Composite materials:
Engineering and science

F. L. Matthews and R. D. Rawlings

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and

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Imperial College of Science, Technology and Medicine, London, UK

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Preface

In order to meet the demand for training in understanding the behaviour of composite materials, the Centre for Composite Materials at Imperial College has, for a number of years, organized a course of lectures entitled 'Introduction to Fibre-Reinforced Composites'. Most of the notes associated with that course have been edited, adapted and expanded here by the present authors, who wish to acknowledge the following colleagues who also contributed to the course notes: Dr P. Cawley, Dr J. M. Hodgkinson, Professor A. J. Kinlock, Professor P. L. Pratt, Professor J. G. Williams (all of Imperial College), Dr P. T. Curtis, Dr D. Dorey (of DERA, Farnborough). Without their original material this text would not have been written.

The purpose of the present book is to give an up-to-date appreciation of the underlying science and the engineering performance of composite materials. The most widely used composites are those with polymer matrices and with fibrous reinforcement and the text inevitably concentrates on these, although two chapters are devoted to metal and ceramic matrix systems. A significant portion of the book deals with methods for calculating stiffness and strength, the text being supplemented where appropriate by worked examples and representative data. The final chapters deal with aspects of mechanical behaviour such as toughness, fatigue and impact resistance and the properties of joints. The important topic of non-destructive evaluation is also dealt with in detail.

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Overview

1

1.1 INTRODUCTION

In the continuing quest for improved performance, which may be specified by various criteria including less weight, more strength and lower cost, currently-used materials frequently reach the limit of their usefulness. Thus materials scientists, engineers and scientists are always striving to produce either improved traditional materials or completely new materials. *Composites* are an example of the latter category.

Not that composites are really new. A composite is a material having two or more distinct constituents or phases and thus we can classify bricks made from mud reinforced with straw, which were used in ancient civilizations, as a composite. A versatile and familiar building material which is also a composite is concrete; concrete is a mixture of stones, known as aggregate, held together by cement. In addition to synthetic composites there are naturally occurring composites of which the best known examples are bone, mollusc shells and wood; bone and wood are discussed later in the chapter.

Within the last forty years there has been a rapid increase in the production of synthetic composites, those incorporating fine fibres in various plastics (polymers) dominating the market. Predictions suggest that the demand for composites will continue to increase steadily with metal and ceramic based composites making a more significant contribution (Figure 1.1).

The spur to this rapid expansion over the last few decades was the development in the UK of carbon fibres and in the USA of boron fibres in the early 1960s. These new fibres, which have high elastic constants, gave a significant increase in the stiffness of composites over the well-established glass fibre containing materials, and hence made possible a wide range of applications for composites. One of the key factors was the very high strength-to-weight and stiffness-to-weight ratios possessed by these new composites (section 1.6). Early employment in aviation has led, in more recent times, to wide use in other areas such as the leisure and sports

