactive processes

The elements with Z > 82 are unstable and that the natural radioactivity is caused by the disintegration (decay) or break up of the unstable nuclei. These nuclei may emit an α (alpha) – radiation, β (beta) – radiation and accompanied by a γ (gamma) – ray. The elements which emit these three types of radiation are called **radioactive elements**.

1. **α–decay**. α–particles are emitted from radioactive nuclei. Atomic number Z of the parent nucleus drops by 2 unit and mass number A drops by 4 unit. (Fig 22.2) The nucleus equation has the form

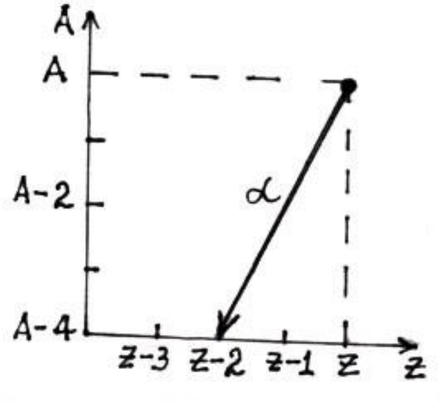


Fig.22.2

$$ {}^{A}_{Z}\mathbf{X} \rightarrow {}^{A-4}_{Z-2}\mathbf{Y} + {}^{4}_{2}\mathbf{He} \tag{22.5}$$

Parent nucleus   Daughter nucleus

Examples:

$${}^{226}_{88}\mathbf{Ra} \rightarrow {}^{222}_{86}\mathbf{Rn} + {}^{4}_{2}\mathbf{He}$$

$${}^{238}_{92}\mathbf{U} \rightarrow {}^{234}_{90}\mathbf{Th} + {}^{4}_{2}\mathbf{He}$$

2. β – decay. The process of β (beta) emission involves no change in mass number A. It does however change the atomic number Z by 1 or +1 depending on whether the particle emitted is

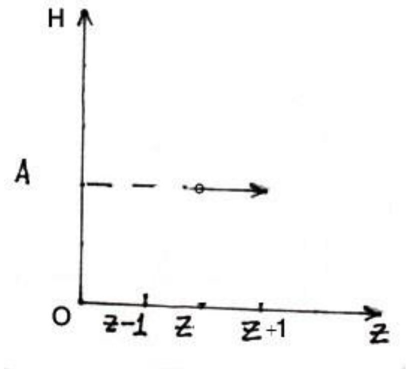


Fig.22.3

negative (β $^{-}$, electron) particle or a positive (**β^{+},** positron).
2.1. β $^{-}$ – decay . The radiation of **β** – particles and antineutrino is occur. Atomic number Z of parent nucleus increases by 1 unit , but mass number A remains the same. (Fig.22.3) An equation of this process is written as

$$ {}_{Z}^{A}\mathbf{X} \rightarrow {}_{\mathbf{Z+1}}^{\mathbf{A}}\mathbf{Y} + \mathbf{e}^{-} + \tilde{\nu}_{\mathbf{e}} \text{ (Antineutrino)} \qquad (22.6) $$

Examples:

$$ {}_{83}^{210}\mathbf{Bi} \rightarrow {}_{84}^{210}\mathbf{Po} + {}_{-1}^{0}\mathbf{e} $$

$$ {}_{90}^{234}\mathbf{Th} \rightarrow {}_{91}^{234} + {}_{-1}^{0}\mathbf{e} $$

2. 2. **β $^{+}$ – decay. β $^{+}$ –** radiation is emitted by nucleus with relatively exceeding number of protons. Positron produced as a result of conversion of proton into neutrino.