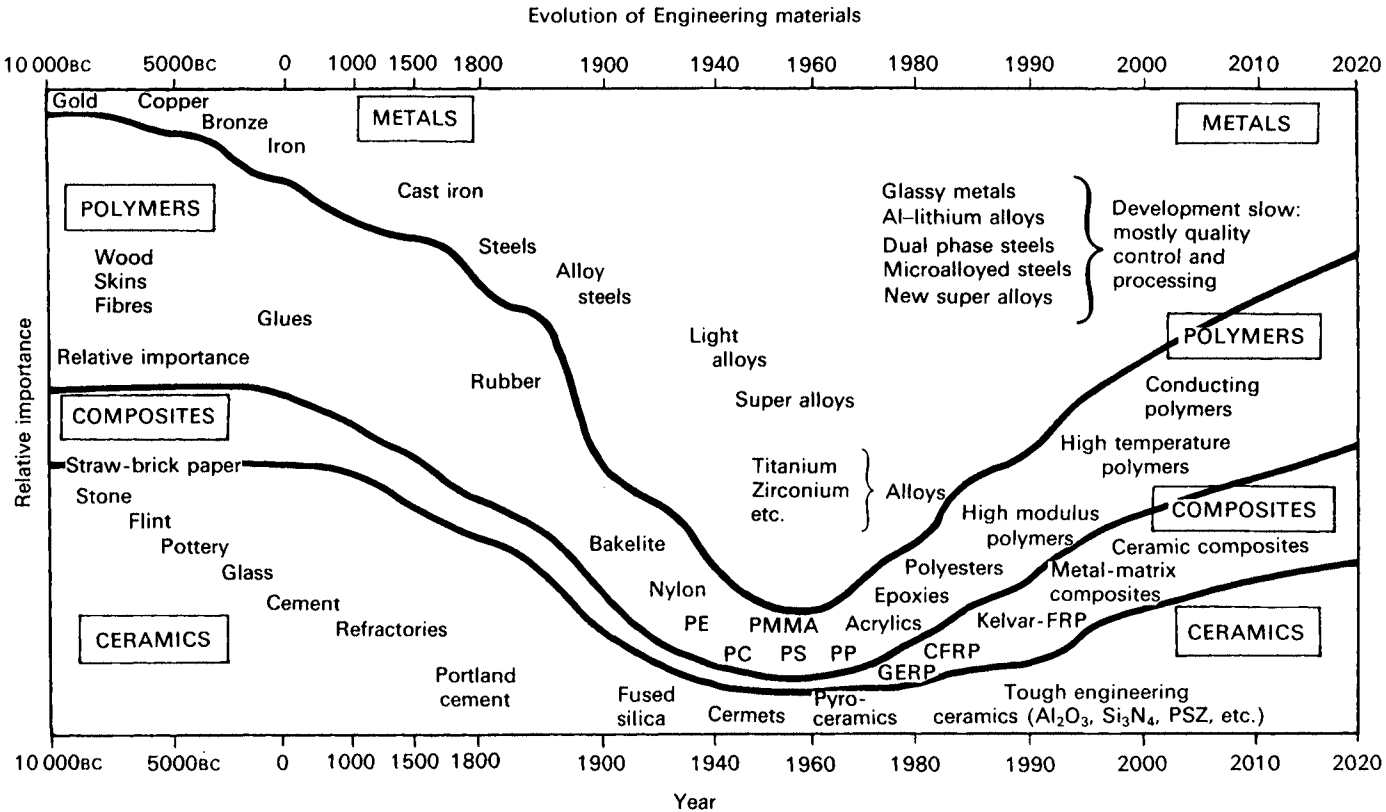


Figure 1.1 Schematic diagram showing the relative importance of the four classes of materials (ceramics, composites, polymers and metals) in mechanical and civil engineering as a function of time. The time scale is nonlinear. (Source: Ashby, 1987.)

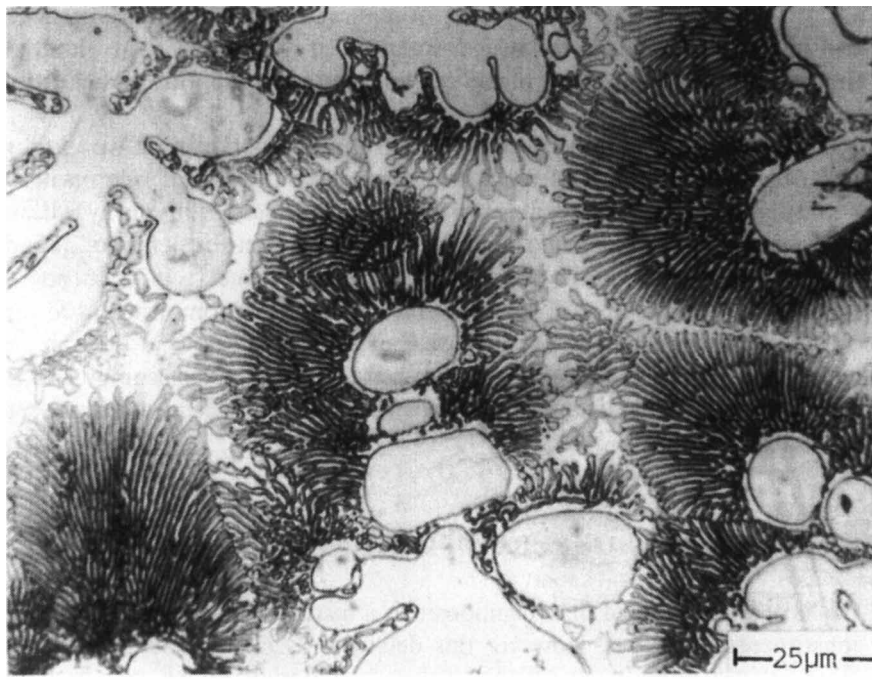
industry. Similarly, the development of high strength silicon carbide, SiC, and alumina, Al_2O_3, fibres which maintain their properties at elevated temperatures initiated much of the current interest in composites based on metal and ceramics.

Composite materials can be studied from a number of different viewpoints each of which requires a different kind of expertise. Thus, the development of a composite material to resist a corrosive environment is primarily within the field of materials science and chemistry. In contrast the design of a load bearing structure requires the expertise of an engineer. It is therefore essential for the full exploitation of currently available composites, and for the future development of composites, that experts in one field are able to understand the problems of those in another. The study of composites is truly interdisciplinary and this will become increasingly clear as the reader proceeds through this book.

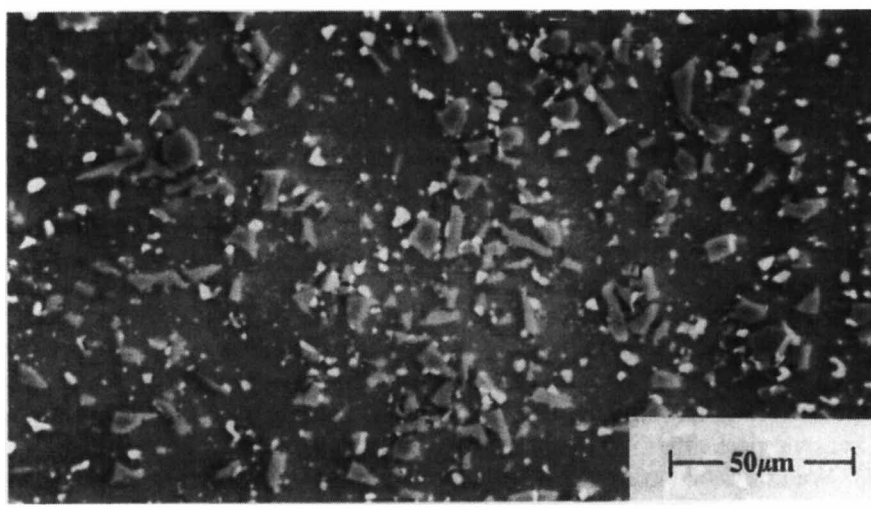
1.2 DEFINITIONS AND CLASSIFICATION

We have already stated that a composite is a mixture of two or more distinct constituents or phases. However this definition is not sufficient and three other criteria have to be satisfied before a material can be said to be a composite. First, both constituents have to be present in reasonable proportions, say greater than 5%. Secondly, it is only when the constituent phases have different properties, and hence the composite properties are noticeably different from the properties of the constituents, that we have come to recognize these materials as composites. For example plastics, although they generally contain small quantities of lubricants, ultra-violet absorbers, and other constituents for commercial reasons such as economy and ease of processing, do not satisfy either of these criteria and consequently are not classified as composites. Lastly, a man-made composite is usually produced by intimately mixing and combining the constituents by various means. Thus an alloy which has a two-phase microstructure which is produced during solidification from a homogeneous melt, or by a subsequent heat treatment whilst a solid, is not normally classified as a composite (Figure 1.2(a)). However if ceramic particles are somehow mixed with a metal to produce a material consisting of the metal containing a dispersion of the ceramic particles, then this is a true composite material (Figure 1.2(b)).

We know that composites have two (or more) chemically distinct phases on a microscopic scale, separated by a distinct interface, and it is important to be able to specify these constituents. The constituent that is continuous and is often but not always, present in the greater quantity in the composite is termed the *matrix*. The normal view is that it is the properties of the matrix that are improved on incorporating another constituent to produce a composite. A composite may have a ceramic, metallic or polymeric matrix.



(a)



(b)

Figure 1.2 (a) Micrograph of a cast Co–Cr–Mo–Si alloy with a multiphase microstructure. (Source: Halstead and Rawlings, 1986.) (b) Scanning electron micrograph of an aluminium alloy (2014) reinforced with angular particles of silicon carbide. The white particles are a second phase in the aluminium alloy matrix. (Courtesy D.J.B. Greenwood.)

The mechanical properties of these three classes of material differ considerably as demonstrated in Table 1.1. As a generalization polymers have low strengths and Young's moduli, ceramics are strong, stiff and brittle, and metals have intermediate strengths and moduli together with good ductilities, i.e., they are not brittle.

The second constituent is referred to as the *reinforcing phase*, or *reinforcement*, as it enhances or reinforces the mechanical properties of the matrix. In most cases the reinforcement is harder, stronger and stiffer than the matrix, although there are some exceptions; for example, ductile metal reinforcement in a ceramic matrix and rubberlike reinforcement in a brittle polymer matrix. At least one of the dimensions of the reinforcement is small, say less than 500 μm and sometimes only of the order of a micron. The geometry of the reinforcing phase is one of the major parameters in determining the effectiveness of the reinforcement; in other words, the mechanical properties of composites are a function of the shape and dimensions of the reinforcement. We usually describe the reinforcement as being either *fibrous* or *particulate*. Figure 1.3 represents a commonly employed classification scheme for composite materials which utilizes this designation for the reinforcement (Figure 1.3, block A).