Atomic number Z of parent nucleus decreases by 1 unit, but mass number A remains unchangeable (Fig. 22.4). This reaction is written in the form

$$ {}_{Z}^{A}\mathbf{X} \rightarrow {}_{\mathbf{Z-1}}^{A}\mathbf{Y} + \mathbf{e}^{+} + \nu_{\mathbf{e}} \quad \text{(neutrino)} \qquad (22.7) $$

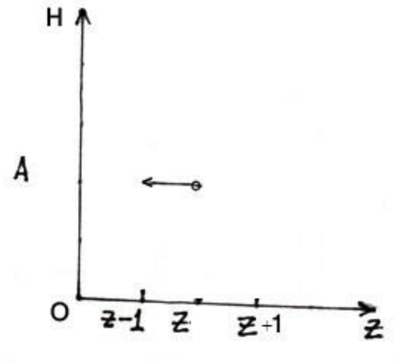


Fig.22.4

Examples of **β^+ – decay.**

$^{30}_{15}\mathbf{P} \rightarrow ^{30}_{14}\mathbf{Si} + ^{0}_{+1}\mathbf{e}$

$^{22}_{11}\mathbf{Na} \rightarrow ^{22}_{10}\mathbf{Ne} + ^{0}_{+1}\mathbf{e}$

3. **γ – radiation**. In this process the short – length electromagnetic wave is radiated. An excited nucleus $X^{\bullet}$ goes back to its ground (unexcited) state, by emitting one or more of gamma ray. Since, **γ –** rays are mass less photon, their emission will cause no change either of A or of Z of the parent **nuclei.**

The equation for the transmutation of a nucleus by the emission of a gamma ray is

$$^{A}_{Z}\mathbf{X}^{*} \rightarrow ^{A}_{Z}\mathbf{X} + \gamma \qquad (22.8)$$

Parent nucleus in excited energy state — Daughter nuclei in lower energy state

Thus, γ – decay does not change a transmutation of one element into another.

4. K - occupation. A nucleus occupies one electron of K - shell. In result one of protons of nucleus transmitted into neutrino. Atomic number of new nucleus decreases by one unit and atomic number remains the same. The equation for this transmutation has the following form:

$$ {}_{Z}^{A}\mathbf{X} + \mathbf{e}^{-} \rightarrow {}_{Z-1}^{A}\mathbf{Y} + \nu_{e} \qquad (22.9) $$

5. Spontaneous fission of nuclei. In this process two or three neutrons are radiated. Nucleus of uranium and plutonium split into two parts with various masses. The masses produced become the same with increasing of mass number **A.** An equation of spontaneous fission is

$$ {}_{Z}^{A}\mathbf{X} \rightarrow {}_{Z_1}^{A_1}\mathbf{Y} + {}_{Z_2}^{A_2}\mathbf{Y} + 2\mathbf{n} \qquad (22.10) $$

6. Proton radioactivity. Two protons are released in this process. Both atomic number Z and mass number A of the parent daughter drops by one unit in a new nucleus. The corresponding equation is written as

$$ {}_{Z}^{A}\mathrm{X} \rightarrow {}_{Z-1}^{A-1}\mathrm{Y} + {}_{1}^{1}\mathrm{H} \qquad (22.11) $$

§22. 4. Law of radioactive decay. Half– life. Activity

It has been observed that, the actual number of atoms which decay at any instant is proportional to the number of atoms present.

A variation of the number of radioactive nuclei is calculated in accordance with the following law: Suppose, number of decays dN over time period Δt is proportional the number of present parent nuclei N:

$$ dN = -\lambda N dt $$

where coefficient of proportionality λ – is called the decay constant. N – is the number of radioactive nuclei present at time t.

The given expression may be re written as

$$\frac{dN}{N} = -\lambda dt$$

whose solution gives:

$$N = N_0 \; \mathbf{e}^{-\lambda t} \qquad (22.12)$$

where N_0 – is the original number of radioactive nuclei at t =0. An equation (22.12) is known as the **law of radioactive decay.**

Half – life. The number of radioactive atoms in a material that decay to one half of the original number in a time is characteristic of a material. Thus, the time taken for the atoms or a radioactive material to decay to half of original number is called the **half– life** of the material. The half life of a radioactive material is a nuclear property and is not influenced in any way by the physical conditions of the material (temperature, pressure or chemical reactions).