Particulate reinforcements have dimensions that are approximately equal in all directions. The shape of the reinforcing particles may be spherical, cubic, platelet or any regular or irregular geometry. The composite illustrated in the micrograph of Figure 1.2(b) has angular particles of about 10 μm as the reinforcement. The arrangement of the particulate reinforcement may be *random* or with a *preferred orientation*, and this characteristic is also used as a part of the classification scheme (block B). In the majority of particulate reinforced composites the orientation of the particles is considered, for practical purposes, to be random (Figure 1.4(a)).

A fibrous reinforcement is characterized by its length being much greater than its cross-sectional dimension. However, the ratio of length to the cross-sectional dimension, known as the aspect ratio, can vary considerably. In single-layer composites long fibres with high aspect ratios give what are called *continuous* fibre reinforced composites, whereas *discontinuous* fibre composites are fabricated using short fibres of low aspect ratio (block C). The orientation of the discontinuous fibres may be random or preferred (Figures 1.4 (b) and (c)). The frequently encountered preferred orientation in the case of a continuous fibre composite (Figure 1.4(d)) is termed *unidirectional* and the corresponding random situation can be approximated to by *bidirectional* woven reinforcement (Figure 1.3, block D).

Multilayered composites are another category of fibre reinforced composites. These are classified as either laminates or hybrids (block E). *Laminates* are sheet constructions which are made by stacking layers (also called plies or laminae and usually unidirectional) in a specified sequence. A typical laminate may have between 4 to 40 layers and the fibre orientation changes

Table 1.1 Comparison of room temperature properties of ceramics, metals and polymers

	Density (Mg/m^3)	*Young's modulus* (GPa)	*Strength*[a] (MPa)	*Ductility* (%)	*Toughness* K_{IC} ($MPa\ m^{1/2}$)	*Specific modulus* [$(GPa)/(Mg/m^3)$]	*Specific strength* [$(MPa)/(Mg/m^3)$]
CERAMICS							
Alumina Al_2O_3	3.87	382	332	0	4.9	99	86
Magnesia MgO	3.60	207	230	0	1.2	58	64
Silicon Nitride Si_3N_4		166	210	0	4.0		
Zirconia ZrO_2	5.92	170	900	0	8.6	29	152
β-Sialon	3.25	300	945	0	7.7	92	291
Glass-ceramic Silceram	2.90	121	174	0	2.1	42	60
METALS							
Auminium	2.70	69	77	47		26	29
Aluminium-3%Zn-0.7%Zr	2.83	72	325	18		25	115
Brass Cu-30%Zn	8.50	100	550	70		12	65
Nickel-20%Cr-15%Co	8.18	204	1200	26		25	147
Steel mild	7.86	210	460	35		27	59
Titanium-2.5% Sn	4.56	112	792	20		24	174
POLYMERS							
Epoxy	1.12	4	50	4	1.5	4	36
Melamine formaldehyde	1.50	9	70			6	47
Nylon 6.6	1.14	2	70	60		18	61
Polyetheretherketone	1.30	4	70			3	54
Polymethylmethacrylate	1.19	3	50	3	1.5	3	42
Polystyrene	1.05	3	50	2	1.0	3	48
Polyvinylchloride rigid	1.70	3	60	15	4.0	2	35

[a]Strength values are obtained from the test appropriate for the material, e.g., flexural and tensile for ceramics and metals respectively.

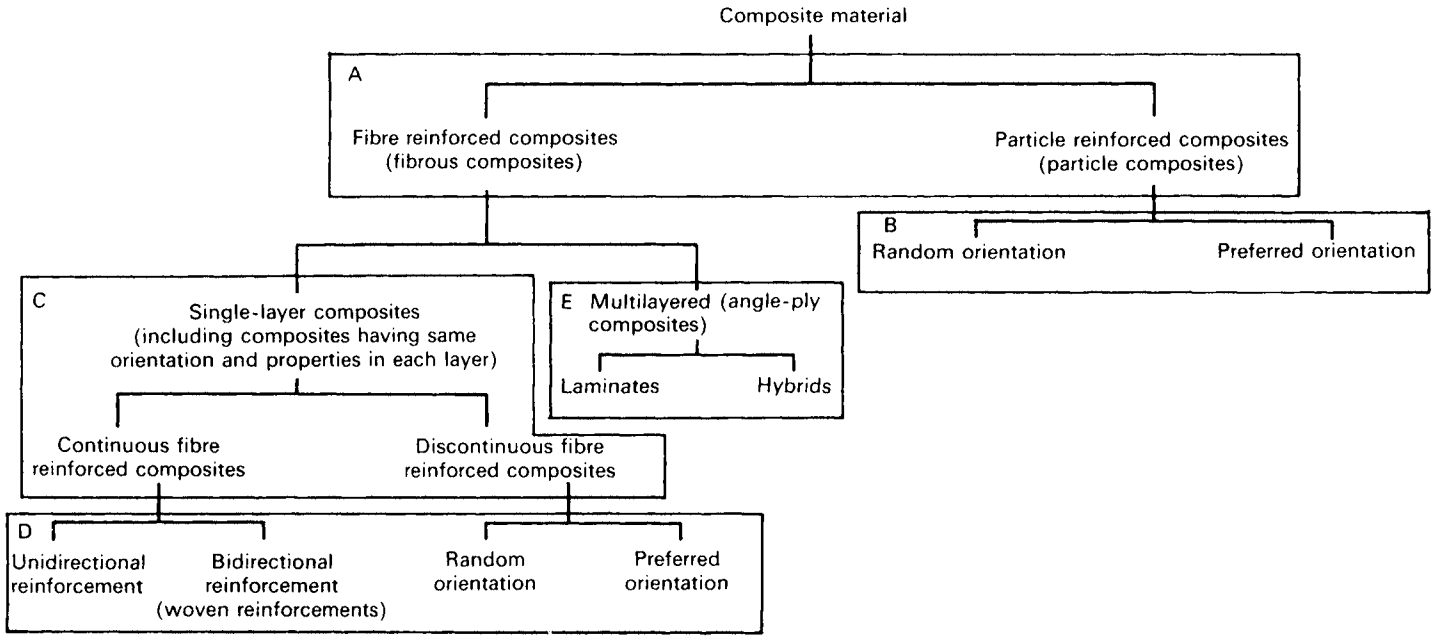


Figure 1.3 Classification of composite materials.

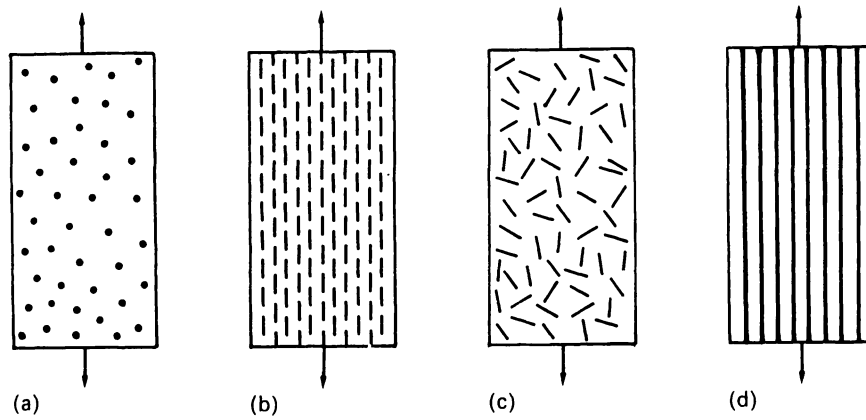


Figure 1.4 Examples of composites: (a) particulate, random; (b) discontinuous fibres, unidirectional; (c) discontinuous fibres, random; (d) continuous fibres, unidirectional.

from layer to layer in a regular manner through the thickness of the laminate, e.g., a 0/90° stacking sequence results in a *cross ply* composite.

Hybrids are usually multilayered composites with mixed fibres and are becoming commonplace. The fibres may be mixed in a ply or layer by layer and these composites are designed to benefit from the different properties of the fibres employed. For example, a mixture of glass and carbon fibres incorporated into a polymer matrix gives a relatively inexpensive composite, owing to the low cost of glass fibres, but with mechanical properties enhanced by the excellent stiffness of carbon. Some hybrids have a mixture of fibrous and particulate reinforcement.

1.3 NATURAL COMPOSITES

Wood is an interesting example of a natural fibre composite; the longitudinal hollow cells of wood are made up of layers of spirally wound *cellulose fibres* with varying spiral angle, bonded together with lignin during the growth of the tree. A model of good design with a composite material is the medieval longbow. In the medieval yew longbow the tensile surface was made by the bowyer from the resilient white sapwood with the bulk of the cross-section from the dark heartwood which is strong in compression. The result was a one-piece self-composite product at two levels, of remarkable strength, flexibility and stiffness.

Bone is a composite material composed primarily of organic fibres, small inorganic crystals, water and fats. The proportions of these components will vary with the type of bone, animal species and age but typically about 35% of the dry, fat-free weight of bone is the organic fibre, *collagen*.