The variation in the number of atoms decaying a particular time can be graphically represented as shown in Fig. 22.5. The half life is represented by $T_{1/2}$ and is determined from the graph as the time taken for the radioactivity to fall to half of the original value N_0 . Substituting $t = T_{1/2}$ and $N= \frac{1}{2} N_0$ from eq. (22.12) we get

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad (22.13)$$

Radioactive elements Technetium (Z=43, $T_{1/2}$ = 2.6 million years) and Promethium (Z=6.1, $T_{1/2}$ = 265 days) have rather short lifetimes compared to the age of our planet (4.5 billion years).

Activity. The activity of radioactive sample is the number of disintegrations per second that occur.

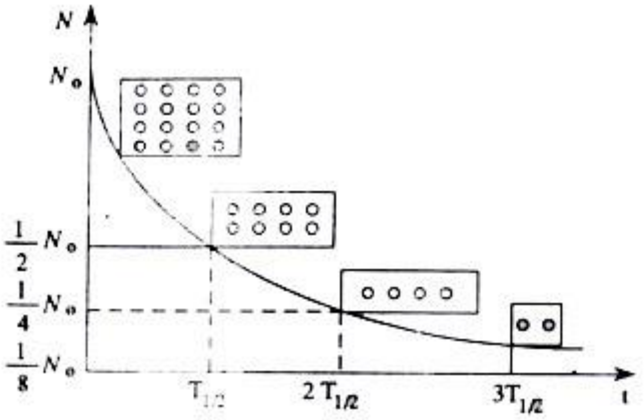


Fig.22.5

From law of radioactive decay we get

$$A = \lambda N = \frac{\mathbf{0.693N}}{\mathbf{T}_{\mathbf{1/2}}} \qquad (22.14)$$

Since , an activity is proportional with the number N we can write also

$$A = A_O\, \mathbf{e}^{-\lambda \mathbf{t}} = \frac{\mathbf{A}_{\mathbf{0}}}{\mathbf{2}^{\mathbf{t/T_{1/2}}}} \qquad (22.15)$$

The SI unit for activity is the becquerel (Bq) : one becquerel equals one disintegration per second. Activity is also measured in terms of unit called curie (Ci) :

1 Ci = 3.70 x 10^{10} Bq .

§22. 5. Nuclear fission and fusion

The fission process was discovered in the irradiation of uranium with neutrons by the chemists Hahn and Strassman in 1938. Fission is the splitting of a nucleus into two or more fragments of almost equal size. The nuclei of U ^{235} were split by incident neutrons into two massive parts of nearly equal masses, almost as if it were a drop of liquid breaking up:

$$\mathbf{{}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n}$$

compound
unstable nucleus

It is occur because the energy liberated in such nuclear reactions was many order greater than in ordinary nuclear reactions. The binding energy of nuclei decreases with the increase of mass number A. On the other hand, the long range repulsive forces in the heavy elements become almost comparable to the nuclear forces. The addition of one more particle in the nucleus easily disturbs the equilibrium and the nucleus becomes unstable. Nuclear fission can be best explained if we imagine an atomic nucleus to be an electrically charged droplet (Fig. 22.6.) An addition of another particle would transform its initial spherical shape into a more or less dumbell shape causes two kinds of forces to act on it:

1. The forces of nuclear surface tension attempting to restore the nucleus to its original spherical shape.
2. Coulomb repulsive forces between the electric charges on the opposite ends of the dumbell attempting to break up the nucleus into two halves.

Nuclear fusion. A process inverse to the fission is called ***nuclear fusion***. The heavier nuclei are formed from two or more lighter nuclei. The energy released in the fusion of lighter nuclei into heavier nuclei is called thermonuclear fusion.

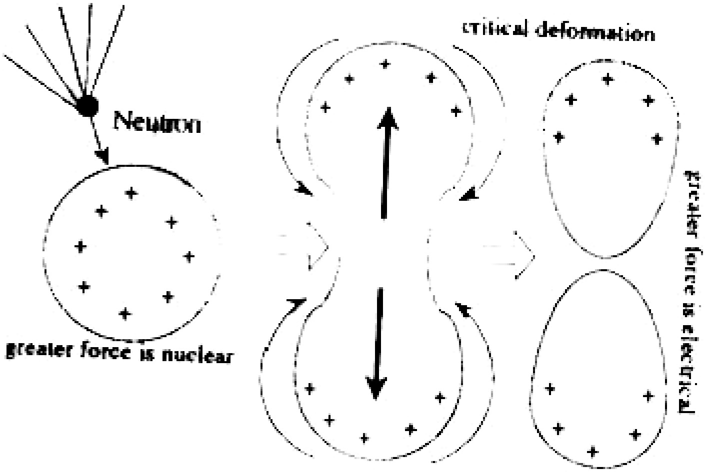


Fig.22.6

Note, that in all processes of nuclear changes or nuclear disintegrations the following physical quantities should remain constant: electric charge, energy, mass, linear and angular momentums, nucleon number.

§22.6. Nuclear reactor

The principle of nuclear reactor is the controlled fission chain reaction. These nuclear reactors are being widely used in many countries of the world for the commercial generation of electricity. Today there are many kinds and sizes of reactors, but they all have three basic components ; fuel element, a neutron moderator, and control rods .

In order that a chain reaction is self sustained it is important to slow down the fast neutrons which appears as fission fragments. This is achieved by placing uranium, the fuel element in the shape of thin rods of about 1 cm in diameter in a medium known as moderator. The medium is made up of nuclei of comparable mass to that of mass of the neutron and of material which does not absorb the precious neutrons and thus takes them out of the fission producing cycle. Graphite is usually used as moderator in nuclear reactors. The fast neutron produced in a fission process thus undergo several successive collisions and are slowed down. In case too many neutrons are produced, the chain reaction becomes very fast and thus must be prevented getting out of hand. This is done by inserting in the reactor control rods of boron or cadmium, which are effective neutron absorbers. These rods can be moved in and out of the reactor from the control room. If the fission chain starts to go too fast, the control rods can be slightly withdrawn. In the case of electrical failure the control rods fall and shut off the reactor automatically. In a large power reactor there may be thousand of fuel element placed closed together, the entire reaction of fuel elements being known as the reactor core.

A charge machine is used to start the reactor. The uranium rods, each of about 1 m long and 2 cm in diameter are lowered into about 1700 fuel channels in the graphite core. The boron steel rods are then raised slowly and placed at certain position as soon as the chin reaction proceed at the required rate. The reactor is now said to have " gone critical " and heat is produced steadily.

The hot gas is led into heat exchanger outside the reactor. It heats up water and gets converted into high pressure and low pressure steam. The steam is used to drive the turbines which tum electrical generators

§22.7. Particle observation and detecting methods

1. Method of scintillations

If the glass plate covered with the thin film of zink sulphide ZnS is placed in the dark and the substance radiating alpha particles or protons bring up nearly to it then by means of microscope separate fluorescences are observed. Each of these fluorescences is due to single particle. Sometime are seen the sections of trajectories, over which these particles move. By recording the number of fluorescences and trajectories one can count the number of particles falling on the layer of illuminating substance.

2. Fluorescence counters

This method presents the technique of scintillations. However the process of particle recording is executed by no eye, but with the help of photocell. Photons emitted upon the illuminations fall on the photocathode of photocell and create the electric current in a circuit of it. By recording the number of current pulses one can detect not only the observed fluorescences produced by alpha particles, but weak fluorescences caused by electrons falling on the illuminating substance as well.

3. Wilson cloud chamber

The particle detecting by cloud chamber is based on the principle that supersaturated vapors condense more readily on ions or dust particles. It is in a way similar to formation of clouds in the atmosphere. Warm air containing evaporated water molecules expands and cools as it rises in the atmosphere. Below a certain temperature the water molecules form tiny droplets. These drops then condense on dust particles around and form a cloud which is visible. Wilson showed that if the air inside a chamber was dust free but contained ions, then the excess water vapors in the air would condense around these ions reflect light and can be photographed. In this way a visible track of the path of ionizing particle can be obtained.