Collagen is a fibrous protein and located around the outside of the collagen fibres are small rod-like crystals of *hydroxyapatite* with dimensions of the order of $5 \times 5 \times 50$ nm. Therefore at this microscopic level we have a hydroxyapatite-reinforced collagen composite and this may be considered to be the basic 'building block' for bone. The different types of bone are simply composites on a macroscopic scale resulting from different arrangements of the collagen fibres; in woven bone the fibres are arranged more or less randomly whereas in lamellar bone the fibres are orientated locally in layers to form lamellae.

A long bone, such as the femur, has an outer shell of high density, low fat content bone which is termed *compact* or *cortical* bone. Cortical bone has good mechanical properties with flexural strength in the range 46–156 MPa and Young's modulus of the order of 20 GPa. Contained within the shell of cortical bone is a softer, spongy bone (*cancellous* or *trabecular* bone) which consists of a three-dimensional network of beams and sheets known as

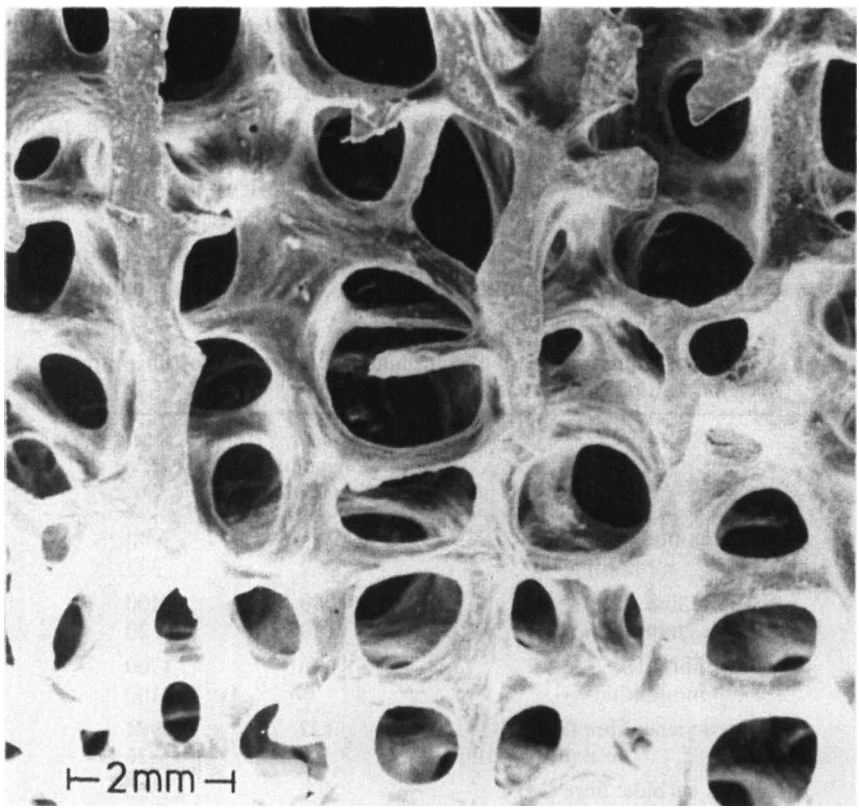


Figure 1.5 Scanning electron micrograph of cancellous bone. (Courtesy I. M. Thomas.)

trabeculae (Figure 1.5). The compressive strength and the Young's modulus of cancellous bone are typically in the ranges 5 to 20 MPa and 0.02 to 1.7 GPa respectively.

There are many more examples of composite materials in nature but the coverage given here on wood and bone is sufficient to illustrate this class of material and to indicate the high standards that synthetic composites have to achieve to be of comparable quality.

1.4 MORE ABOUT THE MATRIX AND REINFORCEMENT

Although there are many examples of composites employed mainly for their physical properties, such as the superconducting composites consisting of continuous Nb_3Sn fibres in a bronze matrix, most composites are designed to exploit an improvement in mechanical properties. Even for composites produced essentially for their physical properties, the mechanical properties can play an important role during component manufacture and service. For these reasons, in this section on the matrix and reinforcement, we will concentrate on mechanical behaviour. The properties of some fibres and their monolithic counterparts are compared in Table 1.2.