The cloud chamber was suggested to be placed in the magnetic field. It permits to determine the charge, speed and mass of flying particles.

4. Geiger - Muller counter

An ionizing particle if enters a Geiger - Muller tube, creates ions and free electrons in the gas through successive collisions with the atoms of the gas.

The electrons are pulled so strongly towards the central positive wire (Fig. 22.8.) that they further strike gas atoms, knocking off other electrons. Thus an avalanche of electrons is formed, which when reaches the positive wire, creates a detectible current pulse through the counting unit such as scaler. For each particle passing through the tube a pulse is recorded. In this way Geiger counter is able to count the particles that come through it.

The gas must be made nonconducting immediately in order to register the entry of the next particle. This is called quenching of discharge. The conduction is terminated by an external unit which momentarily drops the voltage on the anode below the critical value for ionization. The tube thus becomes ready to register the next incoming particle. Likewise a momentary current or pulse is recorded corresponding to each single particle entering the tube. However, the size of the current pulse is independent of both the energy and type of incident radiations.

5. Bubble chamber

This method is based to the observation of gas bubbles formed in result of vaporization of superheated liquid including the ions. In a liquid these ions can be produced due to particles with energy, less than the energy of ions of super-cooled vapor. Therefore, the chamber with superheated vapor permits to observe the trajectories of accelerated charged particles having lower energies as compared to that the particles detected by means of cloud chamber.

§22.8. Measurement of radiation intensity

1. Radiation absorbed dose. Absorbed dose is known as a ratio of absorbed energy to the mass of irradiated substance:

$$D = \frac{W}{m} = \frac{W}{\rho V}, \qquad (22.14)$$

where W the energy absorbed, V the volume of substance and ρ is its density.

The SI unit of absorbed dose is Grey (Gr) = $\frac{Joule}{kg}$

2. Expose dose. Exposition dose is equal to the ratio of charge produced as a result of ionization due to irradiation to the mass of ionized air.

$$\mathbf{D_E} = \frac{Q}{m} \qquad (22.15)$$

The SI unit of expose dose is roentgen.

1 R = 2.58x10 $^{-4}$ C /kg

3. Dose Rate. Dose power is defined as a dose per unit time.

$$\mathbf{P_D} = \frac{D}{t} \qquad (22.16)$$

4. **Equivalent dose**. The radiation equivalent is the biological dose and equals to the absorbed dose multiplied the translational coefficient.

The equivalent dose is measured in sieverts (sv) or 1 ber = 10 $^{-2}$ sv.

Physical constants

Atomic mass unit (a.m.u.)		1.676×10^{-27} kg = 931/147 Mev.
Acceleration due to gravity at sea level, lat.45° g		9.806 m/s^2
Avogadro′s number	N_A	6.02×10^{23} particles/mol.
Boltzman′s constant	$k=R/N_A$	1.3807×10^{-23} J/K
Charge of electron	e	-1.602×10^{-19} C
Constant in Coulomb′ s law $k = 1/4\pi\varepsilon_0$		8.988×10^{9} N m/C^2
Gravitational constant	G	6.670×10^{-11} N m^2 /kg^2
Gas constant	R	8.314 J/mol. K
Mass of a Sun	M_S	1.99×10^{30} kg
Mass of an Earth	M_E	5.98×10^{24} kg
Mass of Moon	M_M	7.35×10^{22} kg
Mass of an electron	m_e	9.109×10^{-31} kg
Mass of a proton	m_p	1.672×10^{-27} kg
Magnetic constant	$k = \mu_0/4\pi$	10^{-7} N/A^2
Mechanical equivalent of heat	J	4.185×10^{3}J/kcal.
Molecular weight of air	μ_A	28.97 kg/kmol
Planck′ s constant	h	6.626×10^{-34} J/Hz = 4.136×10^{-15} eVs
Permitivity of free space	ε_0	8.85×10^{-12} C^2/Nm^2
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Radius of a Sun	R_S	6.96×10^{8}m
Radius of an Earth	R_E	6.37×10^{6} m
Radius of a Moon	R_M	1.738×10^{6} m
Speed of sound in air	v	332 m /s
Speed of light in a vacuum	c	2.997×10^{8} m/s
Faradey′ s number	F	9.65×10^{7} C /kg. ekv
Volume of ideal gas at STP	V	22.415 l/mol