It has already been mentioned that the reinforcement is often a strong, stiff material and this is borne out by the data presented in Table 1.2. The reader may well ask why one should not make use of the high strength and high elastic constants of these reinforcements by producing monolithic components solely from the reinforcement. In fact this is done and, for example, components manufactured from the ceramics silicon carbide

Table 1.2 Comparison of the mechanical properties of fibres and their monolithic counterparts

	Young's modulus (GPa)	*Strength[a] (MPa)*
Alumina: fibre (Saffil RF)	300	2000
monolithic	382	332
Carbon: fibre (IM)	290	3100
monolithic	10	20
Glass: fibre (E)	76	1700
monolithic	76	100
Polyethylene: fibre (S 1000)	172	2964
monolithic (HD)	0.4	26
Silicon carbide: fibre (MF)	406	3920
monolithic	410	500

[a]tensile and flexural strengths for fibre and monolithic respectively.

and alumina are readily available. The problem with these materials is their brittleness; although able to sustain a high stress, when they fail they do so in a catastrophic manner without warning. Moreover the stress at which nominally identical specimens, or components, fail can vary markedly. The brittleness and the variation in strength is due to failure being initiated at flaws which normally occur at the surface. There is always a distribution of flaw sizes and, in an homogeneously stressed material, failure commences at the largest flaw. The larger the failure-initiating flaw the lower the failure stress (Figure 1.6). The smaller the volume of material, the smaller on average the size of the maximum flaw and the greater the strength. Therefore the fracture stress of a small ceramic particle or fibre is greater than that of a large volume of the same material.

We will see in the next chapter that the mechanical properties of fibres of a given type and dimensions are also determined by the structure. For example the properties of the organic fibre, aramid, are dependent on the orientation of the molecular structure whereas for carbon

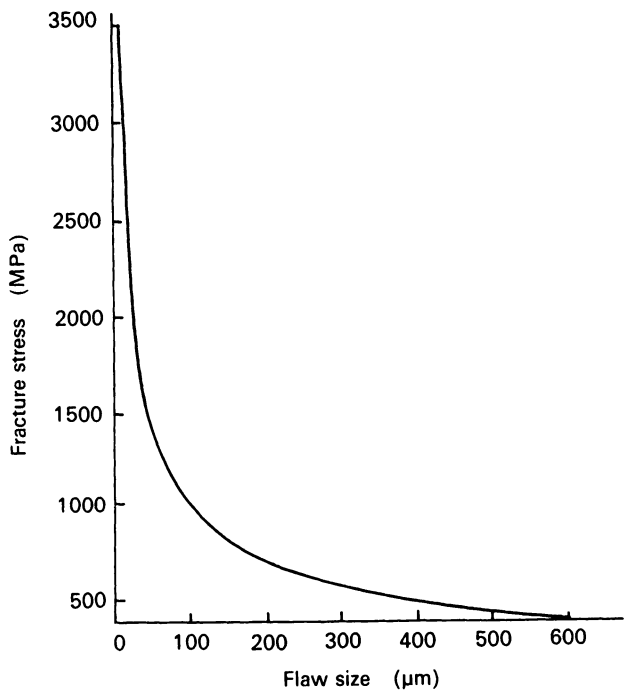


Figure 1.6 Effect of flaw size, a, on fracture stress, σ, of a brittle material with $K_{IC} = 10\,\text{MPa}\,\text{m}^{1/2}$. The fracture stress is calculated from the equation $K_{IC} = Y\sigma a^{1/2}$ with the parameter Y, which depends on geometry and loading, arbitrarily put equal to unity.

fibres the perfection and alignment of the graphite crystals are paramount.

There are of course many properties other than strength that we have to take into account when selecting a reinforcement. In the case of fibres the flexibility is important as it determines whether the fibres may be easily woven or not and influences the choice of method for composite manufacture. The flexibility of a fibre depends mainly on the Young's modulus, E_f, and diameter, D, of the fibre:

$$\text{flexibility} \propto \frac{1}{E_f D^4} \tag{1.1}$$

Thus large diameter fibres with a high Young's modulus are not flexible.

Clearly, single fibres, because of their small cross-sectional dimensions, are not directly usable in structural applications. On the other hand the possibility exists of trying to exploit the high strength of single fibres by physically entwining the fibres in a similar manner to the manufacture of rope from hemp fibres. Unfortunately such a fabrication method would greatly restrict the shape and dimensions of components and furthermore would introduce a considerable amount of surface damage leading to a degradation in strength. These problems may be overcome by embedding the fibres in a material to hold the fibres apart, to protect the surface of the fibres and to facilitate the production of components. The embedding material is, of course, the matrix. The amount of reinforcement that can be incorporated in a given matrix is limited by a number of factors. For example with particulate reinforced metals the reinforcement content is usually kept to less than 40 vol. % (0.4 volume fraction) because of processing difficulties and increasing brittleness at higher contents. On the other hand, the processing methods for fibre reinforced polymers are capable of producing composites with a high proportion of fibres, and the upper limit of about 70 vol. % (0.7 volume fraction) is set by the need to avoid fibre–fibre contact which results in fibre damage.