R E F E R E N C E S

1. **Hugh D. Young., Roger A. Freedman**. University Physics. Pearson International Edition. 12-th edition.P.1551
2. **Douglas C. Giancoli.** Physics for scientists and engineers. Pearson International Edition. P.1016
3. **Jerry D. Wilson.** College Physics **.**p.875
4. **Howe Harley** . Introduction to Physics. First Education 1942., p.560
5. **Stewant B.** Lessons in Elementary Practical Physics. London, 1987, p.500
6. **Irodov I.E.** Fundamental laws of Mechanics. Mir,1980,p.271
7. **Yavorsky B.M., Seleznev Y.A..** Handbook of physics. “Science” ,Moscow 1989, p. 567
8. **N. Koshkin .,M Shirkevich**. Handbook of elementary physics. Mir Publisher, Moscow 1968. p. 214
9. **Wolkenstein V.S**. Problems in general physics. Moscow, Mir,1978,p.349
10. **M.E.Bahchisaraitseva, V.A. Kashirina** . English for power engineering students. “High School “ Publishers.Moscow. 1972.p.140
11. **Smirnova L.A.** Russian-English phrase-book for physicists. “ Soviet Encyclopedia“ Publishing House.1968. p.336
12. **Tolstoy D.M.** English-Russian Physics Dictionary . “ Soviet Encyclopedia “ Publishing House. Moscow- 1968. p.848
13. **Tarasov V.L**. Laser age in optics. Mir, 1981,p.206
14. **Pointing Sc.D.** Text-book of Physics, London,1900,p.460
15. **Amirov Sh.Sh**. Brief Dictionary of terms of general and medico-biological physics. “ Teacher” Publisher. Baku-2005. p.161

Şahmərdan Şahbaz Əmirov

Elementar Fizika

(ingilis dilində)

Shahmerdan Shahbaz Amirov

Elementary Physics

ABOUT THE AUTHOR

Shahmerdan Shahbaz Amirov received his secondary education in Quba region and he is a graduate of the School of Physics at Baku State University(former Azerbaijan State University). Further he graduated with "honored" certificate from the English language department of two years Foreign Languages courses at Azerbaijan University of Languages.Then Sh.Sh.Amirov successfully completed his postgraduate studies at the Institute of Physics of the Azerbaijan National Academy of Sciences . He conducted scientific research in the field of nonlinear optics, and awarded the academic degree of candidate of physical and mathematical sciences by USSR Supreme Attestation Commission . He is an author of numerous articles published in world prestigious journals as the "Quantum Electronics", "Optics and Spectroscopy" and has been awarded a grant of the International Scientific Foundation (New York, USA) in 1993. He was a participant of seminar-lectures in the field of nonlinear optics at the Technical University of Aachen, Germany, in 1999. In 2003 he was elected an individual member of the European Physical Society . Shahmerdan Amirov took part in the International Grant Project funded by Canadian Government(No.3998 240.000$ 3 years) He was experienced in teaching the subject of Physics in English at the University of the “Qafqaz” for two years. At present the subjects of "Medical and Biological Physics", and "Higher Mathematics" at Azerbaijan Medical University as well as the subjects of "Fundamentals of Circuit Theory", "Electronics " , “Introduction to Electronics of Defense Systems “ “Radioengeeniring Equipment“,“Fundamentals of radioengineering “ , and “Physics “ are taught in English by assistant professor Sh.Sh.Amirov

He is an author of 4 textbooks on physics and a dictionary published in English for medical students