Finally, the fact that the reinforcement is bonded to the matrix means that any loads applied to a composite are carried by both constituents. As in most cases the reinforcement is the stiffer and stronger constituent; it is the principal load bearer. The matrix is said to have transferred the load to the reinforcement.

1.5 FACTORS WHICH DETERMINE PROPERTIES

The fabrication and properties of composites are strongly influenced by the proportions and properties of the matrix and the reinforcement. The proportions can be expressed either via the weight fraction (w), which is

relevant to fabrication, or via the volume fraction (v), which is commonly used in property calculations (for example, Chapter 8).

The definitions of w and v are related simply to the ratios of weight (W) or volume (V) as shown below.

Volume fractions: $$v_f = V_f/V_c \quad \text{and} \quad v_m = V_m/V_c. \tag{1.2a}$$

Weight fractions: $$w_f = W_f/W_c \quad \text{and} \quad w_m = W_m/W_c, \tag{1.2b}$$

where the subscripts m, f and c refer to the matrix, fibre (or in the more general case, reinforcement) and composite respectively.

We note that

$$v_f + v_m = 1, \text{ and}$$

$$w_f + w_m = 1.$$

We can relate weight to volume fractions by introducing the density, ρ, of the composite and its constituents. Now

$$W_c = W_f + W_m$$

which, as $W = \rho V$, becomes

$$\rho_c V_c = \rho_f V_f + \rho_m V_m,$$

or

$$\rho_c = \rho_f(V_f/V_c) + \rho_m(V_m/V_c)$$

$$= \rho_f v_f + \rho_m v_m. \tag{1.3}$$

It may also be shown that

$$\frac{1}{\rho_c} = \frac{w_f}{\rho_f} + \frac{w_m}{\rho_m} \tag{1.4}$$

Also we have

$$w_f = \frac{W_f}{W_c} = \frac{(\rho_f V_f)}{(\rho_c V_c)} = \frac{\rho_f}{\rho_c} v_f,$$

and similarly

$$w_m = \frac{W_m}{W_c} = \frac{(\rho_m V_m)}{(\rho_c V_c)} = \frac{\rho_m}{\rho_c} v_m. \tag{1.5}$$

We can see that we can convert from weight fraction to volume fraction, and vice versa, provided the densities of the reinforcement (ρ_f) and the matrix (ρ_m) are known.

Equation 1.3 shows that the density of the composite is given by the volume fraction adjusted sum of the densities of the constituents. This

equation is not only applicable to density but, in certain circumstances, may apply to other properties of composites. A generalized form of the equation is

$$X_c = X_m v_m + X_f v_f, \tag{1.2}$$

where X_c represents an appropriate property of the composite, and, as before, v is the volume fraction and the subscripts m and f refer to the matrix and reinforcement respectively. This equation is known as the *Law of Mixtures.*

Most properties of a composite are a complex function of a number of parameters as the constituents usually interact in a synergistic way so as to provide properties in the composite that are not fully accounted for by the law of mixtures. The chemical and strength characteristics of the *interface* between the fibres and the matrix is particularly important in determining the properties of the composite. The interfacial bond strength has to be sufficient for load to be transferred from the matrix to the fibres if the composite is to be stronger than the unreinforced matrix. On the other hand, as the reader will learn in Chapter 11, if we are also concerned with the toughness of the composite, the interface must not be so strong that it does not fail and allow toughening mechanisms such as debonding and fibre pull-out to take place.

Other parameters which may significantly affect the properties of a composite are the shape, size, orientation and distribution of the reinforcement and various features of the matrix such as the grain size for polycrystalline matrices. These, together with volume fraction, constitute what is called the *microstructure* of the composite.

However it should be noted that even for properties which are microstructure dependent, and which do not obey the law of mixtures, the volume fraction still plays a major role in determining properties. The volume fraction is generally regarded as the single most important parameter influencing the composite's properties. Also, it is an easily controllable manufacturing variable by which the properties of a composite may be altered to suit the application.

A problem encountered during manufacture is maintaining a uniform distribution of the reinforcement. Ideally, a composite should be homogeneous, or uniform, but this is difficult to achieve. *Homogeneity* is an important characteristic that determines the extent to which a representative volume of the material may differ in physical and mechanical properties from the average properties of the material. Non-uniformity of the system should be avoided as much as possible because it reduces those properties that are governed by the weakest part of the composite. For example, failure in a non-uniform material will initiate in an area of lowest strength, thus adversely affecting the overall strength of a component manufactured from that material.